How to Take the SAT

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- *Identify clue words to help find the correct answer*
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Marcia Lawrence with Philip Sorgen

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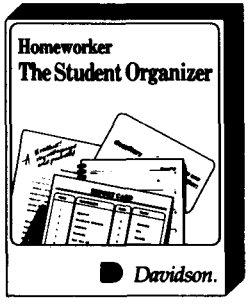

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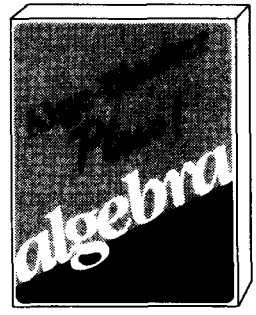

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Budgeting Your Time for the SAT

Budgeting your time is a crucial element in your SAT strategy for success. Everyone is absolutely equal in this one regard: everyone has exactly 30 minutes for each section of the test. What you do with your 30 minutes is up to you.

Planning a time budget in advance will relieve a great deal of test-day stress. Be sure to bring your own watch or stopwatch to the test. (Watches which beep are not allowed, though!)

Verbal Sections

Antonyms, analogies, and sentence completion questions can generally be answered in less than a minute (short questions). Each reading passage question usually averages more than a minute to answer (long questions).

Consider an 18/12-minute breakdown for each 30-minute verbal section, based on the short and long groupings. One verbal section has more reading passage (long) questions than the other, so there are two verbal time plans.

Verbal Section 1 (45 questions)		**Verbal Section 2** (40 questions)	
15 Antonyms	- **18** min.	10 Antonyms	- **12** min.
10 Analogies		10 Analogies	
10 Sentence Completions		5 Sentence Completions	
10 Reading Passage Questions	- **12** min.	15 Reading Passage Questions	- **18** min.

Reading passage questions thus will consume about half your total time on the verbal SAT. These questions are only worth about one-third of the points, though. Generally it is better to tackle the short questions first and save the reading questions until last.

Math Sections

You will have approximately one minute to answer each math question. If you think a question will take much longer than a minute to solve, or you don't know how to do it, put a check mark by it (in the test booklet, not on your answer sheet) and return to it later if you have time. Realize that in your area of strength (algebra, geometry, or arithmetic), even so-called "harder" questions at the end of the section may **not** be hard for you. Make sure that you don't leave any "easy" approachable questions unanswered because you ran out of time. Leave the questions that are harder for **you** until last.

Do

- prepare for the SAT by sharpening reading and math skills.
- approach the test well-rested and relaxed.
- eat breakfast the day of the test.
- bring your identification and your test ticket to the SAT.
- bring two sharpened #2 pencils to the test.
- familiarize yourself ahead of time with the directions to each type of question.
- wear light, comfortable clothing and bring a sweater or jacket.
- arrive early to the test center so that you can choose a seat in a good location.
- save the most difficult questions until last.
- draw a diagram if it will help you answer a math question.
- bring a watch so that you can follow your time plan for the exam.
- answer easy questions first; they count just as much as hard ones.
- use a process of elimination to choose the best answer.
- use your pencil in the exam booklet; underline clue words, strike out incorrect choices and circle the answer you select.

Don't

- try to cram for the test; the SAT measures aptitude, so cramming the night before the test won't help.
- make stray marks on your answer sheet; mark each answer in the correct spot on the answer sheet.
- get panicky if you miss a few questions; most students answer only about half the questions correctly.
- let your attention wander; it's important to stay focused.
- spend too much time on any one question; allow time to consider each one of the questions on the test.
- be afraid to guess if you can eliminate one or two answers.
- forget to consider all the possible answers; even if "C" is good, "E" might be better!
- rely on luck to raise your score; you must work to prepare yourself better if you hope to improve your score. (All of the scores are reported, not just the best ones.)
- bring a watch with a calculator or a beeper.
- waste any extra time you may have at the end of a section; use the time to recheck your answers.

HOW TO TAKE THE SAT

Scholastic Aptitude Test

Marcia Lawrence

with Philip Sorgen

A PLUME BOOK

This book was made possible by the inspiration and assistance of many friends, colleagues, and students.

It is dedicated to Dr. Hans K. Maeder, faithful and encouraging teacher and friend to a generation of college-bound young people.

To my editor, Ted Johnson, for his consummate craftsmanship.

To Diana Freed for her painstaking work on the manuscript and many valuable suggestions.

To my children, Douglas and Suzanne, who motivated this creation.

—And, especially, to Avraham.

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An 80-minute "Audio Cassette Tutor Tape," prepared by Marcia Lawrence to accompany this book, is available from Jeffrey Norton Publishers, Audio Dept., 145 East 49th Street, New York, N.Y. 10017.

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Contents

HOW TO TAKE THE SAT:

Scholastic Aptitude Test

To the Student

Taking a College Board Examination is a skill that can be improved by training. Without this training, under the pressure of normal test-taking anxiety and tension, you are forced to make casual guesses on material that is not as simple as it appears.

As a result, many students do not do justice to their actual abilities on this crucial examination, whose score will determine to which colleges they may reasonably apply. Repeating the test several times or studying questions from prior examinations will not, of itself, improve your basic test-taking skills.

This book offers the test-taker systematic methods for approaching each section of the Scholastic Aptitude Test. It provides you with concrete techniques for analyzing each question, eliminating distractions and concentrating on those alternatives that offer the best possible answer. Armed with these skills, you may approach the examination with confidence and anticipation, knowing that you can give it your best.

The Lawrence Techniques of Test Taking have been thoroughly tested in courses offered by the author and her associates, since 1974, to thousands of college-bound students. They have discovered that this system has helped them to take other examinations more successfully, by teaching them to approach test questions with an organized strategy. If you practice these techniques faithfully, it can do the same for you.

I.
THE VERBAL APTITUDE TEST

Introduction

The Scholastic Aptitude Test is one of the major criteria in the admissions process mounted by the college of your choice. The purpose of the test is to rate your ability to do college work in comparison with one million other high school students who will be taking these examinations at the same time you are.

However, there is little in the traditional high school curriculum that will prepare you for the SAT. It was not designed to test achievement, but how good a test taker you are, how well you understand the examination questions and the intentions of the people who constructed this particular test.

While for millions of students, college acceptances often hang in the balance on the Scholastic Aptitude Test, educators continue to perpetuate the myth that the test can not be studied for. But quite to the contrary, test taking techniques are ACQUIRED SKILLS and scores can be significantly altered by learning the specialized skills that are needed on this examination.

The Scholastic Aptitude Test, like any complex problem, must be approached with an organized strategy. It is important for you to know that each question type not only has a built-in clue which leads to the answer but requires a specific strategy for analyzing the question and narrowing down the answer choices. Consequently, any successful program must teach students how to master the strategies necessary to identify the correct answer.

The purpose of this book is to make maximum use of your potential by making you *test-wise*. You may read extensively and have an excellent vocabulary. In that case, you may need only to master the techniques of test-taking in order to utilize your present information to its fullest. On the other hand, you may be a non-reader with a limited vocabulary, which puts you at a disadvantage even though you may have a natural talent for doing your best on tests. Finally, you may lack both verbal skills and test-taking skills.

We have taken all these possibilities into account in creating this sequential learning program based on years of experience in SAT workshops helping students to improve their scores. Our course will offer you effective, personalized instruction. By the time you have finished this program, you will have learned how to tackle an SAT test, how to check out your answers, and how to zero in on the correct answer.

Once you begin this program, however, you have assumed a shared responsibility. There is no substitute for a good vocabulary, but by following the suggestions in this program regularly and using all of the material we have provided, you can substantially increase your chances of coming up with the right answers. It would be a good idea if you kept pencil and paper ready at all times. You should be constantly writing down clues and information, even when you are not specifically asked to do so.

There are a number of aspects of the SAT you should be particularly aware of:

1. ETS, the Educational Testing Service, which has produced the SAT exam, experiments with each test question for several years before allowing it to appear as a scoring question on a particular test. The purpose of these experiments is to reduce the margin for error in the selection of the right answer. In other words, there must be *clues* that are obvious enough to allow the majority of students taking the test to select the correct answer. It's your job to locate those clues, using the test-taking techniques this book will teach you.

2. What you may not realize, along with 90 percent of the students taking the Scholastic Aptitude Test, is that you not only *can* but *you must write on your exam*

paper. You cross out, underline, circle, and do whatever else you feel you need to do right on your exam, in pencil. Remember, that on the SAT itself you will be allowed no scratch paper.

3. All of the correct answers on the SAT should be arrived at by a *process of elimination*. Never think you've spotted the right answer immediately; there are subtle differences among the answers which need close analysis. If you think an answer is obvious, that might be just the reason the Educational Testing Service chose to include it. So *back into your answer* by first crossing out, with a pencil, all of the obviously incorrect answers.

4. If you narrow your answer down to two possibilities, guessing is usually to your advantage. However, if the entire question leaves you hopelessly baffled, even after you have applied all the appropriate techniques, then *do not guess*. You receive one point for each correct answer and no points for questions omitted. For questions with five answer choices, one-fourth of a point is subtracted for each incorrect answer. (One-third of a point is subtracted for questions with four answer choices, which sometimes appear in the mathematical part of the SAT. You receive both a verbal score and a mathematical score.)

5. It is important to learn to pace yourself through the examination. Questions range from easy to difficult and are often grouped so that the relatively easy ones come first. This is because the SAT is designed to cover a wide range of abilities. Make a notation next to any questions you have real difficulty answering, then skip them and go back to them after you have answered the others, if you still have time.

It is advisable to keep in mind that since you have only 30 minutes to do each section of the SAT, the two reading comprehension passages in the 45 question verbal section should take you no more than 5–6 minutes for each passage or 10–12 minutes total. Since the 40 question verbal section has four reading comprehension passages or 15 questions, you should spend a total of no more than 15–18 minutes on this section of the test.

6. There are several different versions of the SAT examination distributed at the same time, but your scores should not be affected by differences in these tests. Statistical measurements in scoring SAT examinations will adjust any differences among the various tests to ensure that your score is not affected.

7. Students who feel that their test scores are not indicative of their ability should plan to take the SAT examination again in November of their senior year. There is a tremendous maturation over the summer. In addition, the student can use the time to study and internalize these SAT techniques and work on prefixes, suffixes, roots, and vocabulary.

8. One important item to note is that each test booklet contains one experimental section. Depending upon which test you happen to receive, it could be in the verbal section, in the mathematical section, or in the test of standard written English. You will have no way of ascertaining which of your test sections is the experimental one, but *it does not count* toward your scores. This test is used to develop future SAT questions and maintain quality control.

9. One further point you might keep in mind is that there is no fixed rule about the order of the different sections within the verbal section or the order in which you must do them. The reading passages may or may not be the first set of questions on the test. Even when they are the first set of questions, many

students who have not learned the reading comprehension technique offered in this handbook often leave them until last, and then, because these passages are tedious and complex, they either guess at the answers or omit them altogether. But this is the section of the test that depends least on vocabulary, so it is to your advantage to learn the technique until you are proficient. Developing competence in this area will give you greater control over the material in the test and will allow you to move with competence from one section to another. The remaining questions will include word relationships, sentence completions, and antonyms. You might consider alternating between these various groups of test questions, always keeping in mind that you must move at the fastest pace you can.

Particularly if you do skip from section to section and skip more difficult questions with the intention of coming back later, you must be very careful not to get confused on your answer sheet. Each time you answer a questions, make sure the number on the question and the number on the answer sheet correspond. When you skip a question, it is surprisingly easy to forget to skip the corresponding answer space—and, of course, that could mean at best that you waste time correcting your error and at worst that you are scored wrong for all the rest of the questions in the section.

10. Scoring the test. Each correct answer on an SAT verbal test receives one point. One-fourth of a point is subtracted from each incorrect answer.

To score the verbal test use the following formula:

raw score = correct answers $-\frac{1}{4}$ of incorrect answers

To score the quantitative-comparison questions in the math section use the following formula:

raw score = correct answers $-\frac{1}{3}$ of incorrect answers

Do not count any problems that you skipped.

A verbal test designed by the author is shown on the following pages. You will notice that it consists of 45 questions and you are allowed one half-hour in which to complete it. To begin your personal study course in the techniques of how to take the SAT, please find yourself a quiet place where you will have 30 uninterrupted minutes and try the test. Tear out the answer sheet on page 13 and record your answers on it, just as you would in a regular testing situation. Be sure to follow the directions carefully and keep track of your time.

Typical Timetable for the SAT*

Total Time: 3 Hours

9:00 to **9:30**	Part 1	**Verbal Aptitude Test**	15 Antonyms 10 Analogies 10 Sentence Completion Questions 10 Reading Comprehension Questions
9:30 to **10:00**	Part 2	**Verbal Aptitude Test**	10 Antonyms 10 Analogies 10 Sentence Completion Questions 10 Reading Comprehension Questions
10:00 to **10:30**	Part 3	**Test of Standard Written English**	50 Grammar and Usage Questions
10:30 to **11:00**	Part 4	**Mathematics Aptitude Test**	15 Multiple Choice Questions 20 Quantitative Comparison Questions
11:00 to **11:05**		RECESS	
11:05 to **11:35**	Part 5	**Mathematics Aptitude Test**	25 Multiple Choice Questions
11:35 to **12:05**	Part 6	**Verbal Aptitude Test** 40 or 45 Questions or **Mathematics Aptitude Test** 25 or 35 Questions	

*The sections of the test may or may not be presented in this order.

Verbal Aptitude Test 1

Time—30 Minutes
45 Questions

For each question in this section, choose the best answer and blacken the corresponding space on the answer sheet.

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole.

Example:

> Nevertheless, _____ in one area of negotiations does not _____ success in another.
>
> (A) efficiency . . credit
> (B) accomplishment . . guarantee
> (C) justification . . tolerate
> (D) failure . . develop
> (E) experience . . negate
>
> A B C D E
> ☐ ■ ☐ ☐ ☐

1. Though he is an amateur dancer, he has the _____ of a gazelle and the _____ of a professional.

   (A) awkwardness . . strength
   (B) cerebration . . credulity
   (C) agility . . prowess
   (D) delirium . . demeanor
   (E) detriment . . skill

2. Hitler's _____ helped _____ a whole nation to deprive others of their rights.

   (A) demagoguery . . incite
   (B) spirituality . . cajole
   (C) recalcitrance . . prevent
   (D) pessimism . . admonish
   (E) bestiality . . deplore

3. It is a capital offense to _____ a child without the parent's knowledge.

   (A) abate (B) abduct (C) abet
   (D) abjure (E) abominate

4. "That one doesn't try to hide anything, not he; his message is as clear as crystal and his meaning as _____ as day."

   (A) obscure (B) occult (C) manifest
   (D) esoteric (E) erudite

5. "If this drug grants us _____, will it also _____ us freedom from complications?"

   (A) susceptibility . . allow
   (B) eclecticism . . permit
   (C) immunity . . guarantee
   (D) mortality . . enable
   (E) solidity . . deny

Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Select the lettered pair that *best* expresses a relationship similar to that expressed in the original pair.

Example:

> PITCH:BASEBALL:: (A) glove:punch (B) putt:golf (C) kick:football (D) tennis:racket (E) pass:hockey
>
> A B C D E
> ☐ ☐ ■ ☐ ☐

6. YARDS:TIME:: (A) length:seconds (B) towers:courtyard (C) quilt:bedspread (D) realism:platitude (E) description:observation

7. DRIZZLE:RAIN:: (A) storm:wind (B) flurry:blizzard (C) hurricane:tempest (D) cyclone:twister (E) jetty:ocean

8. FOMENT:QUELL:: (A) incense:infuriate (B) taunt:vex (C) escort:accompany (D) allay:subside (E) abet:dissuade

9. COAL:FUEL:: (A) loin:abdomen (B) silver:metal (C) copper:alloy (D) locution:style (E) gold:mercury

10. PEARL:OYSTER:: (A) clam:seal (B) alabaster:abalone (C) milk:cream (D) marina:fish (E) product:factory

11. PITCH:BASEBALL:: (A) hurl:discus (B) slide:javelin (C) punch:fist (D) weigh:reel (E) cast:trawler

12. ETERNAL:EPHEMERAL:: (A) boundless:limited (B) touchy:irascible (C) trenchant:keen (D) tumid:swollen (E) assuaged:satisfied

13. BENIGN:MALIGNANT:: (A) bland:pungent (B) contractual:covenantal (C) supplicant:solicitous (D) salutary:beneficial (E) adverse:contrary

14. DICTIONARY:WORD:: (A) periodical:title (B) Bible:prayer (C) catalogue:item (D) treatise:volume (E) thesis:professor

15. ORGANISM:MUTANT:: (A) caterpillar:butterfly (B) peach:nectarine (C) mosquito:larva (D) apple:grape (E) grass:apple

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

Example:

SOUR: (A) hot (B) sweet (C) straight (D) safe (E) sharp	A B C D E □ ■ □ □ □

16. ABRIDGE: (A) digest (B) prevail (C) enlarge (D) contract (E) journey

17. ENMITY: (A) hatred (B) anonymity (C) attraction (D) witchcraft (E) antipathy

18. DISSEMBLE: (A) take apart (B) disengage (C) reveal (D) unhinge (E) construct

19. RANCOR: (A) malice (B) mischief (C) benignance (D) acrimony (E) status

20. ABROGATE: (A) expunge (B) rescind (C) confirm (D) nullify (E) agitate

21. CAPTIOUS: (A) fault-finding (B) petulant (C) genial (D) enticing (E) splenetic

22. DEBACLE: (A) catastrophe (B) demise (C) boon (D) cataclysm (E) humiliation

23. INORDINATE: (A) restrained (B) unusual (C) excessive (D) chaotic (E) ordinary

24. CHASTISE: (A) extol (B) purify (C) punish (D) pursue (E) discipline

25. AVARICE: (A) penury (B) facsimile (C) prodigality (D) rapacity (E) parsimony

Select the word or set of words that *best* completes each of the following sentences.

26. The professor considered the treatise to be _____ because of its great _____.

(A) libelous . . veracity
(B) harmonious . . ambiguity
(C) recondite . . scholarship
(D) abstruse . . clarity
(E) esoteric . . commonness

27. All our plans are _____; nothing definite has been established so far.

(A) tentative (B) superficial (C) sublime (D) stellar (E) stagnant

28. I despise being near her. She's loud, noisy, boisterous; a real _____.

(A) visionary (B) votary (C) virago (D) zealot (E) wizard

29. At its present rate of _____ the business will soon have to move to larger quarters.

(A) accolade (B) accord (C) accretion (D) acerbity (E) acme

30. The mother warned her children: "Do not involve yourselves with _____ youngsters; they tend to be _____."

(A) aggressive . . pugnacious
(B) benevolent . . kind
(C) generous . . parsimonious
(D) gluttonous . . thin
(E) communist . . reactionary

Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters.

31. DEXTEROUS: (A) saccharine (B) insidious (C) deft (D) interchangeable (E) clumsy

32. LOQUACITY: (A) terseness (B) prolixity (C) volubility (D) quackery (E) verbosity

33. CUPIDITY: (A) altruism (B) acerbity (C) avarice (D) smugness (E) antagonistic

34. DENIGRATE: (A) praise (B) upbraid (C) defame (D) contradict (E) point out

35. CONVIVIAL: (A) jocund (B) mirthful (C) saturnine (D) reveling (E) underhanded

The passages below are followed by questions based on their content. Answer all questions following a passage on the basis of what is *stated* or *implied* in that passage.

The vast current systems, which flow through the oceans like rivers, lie for the most part offshore and one might suppose their influence in intertidal matters to be slight. Yet the currents have far-reaching effects, for they transport immense volumes of water over long distances—water that holds its original temperature through thousands of miles of its journey. In this way tropical warmth is carried northward and arctic cold brought far down toward the equator. The currents, probably more than any other single factor, are the creators of the marine climate.

The importance of climate lies in the fact that life, even as broadly defined to include all living things of every sort, exists within a relatively narrow range of temperature, roughly between 32°F. and 210°F. The planet Earth is particularly favorable for life because it has a fairly stable temperature. Especially in the sea, temperature changes are moderate and gradual, and many animals are so delicately adjusted to the accustomed water climate that an abrupt or drastic change is fatal. Animals living on the shore and exposed to air temperatures at low tide are necessarily a little more hardy, but even these have their preferred range of heat and cold beyond which they seldom stray.

Most tropical animals are more sensitive to change—especially toward higher temperatures—than northern ones, and this is probably because the water in which they live normally varies by only a few degrees throughout the year. Some tropical sea urchins, keyhole limpets, and brittle stars die when the shallow waters heat to about 99°F. The arctic jellyfish Cyanea, on the other hand, is so hardy that it continues to pulsate when half its bell is imprisoned in ice, and may revive even after being solidly frozen for hours. The horseshoe crab is an example of an animal that is very tolerant of temperature change. It has a wide range as a species, and its northern forms can survive being frozen into ice in New England, while its southern representatives thrive in tropical waters of Florida and southward to Yucatan. Shore animals for the most part endure the seasonal changes of temperate coasts, but some find it necessary to escape the extreme cold of winter.

36. The main idea of this selection is best expressed as

    (A) temperature ranges
    (B) ocean currents and their effect on marine life
    (C) current systems
    (D) the importance of climate and animal adaptation
    (E) arctic cold and tropical warmth

37. According to this passage, ocean current systems

    (A) are given to abrupt changes in temperature
    (B) prevent tropical warmth from being carried northward
    (C) determine marine climate
    (D) have little influence in intertidal matters
    (E) bear little resemblance to those found in rivers

38. The author implies that

    (A) animals have a high degree of adaptability
    (B) most tropical animals are highly sensitive to sudden drops in temperature
    (C) sea animals are less hardy than animals living on shore
    (D) most living things cannot tolerate sudden shifts in temperature
    (E) animals learn to adapt within a broad range of temperatures

39. The word *temperate* in the last sentence of the passage most probably refers to

    (A) marine climate
    (B) a well-protected shoreline
    (C) a moderate climate
    (D) an area where temperatures remain stable
    (E) an area where there are sudden shifts in temperature

40. Which of the following statements is *not* true?

(A) The water, which ocean currents transport long distances, retains its original temperature.
(B) The horseshoe crab has a very limited range as a species.
(C) Shore animals exposed to air temperatures at low tide are tough.
(D) Keyhole limpets are sensitive to changes toward higher temperatures.
(E) The Cyanea is able to survive being solidly frozen for hours.

The historical analysis of the Bible is divided into categories: higher criticism, which deals with the organization, authorship and historical background of each book, and lower criticism, which concerns itself with the accuracy of the text itself, which had been transmitted for centuries through handcopied texts, before the advent of printing, and therefore sustained scribal errors, which were recopied from scroll to scroll.

The classic position of higher criticism was developed by Friedrich Wellhausen, a German scholar of the 19th century, who posited the theory that the Bible, as we have it, is a text compiled from previous texts that had been edited, in turn, from earlier recensions. Wellhausen discovered four distinct styles in the *Pentateuch*—the first five books of Scripture—each of which could be distinguished by certain definitive characteristics: J—compiled by an editor in Judea; E—the work of a northern editor, in Ephraim; P—the priestly editor, who reflected the views of the established clergy; and D—the Deuteronomist, whose later additions expressed the official interpretation of the organized establishment.

The final editor, who worked during a period of peace, when literary activity was encouraged, combined all the prior versions into one text. Where the stories or facts were in agreement, only one version was retained; where there were differences, he did not presume to decide between them but laid them out, side by side. Thus, Wellhausen suggested, there were two different stories of Creation in the Bible and two stories of the flood.

41. One development that Friedrich Wellhausen did *not* posit was

(A) that the contribution of D to the *Pentateuch* was as spokesman for the official body
(B) that certain stylistic differences were apparent in the first five books of Scripture
(C) that the *Pentateuch* is a self-contained text with singular characteristics
(D) that P was the official spokesman for the clergy
(E) that the Bible was a synthesis of earlier edited texts

42. According to the author

(A) the Bible is a self-critical unit which concerns itself with authenticity
(B) the Bible is in two major segments which are delineated by subject matter
(C) the Bible is riddled with errors perpetuated by scribes
(D) the historic background of each book in the Bible has been analyzed in depth by Friedrich Wellhausen
(E) the category of lower criticism deals with stylistic differences

43. Which of the following titles best summarizes the content of the passage?

(A) Thoughts About the Bible
(B) The Theories of Wellhausen on the Historical Development of the Bible
(C) Origins of the Bible
(D) Stylistic Interpretations
(E) The Development and Transmission of the Scriptures

44. It is obvious from this passage that the *Pentateuch* is

(A) the critical theories of Wellhausen
(B) a section of the New Testament dealing with lower criticism
(C) that portion of the Old Testament which begins with the story of Creation
(D) the authentic version of the flood
(E) the edited and combined version of the earlier recensions

45. Double versions of the Creation and the Flood exist in the Bible because

(A) careless scribes transmitted the error in their recopying
(B) an editor in Judea felt the inclusion of such diverse versions only added to the interpretive importance of the Bible
(C) an editor synthesized only the stories that were in agreement
(D) stylistic differences exist between the various stories
(E) of the work of Friedrich Wellhausen

STOP

If you finish before time is called, check your work on this section only. Do not work on any other section in the test.

You have just completed a verbal test developed by the author which we believe is comparable to the SAT. What you would probably like to do at this moment is check the correct answers against your answer sheet. But wait, difficult though that may be.

What we are going to do now is to analyze each section of the verbal SAT and teach you the techniques of test-taking. Once you have mastered these techniques, we are going to ask you to retake the test. After you take it the *second* time, you will not only be able to compare your test answers with the results of the test you have just completed, but you will also be able to compare the differences in your approach.

You are now ready to begin your personal study program in the techniques of test-taking.

Answer Sheet—Verbal Aptitude Test 1

First Try

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
25	☐	☐	☐	☐	☐
26	☐	☐	☐	☐	☐
27	☐	☐	☐	☐	☐
28	☐	☐	☐	☐	☐
29	☐	☐	☐	☐	☐
30	☐	☐	☐	☐	☐
31	☐	☐	☐	☐	☐
32	☐	☐	☐	☐	☐
33	☐	☐	☐	☐	☐
34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐
36	☐	☐	☐	☐	☐
37	☐	☐	☐	☐	☐
38	☐	☐	☐	☐	☐
39	☐	☐	☐	☐	☐
40	☐	☐	☐	☐	☐
41	☐	☐	☐	☐	☐
42	☐	☐	☐	☐	☐
43	☐	☐	☐	☐	☐
44	☐	☐	☐	☐	☐
45	☐	☐	☐	☐	☐

Second Try

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
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14	☐	☐	☐	☐	☐
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16	☐	☐	☐	☐	☐
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19	☐	☐	☐	☐	☐
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31	☐	☐	☐	☐	☐
32	☐	☐	☐	☐	☐
33	☐	☐	☐	☐	☐
34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐
36	☐	☐	☐	☐	☐
37	☐	☐	☐	☐	☐
38	☐	☐	☐	☐	☐
39	☐	☐	☐	☐	☐
40	☐	☐	☐	☐	☐
41	☐	☐	☐	☐	☐
42	☐	☐	☐	☐	☐
43	☐	☐	☐	☐	☐
44	☐	☐	☐	☐	☐
45	☐	☐	☐	☐	☐

SENTENCE COMPLETIONS

This type of question tests your ability to use vocabulary and recognize logical consistency among the elements in a sentence. Merely knowing the definition of a word does not guarantee success. Sentence-completion questions deal with one phase of reading comprehension and test your ability to recognize the implications of a sentence as well as your ability to select the word or pair of words that best fits the meaning of the sentence as a whole.

Although there are a variety of basic types of sentence-completion questions, it is important for you to know that every question has a built-in clue which points to only *one* of the five choices as the most appropriate or *best* answer. You should *back into the right answer* by eliminating those answers that are *not possible*—thereby reducing the choices. Whatever word you decide upon for your answer, you should be able to go back to the question and justify your reason for selecting it—that is, to *locate the clue*. The answers on the SAT are *not* left to chance. Both the right and the wrong answers are carefully designed. Therefore, if you are going to make haphazard guesses without carefully examining each answer, you are apt to select an answer that *seems* appropriate on cursory reading only to fall into a carefully laid trap. A closer examination will reveal a *specific clue* for which only one choice is correct. It is important to learn how to locate that clue.

One-Word Sentence Completions

Sentence-completion questions may have two blanks or only one. Although those with two blanks actually contain more clues than those with one, the one-blank questions are easier to understand at first, so we will begin with them.

Step 1

Examine the following sentence-completion question and underline those words which you feel are *key words*. Underline words that you feel provide you with information or clues that will point you in the direction of the right answer. This technique is similar to the one we will teach you to use for reading-comprehension questions, but with one important difference. In reading-comprehension sections of the SAT, which are explained later in this book, the first thing you are advised to do is to look over the questions at the end of the passage. But in the sentence-completion sections, you are advised to analyze the question *before* looking at the choice of answers. That is, you should have a fairly good idea of what you are looking for before you examine the available choices. That is why the answer choices are omitted on the questions below. Please underline key words in the sentence now *before reading ahead*.

Children in poverty-stricken areas have a tendency to be _____ .

Chances are you underlined the words *children*, *poverty-stricken areas*, and *tendency*, because children are a special category of people with special needs; it is important to know that you are talking about a particular group of children,

namely those from poor areas; and *tendency* implies that the children probably are something-or-other, but may not be.

Step 2

Now look at the sentence again and try to single out any word (or words) *of the ones you have underlined* that you think is an especially important clue to the answer. When you have made your choice, circle that word (or words) in the sentence. Do it now, before reading ahead.

Chances are the word you selected is *poverty-stricken,* because if you are looking for an attribute which all of these children have in common, it is going to be their poverty.

Step 3

Reread the sentence and fill in the blank with a word of your own. If you cannot think of a particular word, then ask yourself this question: "Is the word I am looking for *positive* or *negative*?" That is, does the word you want have good connotations or bad connotations? You will probably have little difficulty in deciding that the word you are looking for is a negative word, since something associated with children growing up in an impoverished area is not likely to be good.

Step 4

Go through the choices one at a time, drawing a line through those answers which are *not possible;* that will include, of course, any words you recognize as obviously positive, since you are looking for a negative word. Please do it now before reading further. (You may often find your eyes wandering down the page in search of the answer to such exercises as this. But to get the most out of the exercises, you must control your eyes! If you can't do it any other way, put a file card or small envelope over the page, moving it down only after you've followed the instructions.)

> Children in poverty-stricken areas have a tendency to be _____ .
> (A) overfed (B) opulent (C) ostentatious (D) emaciated (E) affectionate

You probably eliminated (A), (C), and (E) immediately because: (A) a child from an economically deprived area is not likely to be *overfed;* (C) *ostentatious* means "showy," and it is not likely that a deprived child would have anything to brag about; and (E) *affectionate* is a positive word and does not relate to the one clue, which is *poverty-stricken.* You may have left (B) because you did not know that *opulent* means "wealthy"—if you did know the meaning, you certainly eliminated it. But with (B) and (D) remaining, your best choice would have been (D), since most children from poverty-stricken areas do have a tendency to be physically wasted or *emaciated.*

Here is another sentence-completion question for you to practice on. In this exercise, the answers will follow the sentence, but *do not look at the answers* until you have followed the steps in analyzing the question and have an idea of the word you are looking for.

> His clothes are torn, his hair unkempt, his body unwashed: what a _____ sort!
> (A) sporadic (B) spatial (C) slovenly (D) skittish (E) praiseworthy

This sentence completion is an example of a *definition question.* You are looking for an answer which describes the words expressed in the sentence. *Step 1:* Chances are you underlined the words *clothes, torn, hair, unkempt, body,* and *unwashed,*

because each of the words either means a part associated with the body or describes a condition those parts can be in. *Step 2:* You probably circled *torn, unkempt,* and *unwashed,* because these words leave a strong impression. *Step 3:* The word you probably thought of to fill in the blank no doubt had negative connotations, since all of the words that describe the person are negative. *Step 4:* Therefore, you probably eliminated (E) immediately, since *praiseworthy* is a positive word. Chances are you eliminated (A) and (B) next, because (A) *sporadic* means "infrequent" and is not relevant to the person's description, and (B) *spatial* relates to "space" and is also irrelevant. You might have left (D) if you do not know that *skittish* means "lively." But the best choice is (C), since *slovenly* means "untidy"; in fact, if you add the letter *B* to the first three letters, *slo,* you have the word "slob," which is an apt word for the person being described.

In its final diagrammed state, your sentence completion should look like this:

His clothes are torn, his hair unkempt, his body unwashed; what a _____ sort!

(A) ~~sporadic~~ (B) ~~spatial~~ (C) slovenly (D) ~~skittish~~ (E) ~~praiseworthy~~

Step 5

There is one additional sentence-completion technique you should keep in mind. Although recognizing the implication of a sentence and being able to isolate the clues that lead to the correct answer are the important skills you need to develop, the subtleties of usage also play an important part. That is, if you are left with a choice between two words, both of which seem possible (such as *scold* and *chastise,* which are synonyms), read the sentence completion "out loud" with both words in the blank and select the one that *sounds* best. Yes, you can speak the sentence to yourself in a whisper or just pronounce it under your breath so that your neighbor will not be disturbed.

PRACTICE EXERCISE

Here are some sentence completions for you to practice on.

The sentence-completion section of the real SAT has both one-word and two-word omissions, but we will get to the two-word omissions later. Each sentence below has only one blank, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole. Be certain to follow the five steps in analyzing the question *before* you look at the answers. Cover them up with a card, envelope, or sheet of paper if you find it hard to resist sneaking a glance.

1. Although I appreciate his romantic dreams, he is _____, because they are unrealistic.

   (A) robust (B) commendable (C) remediable (D) practical (E) quixotic

2. The contractual system of government in England set a _____ which has been followed faithfully until the present day.

   (A) prelude (B) precept (C) precedent (D) presumption (E) presentiment

3. I was frankly stunned to see such a conservative woman wear so _____ a dress.

   (A) plaintive (B) pious (C) pertinent (D) pert (E) pernicious

4. The exhausted camper was pinned down under the weight of his _____.

   (A) parasite (B) paraphernalia (C) paranoia (D) paradox (E) parable

5. A(n) _____ is like a lake, but the water is generally more brackish.

   (A) lagoon (B) lagniappe (C) interim
   (D) cul-de-sac (E) infirmity

6. Although his speech was _____ and delivered on the spur of the moment, it was one of the most sincere and compassionate I have ever heard.

   (A) infallible (B) indigent (C) inane
   (D) impromptu (E)) impetuous

7. His early experiences as a pioneer _____ his work with a rustic quality.

   (A) impeached (B) impaired (C) imbued
   (D) gouged (E) galvanized

8. Since he wishes to become a lawyer, his curriculum is centered on _____ studies.

   (A) futuristic (B) frenetic (C) formidable
   (D) forensic (E) foppish

9. Much of what is now _____ was once impossible.

   (A) foolhardy (B) fickle (C) fervid
   (D) feasible (E) fatuous

10. Brian loves French food, but he refuses to eat at the stylish restaurant in town because the prices are _____ and far beyond his means.

    (A) fanciful (B) extraneous
    (C) extemporaneous (D) extant (E) exorbitant

ANSWERS AND EXPLANATIONS

1. (E)	2. (C)	3. (D)	4. (B)	5. (A)
6. (D)	7. (C)	8. (D)	9. (D)	10. (E)

1. (E) It is likely that your sentence completion looks like this:

   Although I appreciate his romantic dreams, he is _____, because they are unrealistic.

One of the important clues in this sentence completion is the word *although*, because it indicates that whatever the person states in the first part of this sentence, it is going to be contradicted in the second part. This, then, is another common type of sentence completion; it is based upon a contrast between the ideas expressed in the first part of the sentence and those expressed in the second part. Therefore it is important that you circle all connecting words and phrases in every sentence completion. Upon examination of the question, you probably realized that you were looking for a word that means the same as *unrealistic*. Therefore you probably eliminated (A), (B), (C), and (D) immediately because none of these words means *unrealistic*. The correct answer is (E), because *quixotic* means "idealistic" or "impractical.

2. (C) The contractual system of government in England set a _____ which has been followed faithfully until the present day.

The clue to this question is in the words *system, followed, until present day*. In other words, you are looking for a word which implies that something was established at an earlier time and has continued down to the present. Chances are that when you thought of your own word to fit in the blank space, you came up with the same word that is offered as one of the choices; that is, the word *precedent*, which is the right answer. You probably eliminated (A), (D), and (E) immediately because (A) a *prelude* is an event preceding or coming before a more important event, and in this question we are dealing with the perpetuation of the same event; (D) *presump-*

tion means "something that is assumed," which is an irrelevant answer; and (E) a *presentiment* is a premonition or sense of something to come. Suppose you are left with choices (B) and (C), and do not know that a *precept* is a command or principle intended as a general rule of action. The better choice is (C), because *precedent* is the only word that means an earlier occurrence of an action that may justify a subsequent action of the same kind. In this case, the import of the sentence has to do with something occurring earlier and continuing to the present.

3. (D) I was frankly stunned to see such a conservative woman wear so _____ a dress.

The clue to this sentence completion resides in the word *stunned*. If the woman is *conservative*, what kind of dress would shock or stun? Obviously, a word which means the opposite of *conservative*. You probably eliminated (A), (B), and (C) as irrelevant, because (A) *plaintive* means "unhappy"; (B) *pious* means "religious"; and (C) *pertinent* means "relevant," from the more common word "pertain". You may have left (E) because you did not know that *pernicious* means "deadly." (D) is the right answer because *pert* means "lively" or "saucy."

4. (B) The exhausted camper was pinned down under the weight of his _____.

Most of the words you underlined in this short sentence could have been circled, because each of the words present carries a big information load. You probably arrived at your own word, which was undoubtedly some kind of camping equipment. Therefore, you will be looking for a word which is synonymous with this choice. You probably had no difficulty eliminating every word but the correct one, *paraphernalia*, which means "personal belongings." All of the other choices make perfectly good sentences—and they are all more amusing, at least, than the correct choice—but there *is no clue* to justify them. (A) a *parasite* is an organism that lives in or on another organism; (C) *paranoia* is an excessive or irrational suspicion of others; (D) a *paradox* is an apparent contradiction; and (E) a *parable* is a story that illustrates a moral principle.

5. (A) A(n) _____ is like a lake, but the water is generally more brackish.

Since you might have difficulty with the word *brackish*, it would be a good idea to concentrate on the words you circled in the first part of the sentence, that is, *like* and *lake*. On the basis of the information, you should have eliminated every answer but (A), even though you may not have known that a *lagoon* is a shallow pond near a larger body of water. Only if you had narrowed it down to two answers, both of which were bodies of water, would it have then been necessary to know that *brackish* means "slightly salty." You should have known that (C) an *interim* is an interval of time, and (E) *infirmity* means "feebleness or frailty." You may not have known that (B) a *lagniappe* is a small gift and that (D) a *cul-de-sac* is a dead end, like a dead-end street, but you might have eliminated both choices, since neither sounds like a body of water.

6. (D) Although his speech was _____ and delivered at the spur of the moment, it was one of the most sincere and compassionate I have ever heard.

One of the important clues in this sentence is the word *and*, which implies that whatever is on one side of the word, in this case the words *on the spur of the moment*, there should be a word of similar import on the other side. That is what you will be looking for among your choices. You should have eliminated all of the choices except (D) and (E), since (A) *infallible* means "incapable of error"; (B)

indigent means "impoverished" or "needy"; and (C) *inane* means "empty" or "lacking meaning." You may have been unsure of the choice between (D) and (E), because one often associates *impetuous* with the idea of doing something without thinking, hence something done at the spur of the moment. But on closer scrutiny it is apparent that (D) is the better choice because (D) *impromptu* actually means "made or done on the spur of the moment" or "improvised"; and (E) *impetuous* means "forceful" or "impulsive." Some students will be bothered by the fact that *impromptu* and *on the spur of the moment* mean almost exactly the same thing, because they know that it is usually poor writing style to repeat ideas in a sentence. So they will be drawn toward (E) *impetuous*, because this word does produce a more interesting sentence. But one of the most important test-taking skills is being able to sense what the test makers are trying to test. In this case it is the meaning of the words, not the literary quality of the sentence.

7. (C) His early experiences as a pioneer _____ his work with a rustic quality.

In some way, the early experiences of this person did something to influence his work by giving it a particular quality. The word you are looking for, therefore, is a word that suggests some kind of effect or influence. It should have been easy for you to eliminate (A), (B), and (D), because: (A) *impeached* means "accused"; (B) *impaired* means "damaged"; and (D) *gouged* means "scooped out," or, in a common figurative sense, "brutally extracted." None of these words has the proper suggestion of affecting something except *impaired*. But since the word you are looking for has *positive*, not negative, connotations, (B) *impaired* does not work. Left with choices (C) and (E), there are many students who will recognize that *galvanized*, which means "stimulated or excited by an electric shock," makes it easy to back into the right answer, even if you did not know that the word *imbued* means "penetrated" or "permeated"—words that approximate the meaning we are looking for. Had you known that one of the meanings for the prefix *im-* is "within," this might have helped you in narrowing down your choices.

8. (D) Since he wishes to become a lawyer, his curriculum is centered on _____ studies.

Since you are looking for an adjective to describe the kind of studies you would pursue if you wanted to become a lawyer, the word you are looking for has to relate directly to law. You would probably eliminate (A) and (E) immediately, since (A) *futuristic* means "relating to the future"; and (E) *foppish* means exactly what it sounds like, "foolish" or "silly." It is extremely important in the sentence completion to *read the sentence out loud with your chosen word in the blank space*, since this test-taking section depends to such a large extent on how well the word sounds in the context of the reading. The reason for this is that this is basically a test having to do with usage—that is, how a word is used in a sentence. Again, please keep this note in mind: If you are left with a choice of two words and both words seem possible in terms of their definition, read the sentence *out loud*, using each word choice in the sentence, and select the one that *sounds best*. And here is a bit of unorthodox but practical advice: In most cases, the more difficult word—or in many cases the word you don't know, if you are left with two and they both sound right in the sentence even though you only know the meaning of one—is the right answer. If you did not know what *foppish* meant and did not have the intuitive feeling that it probably related to the word "foolish," reading the word *out loud* in the sentence should have dispelled any notion that it could conceivably be the right answer. *Foppish studies* simply sounds ridiculous.

If you were left with choices (B), (C), and (D) and read them out loud in the sentence, chances are you settled on the right answer, *forensic*, which means "the

art or study of argument." Most students eliminate (C), since *formidable* means "awesome" or "alarming"; and (A) *frenetic* sounds too much like the word "frantic," which is exactly what it means.

9. (D) Much of what is now _____ was once impossible

In this sentence completion it is obvious that the word missing in the blank is the opposite of *impossible;* that is, that something is now possible that was once not possible. You probably eliminated (A), (B), and (E) right away since you knew that (A) *foolhardy* means "excessively bold"; (B) *fickle* means "changeable"; and (E) *fatuous* means "foolish" or "silly." With choices (C) and (D) remaining, it would be a good idea to read the sentence out loud, substituting those choices in the blank space. If you did not know that (C) *fervid* means "full of passion" or "zealous" and (D) *feasible* means "possible" or "suitable," chances are you nevertheless went with (D) because not only did it probably sound better but you would have recognized that the *-ible* ending matches the ending on *impossible.*

10. (E) Brian loves French food, but he refuses to eat at the stylish restaurant in town because the prices are _____ and far beyond his means.

One of the important clues in this sentence completion is the word *and* immediately following the blank. You know that the word you are looking for has something to do with prices. You also know that although Brian loves French food, he is not going to eat at the restaurant because of something to do with those prices. The word *and* moves us closer to the answer because it indicates that whatever word or group of words follows, the word you will be looking for will have to bear a direct relationship to it. Since the words following the word *and* are *far beyond his means,* it is apparent that the word you are looking for will have something to do with the word "expensive." If you are following the instructions on how to do the sentence completion, remember that one of those instructions was to think of your own choice of word before looking at the answers. Chances are the word "expensive" was exactly what you came up with. You probably eliminated (A), (B), and (C) immediately, since (A) *fanciful* means "imaginary"; (B) *extraneous* means "not necessary" or "accidental"; and (C) *extemporaneous* means "done on the spur of the moment." You may have left (D) *extant* because you did not know that it means "existing" but still chose (E) *exorbitant,* because like most students you knew the meaning of this word, which is "excessive." In addition, reading the word out loud, substituting each of the choices in the space provided, chances are you would have gone with (E) just because it sounds right in the sentence.

Two-Word Sentence Completions

The technique for dealing with the two-word sentence completion is basically the same as that for dealing with the one-word completion, but there are some special variations.

Step 1

Examine the sentence-completion question and underline key words.

Step 2

Single out any word, or group of words, of the ones you have underlined which you think is more important in providing clues to help you locate the answer. When you have made your choice, circle the word or group of words.

Step 3

Reread the sentence and fill in the blank with words of your own *before looking at the answer choices*. Determine if the words you are looking for are positive or negative—that is, have good connotations or bad connotations.

Step 4

Back into the right answer by going through the choices one at a time, drawing a line through those answers that are *not possible*. Leave words you do not know. Use "word attack" skills such as focusing your attention on prefixes and familiar roots to narrow down your choices.

Step 5

When you have narrowed down your choices, read the sentence out loud, substituting the remaining answers in the space provided, and select the answer that sounds best.

Examine the following sentence-completion question. Underline and circle the key words in the sentence. Do it *now* before reading ahead. Try to think of your own words to put in the blanks and determine for yourself whether the words you are looking for are positive or negative.

> The military tribunal _____ the otherwise illegal action because the bravery of the soldier and the emergency of the situation _____ his guilt.
> (A) enforced . . heightened (B) approved . . condemned
> (C) condoned . . extenuated (D) extended . . retracted
> (E) extradited . . increased

Try eliminating choices, dealing with one side of the sentence completion at a time. In other words, work with just the first omission as if it were a single completion and see how many choices on the left side you can eliminate. It is important to decide what kind of a word you need in the first blank. It is obvious that the important word here is *otherwise*, indicating that the military tribunal is treating in a less serious way something which ordinarily carries severe penalties. You are therefore looking for just such a word. That would eliminate (A), (D), and (E) immediately, because (A) *enforce* means "compel obedience"; (D) *extend* means "advance"; and (E) *extradite* means "surrender an alleged criminal to another authority for trial." None of these words refers to treating an offense in a less serious manner. You realize, of course, that by eliminating *only one side*, you have eliminated the entire choice. That should simplify things.

If you are left with (B) and (C) as possible answers, and do not know that *condoned* means "excused," it should still seem highly unlikely that a military tribunal would *approve* an illegal action under any circumstances, so you would have backed into the right answer, which is (C). But leaving those two for a moment, let us see if we can work with the right side of those two choices and eliminate one of the answers more reasonably. You probably realized that the word you need in the second blank has something to do with lessening the soldier's guilt because of his bravery and because of the emergency. That would eliminate (B) immediately, since *condemned* is the exact opposite of what you are looking for. Therefore, even if you did not know that *extenuated* means "les-

sened," you would have backed into the right answer. Now if you read the sentence completion with your answer choice in the blanks, it should sound as though they were made for each other.

Your final diagrammed sentence completion should look something like this:

The military tribunal _____ the otherwise illegal action because the bravery of the soldier and the emergency of the situation _____ his guilt.
(A) ~~enforced~~ . . heightened (B) approved . . ~~condemned~~
(C) condoned . . extenuated (D) ~~extended~~ . . retracted
(E) ~~extradited~~ . . increased

What you have done is first use one set of clues—the left-hand word in each pair—and then use the second set of clues—the right-hand word in each pair. It is because two-word sentence completions offer two sets of clues that they are actually easier than one-word sentence completions, permitting you to eliminate all the wrong answer choices even though you may not know what all the words mean.

Here is another two-word completion for you to practice on. Try eliminating the choices *one side at a time*. When you have narrowed your choices down as far as you can, try reading the sentence *out loud* with the choices in the blanks and select the one that sounds best.

How can two brothers of the same family be so different; the one kind and _____, the other _____ and bitter.
(A) mild . . truculent (B) mercantile . . avaricious (C) paranoid . . deluded
(D) lackadaisical . . languid (E) unhealthy . . jaundiced

Note the importance of the word *and* following each of the blanks, which indicates that whatever word you select for the blank space must be similar to the word preceding the word *and*. The first blank, therefore, is asking for a word that is similar to *kind;* it will be a positive word. Therefore, you can eliminate (C), (D), and (E) immediately because they all have negative connotations. (B) can be eliminated because *mercantile* means "commercial" and is totally irrelevant; you should be able to see the connection between this word and "merchant." The answer, therefore, is (A), because *mild* means "calm" and *truculent* means "fierce" and "destructive," a negative word, which is what is needed for the second omission.

Your final diagrammed sentence completion should look like this:

How can two brothers of the same family be so different; the one kind and _____, the other _____ and bitter.
(A) mild . . truculent (B) ~~mercantile~~ . . avaricious (C) ~~paranoid~~ . . deluded
(D) ~~lackadaisical~~ . . languid (E) ~~unhealthy~~ . . jaundiced

Note that in this case, if you started trying to eliminate right-hand words in each answer choice, you doubtless had trouble, because all the right-hand words are negative and make sense in the sentence. You should quickly have realized this and switched your attention to the left-hand words in each answer choice.

PRACTICE EXERCISE

Here are some sentence completions for you to practice on. Each sentence has two blanks, and beneath the sentence are five lettered sets of words. Choose the set of words that *best* fits the meaning of the sentence as a whole.

1. The _____ senses were cultivated by the Greeks because of their appreciation of _____.

   (A) cretinous . . knowledge
   (B) esthetic . . beauty
   (C) derisive . . competition
   (D) lascivious . . abstinent
   (E) derogatory . . color

2. Shirley's emotions were ambivalent; she _____ Henry for his kindness in pubic but _____ him for his pettiness with her.

   (A) respected . . reproached
   (B) taunted . . tormented
   (C) deluded . . misled
   (D) imprecated . . cursed
   (E) immolated . . destroyed

3. Whereas it is not proper to conduct oneself in a(n) _____ manner, it is also true that to be _____ invites manipulation by others.

   (A) bumptious . . meek
   (B) civil . . repugnant
   (C) benign . . malicious
   (D) benevolent . . kind
   (E) arrogant . . brusque

4. Unlike her _____ and callous sister, Lurleen was _____ toward the victims of the flood.

   (A) apathetic . . compassionate
   (B) loving . . harsh
   (C) inclement . . vapid
   (D) complaisant . . incorrigible
   (E) carping . . vicious

5. We hope that our skin is not _____ to disease, just as a raincoat should be _____ to rain.

   (A) immune . . penetrable
   (B) pervious . . impermeable
   (C) brusque . . impolite
   (D) anticlimactic . . apocryphal
   (E) altruistic . . charitable

6. While talk of antiballistic missiles would surely have been _____ in the days of the Ancient Greeks, references to law and culture would have been as _____ then as they are today.

   (A) likely . . unusual
   (B) current . . rare
   (C) anachronistic . . relevant
   (D) mundane . . spiritual
   (E) holy . . blasphemous

7. In order to ascertain the composition of a substance, the scientist uses _____. With his knowledge of chemical composition, he can recombine the component parts and create a new substance by means of _____.

   (A) alchemy . . reconstruction
   (B) analysis . . synthesis
   (C) attrition . . erosion
   (D) titration . . salivation
   (E) sucrose . . lactose

8. "That _____ has no respect for our ancient traditions," claimed the leader of the cult, "and wishes only to create _____ among the pious."

   (A) snob . . bliss
   (B) termagant . . chatter
   (C) apostate . . imagery
   (D) iconoclast . . chaos
   (E) alien . . xenophobia

9. The dancer's marvelous performance was unlike anything we had ever seen before: it was not only _____, it was truly _____.

   (A) different . . singular
   (B) disheveled . . disgruntled
   (C) frail . . frenetic
   (D) preeminent . . pugnacious
   (E) propulsive . . vituperative

10. "I find nothing wrong with Sandy's nature of being _____; I fear only that she may repel others by her apparent _____."

    (A) fatalistic . . utopianism
    (B) complimentary . . obsequiousness
    (C) virile . . effeteness
    (D) lackadaisical . . hyperactivity
    (E) malicious . . benevolence

ANSWERS AND EXPLANATIONS

1. (B)	2. (A)	3. (A)	4. (A)	5. (B)
6. (C)	7. (B)	8. (D)	9. (A)	10. (B)

1. (B) The senses were cultivated by the Greeks because of their appreciation of _____.

In this sentence completion the words *because of* indicate that the word missing in the first blank depends upon the word in the second blank. We can assume the words are positive because of the word *appreciation:* because the Greeks appreciated something good, they developed certain positive senses. Working with this first omission, chances are you eliminated (C) and (E) immediately, since they are both negative words. Left with (A), (B), and (D), you then moved to the second omission and probably eliminated (D), even if you didn't know that *abstinent* means "self-denying." It is likely that you associated *abstinent* with the word "absence" and reasoned that *abstinent* is not something one would appreciate. Besides, *abstinent* is an adjective, and a noun seems called for. (Usually the SAT will not include choices that are the wrong part of speech, but you can never tell. After all, some nouns end in *-ent,* such as "talent," and some end in *-ic,* such as "ethic." If you were completely unfamiliar with "talent" and "ethic" you might think they were adjectives, and the makers of the SAT could lay a trap for you.) Now you have narrowed your choices to (A) and (B). Reading them both in the sentence, you might have selected (B) as the correct answer since it *sounds best,* without knowing that the word *esthetic* means "pertaining to beauty." Furthermore, there are students who will recognize that the word *cretinous* has negative associations without being able to define the word precisely (it describes a person with a certain type of mental deficiency).

2. (A) Shirley's emotions were ambivalent; she _____ Henry for his kindness in public but _____ him for his pettiness with her.

It should be clear from your diagramming that there are several important clues in this sentence completion. The most important clue is the word *ambivalent.* Even if you don't know the word, you should know from the prefix *ambi-* that the word has some meaning of "both." Therefore you know that Shirley has mixed feelings. The word *but* is your second major clue, since it indicates that whatever precedes the word is going to contrast with whatever follows it. You know what to look for in the first omission, since the clue is the word *kindness;* you are looking for a positive word. You would therefore eliminate (B) and (C) immediately, since they are negative words. You may have let (D) and (E) remain, if you do not know that *imprecated* means the same as "cursed" and *immolated* means "sacrificed." Turning to the second omission, you probably eliminated (E) even though it is negative, because if you read it in the sentence, you realized that *destroyed* is too intense a word. If you are left with choices (A) and (D), (A) should seem the correct answer, since *respected* is definitely a positive word, and *reproached* contrasts with it.

3. (A) Whereas it is not proper to conduct oneself in a(n) _____ manner, it is also true that to be _____ invites manipulation by others.

The clues to the two omissions in this sentence completion are well defined. In the first blank it is clear that we are looking for a word that is negative, since the *manner* is apparently an improper one. That would eliminate choices (B), (C), and

(D), since they are all positive words: (B) *civil* means "polite"; (C) *benign* means "kind" or "harmless"; and (D) *benevolent* means "kind." Left with (A) and (E), we turn to the second omission, where it is apparent that the word we are looking for describes some quality that will allow others to manipulate one. You may not know that *bumptious* means "pushy," but most students know the meaning of the word *meek,* which is "gentle"; it is usually associated with the word "mild." A *meek* person can be easily manipulated. Besides, even if you did not know that the second word of choice (E), *brusque,* means "sharp" and "abrupt," it doesn't sound as if it could describe someone who is easy to manipulate.

4. (A) Unlike her _____ and callous sister, Lurleen was _____ toward the victims of the flood.

The word *unlike* at the beginning of the sentence tells us immediately that the missing words are going to contrast with each other. It is also obvious that the first omission is negative because of the word *and,* which indicates that the missing word is going to be similar to the word *callous,* which means "feeling no emotion." That would eliminate (B) and (D), since both *civil* and *benevolent* are positive words. Since you are looking for a contrasting word in the second blank—that is, a positive word—(E) would be eliminated, because *vicious* is a negative word. With (A) and (C) remaining, (A) seems the likely way to go; *vapid* sounds like a negative word even to those who do not know its meaning, which is "empty." When you read the two choices out loud in the sentence, (A) sounds better.

5. (B) We hope that our skin is not _____ to disease, just as a raincoat should be _____ to rain.

It should be clear from your diagramming of this sentence completion that you are looking for words that mean essentially the same thing; that is, words that have something to do with "keeping out" or "keeping away." Yet if you read the sentence carefully you will see that because of the wording, specifically the word *not,* you are going to want contrasting forms of the same word. Skin is *not going to let in* disease just as a raincoat is *going to keep out* the rain. For the first blank, therefore, we are looking for a word that means the same as "letting in." That would eliminate (A), (C), (D), and (E), since none of these words has such a meaning. If you did not know the meaning of *altruistic,* which is "concerned for the welfare of others," you still should know that the familiar word *charitable* doesn't fit in the second half of the sentence. That leaves only choice (B).

6. (C) While talk of antiballistic missiles would surely have been _____ in the days of the Ancient Greeks, references to law and culture would have been as _____ then as they are today.

If you were substituting your own words in the blanks before consulting the available choices, it is more than likely that you came up with some variation of the idea that it would be strange, out of place, or impossible for the Ancient Greeks to be talking about antiballistic missiles. Therefore, looking over your choices for the first omission, you probably eliminated (A), (B), and (E), since none of them is a remote possibility. Of course, you may have some difficulty with the words in (C) and (D); you may not know that *anachronistic* means "chronologically out of place" and *mundane* means "earthly" or "ordinary." However, the word that you probably substituted in the second omission was probably some variation of the idea that it was appropriate for the Greeks to be discussing law and culture. So your choice would naturally be (C), for the word *relevant* is the most suitable choice.

7. (B) In order to ascertain the composition of a substance, the scientist uses _____. With his knowledge of chemical composition, he can recombine the component parts and create a new substance by means of _____.

You know that the word you are looking for in the first blank has something to do with what a scientist uses in figuring out what a substance is made of. You would probably have no trouble eliminating (A) and (E) because (A) *alchemy* is a superstitious medieval science; and (E) *sucrose* is the sugar that occurs naturally in most land plants. With choices (B), (C), and (D) remaining, it is clear by examining your second blank that the word you are looking for has something to do with the clues *recombine* and *create a new substance*. That would eliminate (C) and (D) immediately, since (C) *erosion* is a wearing away, not a recombining; and (D) *salivation* is the production of saliva and is totally irrelevant. If you know that (C) *attrition* means "weakening" and (D) *titration* is a method of determining the strength of a solution, you could have eliminated (C) and (D) the first time through. (B), of course, is the correct answer.

8. (D) "That _____ has no respect for our ancient traditions," claimed the leader of the cult, "and wishes only to create _____ among the pious."

The clues for the first omission in this sentence completion are the words *no respect* and *traditions*. Almost any negative word would make sense in this first blank, but remember that there should be some kind of clue to the correct one. This is another example of a definition clue; you are expected to find a word which means someone with *no respect for our ancient traditions*, not just any negative word. That would eliminate (A) and (E), since you know that neither the word *snob* nor the word *alien* means "breaker of tradition." Turning to the second omission, we see that a negative word is needed, because someone who has no respect for traditions seems to want to create trouble of some kind among the pious. That would eliminate (B) and (C), since neither of the words fits the description of what we are looking for. The answer, of course, is (D): *iconoclast* means "idol breaker," and *chaos* means "great upheaval." (*Termagant* means "scolding woman"; an *apostate* is one who revolts, generally against his religious beliefs; and *xenophobia* means "fear of strangers.")

9. (A) The dancer's marvelous performance was unlike anything we had ever seen before; it was not only _____, it was truly _____.

From the clues you have isolated, it should be apparent that the words you are looking for in this sentence completion are not only both positive words, but are close to being synonyms, differing only in intensity. We know they are positive because of the word *marvelous*. If you substituted your own words in the blanks, you probably came up with something like "it was not only great, it was truly unique." You probably had no difficulty eliminating (B) and (C) immediately, because they are all negative words. Left with choices (A), (D), and (E), you were probably able to eliminate (D) and (E), since (D) *pugnacious* sounds negative (it means "belligerent"); and (E) *vituperative* also sounds negative (it means "abusive"). (A) is the correct answer; *singular* does give the idea of the uniqueness of the performance.

10. (B) "I find nothing wrong with Sandy's nature of being _____; I fear only that she may repel others by her apparent _____."

The words *nothing wrong* should clue you in to the fact that the word you are looking for, although not necessarily a strong positive word, is certainly not negative. You can begin by eliminating all negative words for the first omission. That would eliminate (E), since *malicious* is a negative word. Turning to the second omission, it is apparent from the words *may repel* that the word you are looking for, although not necessarily a strong negative word, is certainly not positive. So you can begin by eliminating (A) because *utopianism* is not a quality; it is a philosophical point of view, a belief in a perfect society. With choices (B), (C), and (D) remaining, it is important to go back to the sentence completion and reexamine the relationship between the two words you are looking for. The clues *nothing wrong* and *may repel* seem to indicate that you are looking for synonyms based on their degree of intensity; that is, it appears that the second missing word is more intense than the first missing word. Looking over your answers, it would be possible to eliminate (D), because you probably know both words and realize that they are opposites: *lackadaisical* means "lacking life or spirit," and *hyperactivity* means "excessive activity." You may have difficulty with the word *effeteness* in (C) (it means "lack of vitality" or "decadence"), but you can reconsider the first word in that choice, which is *virile*. You probably know that "virility" is a kind of manly vigor; most people associate it with sexual vigor. If you read the sentence with that word in the first blank, you will realize that it doesn't suit the meaning or spirit of the sentence. You will then have backed yourself into the right answer. *Complimentary* is the positive word you are looking for, and *obsequiousness* means "subservience" or "excessive attentiveness." Thus (B) has for its choice two words that not only pertain to the same idea but have different degrees of intensity—just what you're looking for.

ANALOGIES

The type of question treated in this section requires you to analyze word relationships and to recognize *analogies*, that is, things that are parallel in some way. The student is given two words that have a specific relationship, followed by five pairs of words, one pair of which clearly imitates the relationship between the first two.

To be able to choose the best parallel relationship from among the answers, the student must establish the relationship not only between the first two words but between the words in the answer choices. This part of the test calls on your knowledge of vocabulary, but there are also important ways of approaching words that you don't know at all or perhaps know only vaguely. First we will explain the basic techniques to use when you *do* know the words, and then techniques you can resort to when you *don't* know the words.

If You Know Both Words

Step 1

Examine the following analogy question, paying particular attention to the relationship between the first two words.

> FEATHER:BIRD: : (A) fur:beaver (B) zebra:stripes (C) sky:cloud (D) goose:down (E) skin:man

Let's assume that you know the meaning of the first two words, *feather* and *bird*. This, then, is what you are to do. On a separate sheet of paper, *make a sentence*, using the words *feather* and *bird*. Make your sentence as *simple* as possible and as *specific* as possible. Make certain that it expresses the relationship between the first two words. Think about it for a minute. What does a bird have to do with feathers? Write your sentence in the space below, *before* reading further.

Chances are that you came up with either one of two sentences: *A bird has feathers* or *Feathers cover a bird.* (Of course, variations of either sentence are perfectly acceptable.)

Step 2

Now take your sentence and read it exactly the same way as you have written it, only instead of the words *bird* and *feather*, substitute the choices given in A through E.

But this is important: The words must be in the same order. For example, if you had used the sentence *A bird has feathers*, you would have to say: (A) *A beaver has fur*, (B) *A stripes has zebra*, (C) *A cloud has sky*, (D) *A down has goose*, and (E) *A man has skin*.

As you read through your choices for the first time, take your pencil and *draw a line* through all those choices that do not seem to make any sense or are impossible. *Go back* to the analogy and try it now. Do your crossing out right on this sheet.

Your analogy should look like this:

FEATHERS:BIRD:: (A) fur:beaver (B) ~~zebra:stripes~~ (C) ~~sky:cloud~~ (D) ~~goose:down~~ (E) skin:man

You probably eliminated (B), (C), and (D) because stripes do not have zebra, cloud does not have sky, and down does not have goose.

Step 3

Look at your remaining choices, (A) and (E). Now that you have narrowed your choices down to two, it is a good idea to work with one side of the analogy at a time.

Suppose you look at the left side first. (A) *fur* is more analogous to *feathers* than (E) *skin* is—after all, birds, beavers, and men all have skin, so *skin* is probably too broad a word to express the analogy. But a *beaver* has *fur* in exactly the same way as a *bird* has *feathers*. So (A) is the right choice.

Suppose you look at the right side first. (A) *beaver* seems to relate pretty well to *bird*—better than (E) *man*, since beavers and birds are both animals, and men are usually not thought of as animals. This gives you the right answer too. But be careful in such situations! The distinction between man and animal, human and nonhuman, *may* be important in some analogies, but it is by no means always the determining distinction. Suppose the answer choices had been (A) *tail:beaver* and (E) *hair:man*. In this case (E) would be correct.

Thus, by making a sentence which expresses the relationship between the first two words, it is usually possible to eliminate several of the analogy choices immediately without having to examine each one studiously; they will simply sound ridiculous in your sentence. If you have narrowed your answers down to two, both of which sound possible in your sentence, examine each part of the analogy individually. In some cases it may also help a lot to revise your original sentence to make it more specific. In the example above, revising your sentence to *Feathers grow from the skin of a bird* would quickly lead you to the right answer choice.

PRACTICE EXERCISE

Here are some analogies for you to practice on. Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Practice making specific sentences. Substitute the choices in A through E in your sentence, eliminating those that are not possible. Then select the lettered pair that best expresses a relationship similar to that expressed in the original pair.

1. TREATY:WAR:: (A) achievement:award (B) strike:deadlock (C) invasion:rebellion (D) agreement:altercation (E) law:protection

2. SHELF:BOOK:: (A) plant:soil (B) duplication:copy (C) vase:rose (D) itinerary:hour (E) author:guild

3. ALPHABET:LETTERS:: (A) desk:chairs (B) insect:wings (C) grapefruit:sections (D) body:head (E) flora:fauna

4. CHALK:ERASER:: (A) camera:pose (B) bed:evening (C) stain:detergent (D) crevice:fissure (E) acclivity:hill

5. MANE:LION:: (A) wine:chalice (B) tile:pattern (C) calyx:flower (D) map:navigator (E) dictaphone:stenographer

6. SILVER:TARNISH:: (A) bronze:patina (B) copper:alloy (C) gold:crown (D) nickel:metal (E) pigment:aluminum

7. FROG:AMPHIBIAN: : (A) dog:primate (B) caterpillar:arachnid (C) lizard:toad (D) snake:reptile (E) simian:antelope

8. PROBLEM:SOLUTION: :
(A) erudition:knowledge (B) enigma:puzzle
(C) cellblock:prisoner
(D) dilemma:alternatives
(E) incarceration:convict

9. HYGIENE:HEALTH: :
(A) washcloth:cleanliness (B) talisman:charm
(C) antitoxin:poison (D) serenity:war
(E) gneiss:geology

10. LACERATION:SCAR: :
(A) congelation:summer (B) bombast:creation
(C) card:invitation (D) captivity:capacity
(E) aridity:wilting

ANSWERS AND EXPLANATIONS

1. (D)	2. (C)	3. (C)	4. (C)	5. (C)
6. (A)	7. (D)	8. (D)	9. (A)	10. (E)

1. (D) Your sentence was probably some variation of the following: *A treaty is made to prevent or end a war.* Substituting the choices in that sentence, you should be able to eliminate (A), (B), (C), and (E). (A) an *achievement* is not made to prevent or end an *award,* but rather to earn one. (B) a *strike* is not made to prevent or end a *deadlock;* it may, in fact, arise out of a deadlock. Although (C) an *invasion* could be made to prevent or end a *rebellion,* it seems at least as likely to start one; the ideas are not clearly opposed. (E) *laws* are not made to prevent or end *protection,* but rather to provide it. Also, *protection* is not a state of being that can come to an end; it is an abstract idea. (D) is the right answer, because *altercation* means "disagreement" or "quarrel," and an *agreement* is indeed made to prevent or end a quarrel.

Note that by *backing into* the right answer—by eliminating all those that are *not possible*—you are left with the right answer, even if you did not know what *altercation* meant. The other choices simply make no sense.

2. (C) When we examine this analogy, we see that a sentence that seems to express the relationship well is: *A book is generally put in a bookshelf.* Using this sentence, you should have been able to eliminate (B), (D), and (E) immediately. (B) a *copy* is generally not put in a *duplication;* they mean the same thing. (D) an *hour* is generally not put in an *itinerary;* an itinerary is the outline of a journey. (E) a *guild* is generally not put in an *author;* a guild is an association of men. Why might (A) have seemed like a possibility? Because a *plant* is generally put in *soil*—but the words are not in the right order; *soil* is not generally put in a *plant.* You have left choice (C), because a *rose* is generally put in a *vase.*

Suppose choice (A) had been *soil:plant?* You would have had some difficulty deciding between (A) and (C), but by examining the relationship between *shelf* and *book* more closely, you still might have found the better answer. A plant grows in soil, it is not put there for storage or display.

3. (C) This question should not be difficult. A likely sentence is: *An alphabet is made up of letters.* You should have eliminated (A), (B), and (D) rather quickly. (A) a *desk* is not made up of *chairs.* (B) an *insect* is not made up of *wings;* wings are only part of an insect, and there are many insects that do not have wings. (D) a *body* is not made up of *heads;* again, a head is only part of a body, but an alphabet is made up completely of letters. (E) *flora* means "plant life" and *fauna* means "animal life"; one is certainly not made up of the other. So (C) is the best answer

because a *grapefruit* is made up of *sections*. Of course, you could have narrowed it down to (C) and (E), but had no idea what *flora* and *fauna* mean. Still, since (C) fits so well in the sentence, you could feel quite safe in choosing it. You have been advised to look at *all* the choices before choosing, since often a choice that looks right at a quick glance is just a trap. But nevertheless, in a situation like this, where you have narrowed it down to one choice that seems right and another that you don't understand, take the one that seems right.

4. (C) *An eraser removes chalk* is probably something like the sentence you came up with. That should have eliminated choices (A) and (B) immediately. (A) *pose* does not remove *camera;* and (B) *evening* does not remove *bed*. With choices (C), (D), and (E) remaining, it might be a good idea to study all the words on the left side first to see what you might eliminate. Chances are you eliminated (E) because it would seem unlikely that a *hill* would remove anything. Of course, if you knew that *acclivity* means "upward slope" you would have eliminated it anyway; the two words are almost synonyms. The same holds true for (D); *crevice* and *fissure* are synonyms, and neither removes the other. Chances are that you selected (D) as your answer; a *detergent* does remove a *stain*.

5. (C) You may have started with quite a simple sentence, such as: *A lion has a mane*. You would then be able to eliminate (A) and (B) immediately because: (A) a *chalice* does not have a *wine;* a *chalice* is a cup. (B) a *pattern* does not have a *tile;* it is generally the other way around. But choices (C), (D), and (E) all do seem to fit the sentence fairly well. So you had better take another look at your original sentence and try to make a new, *more specific* sentence. One likely possibility might be: *A mane grows on a lion*. That would eliminate (D) and (E), because (D) a *map* does not grow on a *navigator;* and (E) a *dictaphone* does not grow on a *stenographer*. (C) is the correct answer, since a *calyx* does grow on a *flower;* the calyx is the external green or leafy part of a flower. If you do not make a new sentence, you might follow another line of reasoning when left with choices (C), (D), and (E). You could examine one side at a time as suggested earlier. A closer look at the relationship between *mane*, *calyx*, *map*, and *dictaphone* would probably prompt you to eliminate *map* and *dictaphone* because a *mane* is a part of the *lion*, and a *map* and *dictaphone* are not part of a *navigator* or *stenographer*. So you would go with (C).

6. (A) A simple way to make a sentence with this analogy is: *Tarnish takes the shine away from silver*. Substituting your choices in this sentence, you more than likely backed into the correct answer, which is (A), even though you may have had no idea what patina means. (B) *alloy* does not take the shine away from *copper;* (C) *crown* does not take the shine away from *gold;* (D) *metal* does not take the shine away from *nickel;* and (E) *aluminum* does not take the shine away from *pigment*.

7. (D) There should be little problem making a sentence, since as most students know: *A frog is an amphibian*. That should easily eliminate (A) and (C), because (A) a *dog* is not a *primate;* and (C) a *lizard* is not a *toad*. (E) would also be eliminated immediately, since the words are in the reverse order. Many students would not know the meaning of *arachnid* and would consider choice (B) a possibility for that reason. Actually, arachnids are spiders, ticks, and similar creatures; a *caterpillar* is not an *arachnid*. But anyway, (D) a *snake* is a *reptile*—these words are familiar to almost all students, and they fit the sentence, so choice (D) is quite safe even if choice (B) puzzles you.

8. (D) The sentence which best expresses the relationship in this question is: *One finds a solution to a problem*. That would eliminate (C) and (E). (C) one does

not find a *prisoner* to a *cellblock;* and (E) one does not find a *convict* to an *incarceration.* As for (B), if you did not know that *enigma* means the same thing as *puzzle,* you still might have eliminated it because the idea of finding a puzzle seems very unlike the idea of finding a solution. With choices (A) and (D) remaining, the decision should be an easy one, since in (D) one finds *alternatives* to a *dilemma.* In (A), one could find *knowledge* as one might find a *solution,* but knowing that *erudition* means the same thing as *knowledge* eliminates that as a possibility. If you didn't know that, still you should have been drawn toward (D).

9. (A) Your sentence for this analogy was probably something like: *Hygiene leads to health.* Using this sentence, you would have to eliminate (D) immediately, because *serenity* does not lead to *war.* It would seem logical to eliminate (E) even if you do not know that *gneiss* is a type of rock; it does not seem probable that a particular thing, whatever it is, could be responsible for producing an area of scientific study. In (C), an *antitoxin* is something that counteracts *poison,* not something that produces it. If you didn't know the word *antitoxin,* perhaps you were able to figure it out: a *toxin* is a poison, and the prefix *anti-* means "against." This leaves us with (A) and (B). If you did not know that (B) a *talisman* is an object which acts as a charm to avert evil—in other words, *is* a charm rather than something that produces one—it would still seem to make the most sense to select (A) as your answer since you know that if you use a *washcloth* you produce *cleanliness.*

10. (E) You probably came up with some variation of the sentence: *Lacerations produce scars.* That would eliminate (C) and (D) right away. (C) *cards* do not produce *invitations;* invitations are written on cards. (D) *captivity* does not produce *capacity;* they are unrelated to each other. With the remaining three choices, by just looking at the words on the right side—that is, *scar, summer, creation,* and *wilting*—*wilting* seems the most appropriate, since, like a *scar,* it is an effect, while *summer* and *creation* have no such relation to *scar.* On the left side, in (A) the word *congelation* seems to be made up from the word "congeal," which means "to make into a solid state." That would eliminate (A), since *congelation* does not produce *summer.* In (B), *bombast,* which is pompous speech or writing, does not produce *creation.* If you were shaky on the meaning of *bombast,* and thought it meant some kind of explosion, you might have been attracted by this answer; according to the "big bang" theory of the universe, creation began with an explosion that is still going on. But could the makers of the SAT be expecting you to make as labored a connection as this? It's unlikely. The best answer is, of course, (E), since *aridity* means "dryness" and does produce *wilting.*

If You Know Only One of the Words

Examine the following analogy question, again paying particular attention to the relationship between the first two words.

BIRD:AVIARY:: (A) tomb:headstone (B) actuary:lion
(C) bridge:gully (D) prisoner:cell (E) money:safe

If you know what the word *aviary* means, make a specific sentence using both *bird* and *aviary* in the space below, *before reading further.*

What if you do not know what an aviary is? One way to discover what relationship *aviary* might have to *bird* is to *look over the choice of answers*. What choices do you have in this question? Two relationships that you certainly know: a *prisoner* is locked up in a *cell* and *money* is locked up in a *safe*. Trusting yourself on this limited information, *you have to take a chance*. You have to make up a sentence with *bird* and *aviary*, using these clues. Please do it now, using the space provided. You probably arrived at the same sentence as you would have if you had known what the word *aviary* meant: *An aviary is where a bird is kept*. Now go back to the question and substitute the choices in your sentence, eliminating those that are *not possible*. Again, remember, *the words must be in the same order as in your original sentence*. Work on your analogy now, without reading ahead. The question should now look like this:

BIRD:AVIARY: : (A) ~~tomb:headstone~~ (B) ~~actuary:lion~~
(C) ~~bridge:gully~~ (D) prisoner:cell (E) money:safe

You probably eliminated (A), (B), and (C). (A) a *headstone* is not where a *tomb* is kept. (B) a *lion* is not where an *actuary* is kept; even if you did not know that an *actuary* is one who calculates insurance premiums, you would realize that a *lion* is not a place. (C) a *gully* is not where a *bridge* is kept; again, even if you were not aware that a *gully* is a large trench worn in the earth, you would recognize immediately that a *bridge* is not kept anywhere.

With your remaining choices being (D) and (E), you may be puzzled because they both seem like good analogies. This *does* happen on actual SAT questions; not just students but their teachers may disagree on a particularly subtle question. You have to look for further distinctions that will make one choice better. For example, it should be apparent that (D) has an edge because *prisoner* is animate and *money* is inanimate. And there is another important distinction to keep in mind between (D) and (E) aside from animate and inanimate. That consideration aside, there are many things that are put into a safe besides money, such as documents, jewels, etc., but *only* a prisoner is kept in a cell. So (D) is the *more specific* answer.

So far, you should have noted several important test-taking devices to keep in mind:

1. Make a *specific* sentence.
2. When substituting choices in (A)–(E), make certain words are in the correct order.
3. Check for human and nonhuman, animate and inanimate—but remember that these are not primary considerations, they just help tip the balance where you have two choices that both seem good.
4. When left with a choice of two possible answers, be sure you select the more specific answer.
5. Look over your choices for clues to the relationship of the analogy being tested.

PRACTICE EXERCISE

Here are some analogies for you to practice on. Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Practice making specific sentences. In many of the original analogies, you will not know one of the words, or even though you may know the words, the relationship will not seem clear, so check over your answers for clues to what the relationship could be. Then substitute the choices in (A) through (E) in your sentence, eliminating those that are not possible. Select the lettered pair that *best* imitates the relationship of the original pair.

1. ORACLE:PROPHECY:: (A) judge:hearing (B) servant:means (C) meteorologist:forecast (D) clerk:price (E) announcer:advertisement

2. CANTALOUPE:VINE:: (A) wheat:chaff (B) carrot:root (C) grass:plant (D) orchid:bush (E) quince:tree

3. MOON:FIRMAMENT:: (A) airplane:troposphere (B) flower:bee (C) snail:lobster (D) catnip:cat (E) dice:diversion

4. LOUSE:PERSON:: (A) army:enemy (B) tick:dog (C) bee:flower (D) cat:bird (E) aphid:ant

5. QUILT:BED:: (A) guest:mean (B) torch:flame (C) stigma:burden (D) succor:help (E) glove:hand

6. CALLIGRAPHER:QUILL:: (A) record:phonograph (B) baby:candy (C) businessman:penitentiary (D) stenographer:typewriter (E) astronaut:wheelchair

7. TALKATIVE:VOCIFEROUS:: (A) vituperative:acrimonious (B) vicious:ferocious (C) tame:docile (D) bright:blinding (E) tangent:contiguous

8. DOOR:PORTAL:: (A) carton:container (B) gruel:porridge (C) hallucination:delusion (D) hiatus:gap (E) hole:abyss

9. PECCADILLO:FELONY:: (A) quilt:bedspread (B) meaning:significance (C) immolation:sacrifice (D) puddle:sea (E) imprecation:curse

10. ELASTIC:PLIABLE:: (A) ventral:dorsal (B) vernal:hibernal (C) valiant:brave (D) hungry:anorexic (E) confused:stable

ANSWERS AND EXPLANATIONS

1. (C)	2. (E)	3. (A)	4. (B)	5. (E)
6. (D)	7. (D)	8. (E)	9. (D)	10. (C)

1. (C) Perhaps you did not know that an *oracle* is a person (as a priestess of Ancient Greece) through whom a deity is believed to speak, revealing hidden knowledge. But by examining choices (A) and (C), you might have concluded that (A) a *judge* grants a *hearing* to someone; and (C) a *meteorologist* gives the *forecast*. Using this information, take a chance and create a sentence that expresses the relationship in the original analogy. Try the sentence: *An oracle makes a prophecy*. Substituting the choices in your sentence, you should have been able to eliminate (B), (D), and (E). (B) a *servant* does not make a *mean*. (D) a *clerk* does not make a *price*—he may give a customer a price, but he is merely transmitting ordinary information, not saying anything about the future. (E) an *announcer* does not make an *advertisement*, though, again, he may deliver one. Left with choices (A) and (C), you probably did not hesitate to choose (C) as the right answer, since *prophecy* and *forecast* are more directly related than *prophecy* and *hearing*.

2. (E) This analogy may have presented some difficulty. You probably used some variation of the sentence: *A cantaloupe grows on a vine*. That would have eliminated (A), (B), and (C). (A) *wheat* doesn't grow on a *chaff*; (B) *carrot* doesn't grow on a *root*—it is a root; (C) *grass* doesn't grow on a *plant*—it is a plant. You doubtless left (D), because an *orchid* grows on a *bush*, and you left (E) because a *quince* grows on a *tree*. So you have to look for distinctions between (D) and (E) to determine which is the better analogy. Obviously, *cantaloupe* and *quince* are both fruits, whereas *orchid* is a flower—so (E) is the *more specific* answer. This distinction is unusually subtle, of course—but occasionally SAT questions are somewhat subtle.

3. (A) *The moon is in the firmament* is probably the simplest way to deal with this relationship. That would have eliminated (B), (C), (D), and (E). (B) a *flower* is not in the *bee*. (C) a *snail* is not in the *lobster;* they are two different species of marine life. (D) a *catnip* is not in the *cat;* catnip is strong-scented mint. (E) *dice* is not in the *diversion;* dice are used for the purpose of diversion. (A) is the right answer, because an *airplane* can fly in the *troposphere*. Even if you did not know that the *troposphere* is the portion of the atmosphere that extends outward about 7–10 miles from the earth's surface, you should have been able to deal with this word because of its root, *sphere,* which suggests some kind of area designation and, therefore, the likely answer. Besides, the others have been eliminated as not possible.

4. (B) This could be tricky, since most people associate the word *louse* with a kind of person and so may make up a sentence such as: *A person is a louse.* If you did that, then you probably eliminated (A), (B), (C), and (D), and chose (E) as the correct answer because (A) an *army* is not an *enemy;* (B) a *tick* is not a *dog;* (C) a *bee* is not a *flower;* and (D) a *cat* is not a *bird*. The problem with (E) is that *louse* is a word that *describes* a person, but an *aphid* is an insect, like an *ant*. If you thought that might be the case or had some doubts about locking in with (E) as your answer, you might have gone back to the original analogy and reworked your sentence to read: *A louse is a parasitic insect that can be found on a person.* The answer would be (B), because a *tick* is a parasitic insect that can be found on a *dog*. (C) a *bee* can be found on a *flower,* but the relationship is not as good, because a bee is not a parasite.

5. (E) This question should have been easy, since most people know that: *A quilt covers a bed.* Using this as your sentence, you would be able to eliminate (A) and (B), since (A) a *guest* does not cover *mean;* and (B) a *torch* does not cover a *flame*. If you did not know that (C) a *stigma* is an identifying brand or mark of shame, you might have left this answer choice for the moment. (D) *succor* means "help" or "relief"; if you didn't know that, still, the word *help* is an abstract idea and is not likely to be covered. The best answer is, of course, (E): a *glove* does cover a *hand*. You certainly know these words, and they make a convincing choice, so you should take (E) even if you do not completely understand (C) or (D).

6. (D) If you do not know what a *calligrapher* is, your choices should provide you with a clue. There are two that you should be able to deal with: (D) it is possible that a *stenographer* uses a *typewriter;* but (E) an *astronaut* does not necessarily use a *wheelchair*. Perhaps, then, a *calligrapher* is someone who uses a *quill*. You can also see the root, *-graph-*, in the word—it means "write," which should give added support to your sentence. If you use the sentence *A calligrapher uses a quill,* you can eliminate (A), (B), (C), and (E) immediately. You are then left with (D), the only possible answer.

7. (D) This will be a difficult analogy if your vocabulary is weak. If you have trouble with the word *vociferous* you should move to the choices to see if they can provide you with some clues. Choices (B), (C), and (D) should provide you with some help, since each analogy deals with words that are similar. (D) is different from the other two, since *blinding* is more intense than *bright*. You may assume then, that *talkative* and *vociferous* mean the same thing. If you look at the first few letters of *vociferous,* you may recognize the beginning of the word "vocal," which has to do with speech. Then if you know that the suffix *-ous* means "full of," the word *vociferous* becomes easy to define; it simply means "full of talk"—that is, especially talkative. Now you have an analogy that is asking for synonyms as an

answer. Whenever you have synonyms one of the things you should be looking for is the *degree of intensity*—one word may be more intense than the other. If you know that both words in (A) mean "caustic" or "biting" and both words in (E) mean "adjacent" or "near to," the only possible answer is (D), because it is testing the difference in *intensity* between the two words. Your original sentence could thus have been: *Vociferous is more intense than talkative*. And therefore, (D), *blinding* is more intense than *bright*.

8. (E) Again, looking over your choices for some clue as to the meaning of the word *portal*, assuming you do not know it means a very large and imposing door, you should have no trouble recognizing that you are again dealing with synonyms. You should know enough now to ask yourself if the question is testing the degree of intensity between the two words. If it is, very often it is the word you are *less familiar with which is the more intense word*. So why not try that out in your sentence? *A portal is a large door* is the sentence you would be most likely to come up with. You would then be able to eliminate (A), because a *container* is not necessarily a large carton; (B), because *gruel* is just another word for *porridge;* (C), because a *hallucination* is a *delusion*—there is no question of intensity here; and (D), because a *hiatus* is a *gap*, and again there is no difference of intensity. (E) is the correct choice, because an *abyss* is a very large *hole*.

9. (D) If you do not know that a *peccadillo* is a slight offense, chances are that you know that a *felony* is a serious crime. Therefore, you will look at the choices for clues to the relationship being tested. Even if you're not sure of the meanings of all the words in (C) and (E), the other choices, (A), (B), and (D), should provide you with the information that you need—namely, that you are dealing with synonyms. Again, you should assume first that *peccadillo* and *felony* differ in intensity. Your sentence would then be: *A felony is greater than a peccadillo*, because you know that a felony is serious. That would eliminate (A), because a *bedspread* is not greater than a *quilt*, they are two different things; and (B), because *significance* is not greater than *meaning*, they are really synonyms. Left with choices (C), (D), and (E), and not knowing that *immolation* means a "sacrifice" and *imprecation* means "curse," your best choice would be (D), since a *sea* is greater than a *puddle*.

10. (C) *Something which is elastic is pliable* is probably something like the sentence you used. You are probably also aware of the fact that *elastic* and *pliable* do *not* differ significantly in intensity. That should make it easy for you to eliminate (A), since something which is *ventral* is not *dorsal*—*ventral* means "pertaining to the stomach" and *dorsal* means "pertaining to the back." (E) contains opposites: something which is *confused* is not *stable*. (D) also contains opposites; *anorexic* means "without appetite." Even if you do not know the meaning of (B) *vernal* ("relating to the spring") and *hibernal* ("relating to the winter"), the better choice would be (C), because you do know that something which is *valiant* is *brave*.

If You Do Not Know Either of the Words

Examine the following analogy question, noting the relationship between the first two words:

INTROVERT:EXTROVERT:: (A) withdrawn:outgoing
(B) loquacious:talkative (C) injurious:jinxed
(D) despondent:depressed (E) arrogant:bumptious

Make a specific sentence using both *introvert* and *extrovert* and write it in the space below, *before* reading further.

Are you having difficulty with the words? Is it because you don't know what they mean? Well, you don't have to. Here is a perfect example of determining whether the words are positive or negative simply by examining the prefixes.

Ex- means "out of" and *in-* means "into." And you could have figured that out by thinking of words beginning with those prefixes and thinking about what they mean—words such as "introduce," "invite," and "intramural," and "extravagant," "expel," and "extract." What is the difference between the words beginning with the prefix *ex-* and those beginning with the prefix *in-*? Do they have positive (good) or negative (bad) associations? Now that you have thought about the nature of words beginning with *in-* and *ex-*, you are ready for your sentence. Try to make one, writing it in the space provided above.

You probably arrived at the most natural way to deal with an analogy of this type: *If I am an extrovert, I am not an introvert*, or *An extrovert is the opposite of an introvert*. Now go back to the question and substitute the choices in your sentence, eliminating those that are clearly not possible. *Do it now without looking ahead*. The question should now look like this:

INTROVERT:EXTROVERT: : (A) withdrawn:outgoing
(B) loquacious:talkative (C) ~~injurious:jinxed~~
(D) ~~despondent:depressed~~ (E) arrogant:bumptious

You probably eliminated (C) and (D) immediately. You know that both words in (C) have negative connotations—in other words, *injurious*, which means "tending to inflict injury," is similar to *jinxed*, which means "tending to bring bad luck." They are certainly not opposites. You probably also recognized that the prefixes of the two words in (D) are similar; therefore, you are probably not dealing with opposites here either (though *de-* can be a positive prefix, as in "delightful"). In fact, *despondent* means "hopeless."

Choice (E) should not have presented a problem, because the word "bump" in *bumptious* should give you an obvious clue that it is a word with negative associations and therefore it is unlikely that it is an opposite of the word *arrogant;* in fact, the words mean much the same thing. Left with choices (A) and (B) and not having many clues to go on as to the meaning of *loquacious*, you still might naturally select (A), since it is an obvious contrast. However, it is possible to deal with *loquacious* in two ways. The first is if you know that the root *loq-* means "talk." If you are not familiar with that root, it is still possible to work with the suffix *-ous*, which means "full of." Since *talkative* means "full of talk," there is a strong indication that *loquacious* is a synonym, not an antonym.

This is a good time to expand on the concept of positive and negative words. Positive words generally have good connotations; negative words generally have bad connotations. You can usually determine which type you are dealing with by the prefixes, such as: *retro-*, "backward"; *dis-*, "apart"; *ex-*, "out of." These are essentially negative words, while *pro-*, "forward"; *syn-*, "with"; *ad-*, "forward" are essentially positive words (at least for our purposes, since there will obviously be significant exceptions to the suggestions developed in this book to help you make "educated guesses" on the SAT).

You will also have to make intuitive decisions. Trust your instinct as to whether a word is positive or negative. A word like "irascible," for example—how does it sound to you? Good or bad? Positive or negative?

Words are built on foundations. It is not an accident that "disturb" and "dissatisfy" begin with the same prefixes. So another important technique for you to utilize in trying to define a word is to see if you can think of words beginning with

the first few letters. In the case of "irascible," such words include "irritate" and "irate." These are all negative words.

Another technique you might try is putting the word in some context where you may have heard it before. Consider the word "quixotic." There is an entire range of associations, which could lead you to the meaning of the word. Do you remember Don Quixote in the Broadway show *Man of La Mancha?* The hit song from that show was *The Impossible Dream*—the hero was dreamy, or more specifically, impractically idealistic. And that is just what "quixotic" means.

Needless to say, the more roots you know, the easier time you will have identifying words. For example, the root *-lev-* is the basis for a great many words that have the sense of "lighten," such as "levity," "elevator," and "levitation." What is most important is to utilize all the possible techniques available to help identify the correct answer.

PRACTICE EXERCISE

Here are some analogies for you to practice on. Each question below consists of a related pair of words or phrases. Practice making specific sentences. In many of the original analogies, you may not know the meanings of the words being tested, but you may find some clues as to how best to deal with the relationship by examining the prefixes of the words. The first few letters can also offer you information about its definition.

1. IMPERTURBABLE:DISTRAUGHT::
   (A) palpable:perceptible
   (B) comprehensible:inconceivable
   (C) ambivalent:ambiguous
   (D) evanescent:ephemeral (E) fulgent:radiant

2. EXTERN:INTERN:: (A) hoyden:hussy
   (B) extirpation:implantation
   (C) stipend:scholarship
   (D) feculence:fetidness
   (E) impartiality:objectivity

3. DISCREET:INJUDICIOUS::
   (A) perturbed:agitated
   (B) pertinacious:obstinate
   (C) polluted:corrupt (D) qualified:competent
   (E) inhibited:exhibitionist

4. VERACITY:IMPOSTURE::
   (A) precision:inaccuracy (B) hill:mound
   (C) perjury:falsehood (D) childbirth:delivery
   (E) chief:leader

5. SUPERNUMERARY:INSUFFICIENT::
   (A) bombastic:pompous
   (B) lachrymose:tearful
   (C) sophomoric:immature
   (D) superfluous:inadequate
   (E) valance:drapery

6. PERSEVERING:PERSISTENT::
   (A) prodigal:squandering
   (B) inherent:acquired
   (C) mediocre:outstanding
   (D) popular:aristocratic (E) heavenly:infernal

7. VERACIOUS:VERIDICAL::
   (A) languid:active (B) tortuous:cruel
   (C) transient:lasting (D) prolific:fecund
   (E) trite:original

8. PROXIMITY:PROPINQUITY::
   (A) garb:attire (B) recession:promulgation
   (C) reticence:conviviality (D) rift:juncture
   (E) revelry:sobriety

9. CENSURE:ACCLAIM:: (A) slattern:slut
   (B) precedence:anteriority
   (C) precursor:herald (D) obduracy:concession
   (E) sequel:supplement

10. VARIEGATION:COLOR::
   (A) surveillance:liberty (B) tirade:speech
   (C) medley:music (D) harangue:discourse
   (E) democracy:absolutism

ANSWERS AND EXPLANATIONS

1. (B)	2. (B)	3. (E)	4. (A)	5. (D)
6. (A)	7. (D)	8. (A)	9. (D)	10. (C)

1. (B) It should be apparent to you that the word *distraught,* which means "harassed," is a negative word. Not only does the prefix *dis-* mean "apart from," but the word sounds like the word "distressed," which is still another meaning. With the word *imperturbable* you must tread cautiously. You should recognize the word "perturb," which even sounds like the word "disturb," which is its definition. The prefix *im-* means "not"; hence the word means "not perturbable." Therefore, you are dealing with antonyms. Your sentence may read: A *person who is imperturbable is not distraught.* You should then have been able to eliminate (A), (C), and (E) without too much difficulty. (A) *palpable,* which is related to the word "palpate," means "tangible" or "able to be touched." (Notice the first three letters of the word "palm," which happens to be the part of the hand that does the touching.) *Perceptible* means nearly the same thing, "able to be recognized or perceived." Therefore, something which is *palpable* is *perceptible.* They are obviously not the antonyms you are looking for. In (C), even if you did not know that *ambivalent* means "uncertain" or "confused" and that *ambiguous* means "uncertain" or "vague," the prefix *ambi-* should make it easy since you know that you are probably not dealing with antonyms; *ambivalent* is not the opposite of *ambiguous.* (E) *radiant* should have presented no problem since you know the word means "shining brightly." Chances are you did not know that the word *fulgent* also means "shining" or "bright," but certainly the letters *ful-* should have given you some indication that you were dealing with a positive word, and that *radiant* and *fulgent* are not antonyms. With choices (B) and (D) remaining, you no doubt realize that (B) is the correct answer, because *comprehensible* means "able to be understood" and *inconceivable* means "not able to be understood"; therefore, something that is *comprehensible* is not *inconceivable.* You could have made the correct choice even if you do not know that in (D) the words *ephemeral* and *evanescent* both mean "short-lived."

2. (B) This analogy should have presented you with little difficulty, since you know that the prefix *ex-* and the prefix *in-* suggest that you are looking for antonyms. Your sentence was probably some variation of: *Extern is the opposite of intern.* Your probably eliminated (A), (C), and (E). In (A), you may have had a hunch that *hoyden* and *hussy* are synonyms, not antonyms; they not only look similar but both sound negative, even if you don't know that both mean "boisterous girl." In (C), most students know that a *stipend* is similar to a *scholarship;* both are sums of money that are paid regularly. (E) *impartiality* and *objectivity* also mean the same thing. Left with (B) and (D), it would be surprising if you did not take (B), since you are again dealing with contrasting prefixes, which suggest antonyms even if you don't know that *extirpation* means "rooting out" and *implantation* means "planting firmly"; *extirpation* is the opposite of *implantation.* You are right if you assume that *feculence* and *fetidness* suggest synonyms simply from the similarity of the initial letters and the way the words sound (both mean "smelly" or "foul").

3. (E) You probably know the definition for *discreet,* which is "cautious" or "having good judgment" (as distinguished from the other "discrete"—notice the spelling—which means "separate"). If *injudicious* presented you with a problem,

take a closer look at the word. You should recognize the beginning of the words "judge," "justice," and "judicial," all referring somehow to "judgment." The prefix *in-*, however, makes the word negative: "not having good judgment." So you are obviously dealing with antonyms. That should have eliminated (A), (C), and (D), because (A) someone who is *agitated* is *perturbed;* (C) something which is *polluted* is *corrupt;* and (D) someone who is *qualified* is *competent.* Left with choices (B) and (E), your natural selection was no doubt (E), since regardless of any other consideration, the prefixes *in-* and *ex-* suggest that you are dealing with opposites. Perhaps you knew the meaning of *pertinacious,* which is quite close to *obstinate*—so this pair are not opposites. The best answer, then, is (E), since someone who is *inhibited* is not an *exhibitionist.* Also, *discreet* and *inhibited* both suggest refraining from some action; *injudicious* and *exhibitionist* both suggest engaging in some probably foolish action.

4. (A) Most students are familiar with the words *imposture,* which means "fraud" or "deception," and *veracity,* which means "truthfulness." Your sentence was probably some variation of: *Veracity is the opposite of imposture* or *Veracity means "truth" and imposture means "falsehood."* You should have quickly eliminated (B), (C), (D), and (E). (B) a *hill* is not the opposite of a *mound;* a hill is a mound. (C) *perjury* is not the opposite of *falsehood; perjury* means "lying" and is therefore a synonym. (D) a *childbirth* is a *delivery.* (E) a *chief* is a *leader.* (A) is obviously the correct answer since it is the only pair of antonyms; *precision* is the opposite of *inaccuracy.*

5. (D) It is fairly easy to deal with this analogy by breaking down the word *supernumerary* in terms of its relationship with *insufficient.* The prefix *super-* means "going beyond" and you should have had no trouble recognizing the word "number." Thus the definition is simply "going beyond the number" or "more than enough," which contrasts with *insufficient,* which means "not enough." Again we are dealing with antonyms.

Please make a note. It is a good idea to use your analogy with the *definitions of the words* in addition to a general sentence such as: *Supernumerary is the opposite of insufficient.* Your sentence should read: *Supernumerary means more than enough and insufficient means not enough.* You can eliminate (A), (C), and (E), since: (A) *bombastic* and *pompous* are synonyms; (C) *sophomoric* and *immature* are synonyms; and (E) an analogy with *drapery* could hardly have anything to do with "more than enough" and "not enough," and besides, a *valance* is something that a *drapery* is hung behind, not an opposite. If you did not know that (B) *lachrymose* and *tearful* are synonyms, you might have had a problem. But probably you eased yourself into the obvious answer, which is (D). It fits so nicely; *superfluous* means "going beyond the necessary" and *inadequate* means "less than necessary."

6. (A) It is obvious that *persevering* and *persistent* are synonyms if just from the prefix alone. Therefore, you probably eliminated (C), (D), and (E) without any difficulty. (C) *mediocre* is not the same as *outstanding; mediocre* means "ordinary" and is therefore the opposite. (D) *popular* is not the same as *aristocratic,* since *popular* refers to the masses of people and *aristocratic* refers to the upper class. (E) *heavenly* is not the same as *infernal;* they are opposites. The remaining choices, (A) and (B), might present a problem, since it is likely that you know the meaning of the word *squandering* ("wasteful" or "extravagant") and the word *acquired* ("obtained"), but you may not know the meaning of the other words. If you look at the word *inherent,* you will note the prefix *in-*, which means "into." You may also recognize it as related to the word "adhere," which means "stick to" (like adhesive tape). Chances are you worked out the meaning of *inherent* which is "existing in someone or something as a natural quality"—that is, not *acquired,* which sug-

gests something added on. Since analogy (B) is based on antonyms, the right answer is, of course, (A), and in fact *prodigal* means "wasteful," which is the same as *squandering*.

7. (D) You may have nothing more to go on except the hunch that two words beginning with *ver-* are synonyms. Your sentence probably is: *Veracious is the same as veridical*. You probably eliminated (A), (C), and (E) immediately because (A) *languid* means "sluggish" and *active* is just the opposite; (C) *transient* means "temporary" and *lasting* is just the opposite; and (E) *trite* means "commonplace" and *original* is just the opposite. Of your two remaining choices, (D) may have stumped you completely, unless you know that *fecund* means "productive" or "fertile." Then, without knowing that *prolific* means "fruitful" or "producing many of a kind," you might consider the prefix *pro-*, which means "forward," and take a chance on (D) as your answer, which would be right. If you did not know what *fecund* meant, you might have turned to (B) as your first consideration. Take a closer look at the first word. In all likelihood you read this word as a form of the word "torture," which is a common mistake. Look again. The word is not "torturous," which means "causing torture." You are dealing with a completely different word, *tortuous*, which means "winding" or "twisted." Therefore, what you have in analogy (B) are actually two totally unrelated words. You have therefore backed yourself into the right answer, which is (D).

8. (A) Again the *pro-* prefix in the original analogy should have clued you in to the fact that you are looking for synonyms. Both the words *proximity* and *propinquity* happen to mean "nearness." *Proximity is the same as propinquity*. You were probably able to eliminate (B) and (D) without much difficulty. In (B), without knowing that *recession* means "going backward" and that *promulgation* means "announcing" or "making known," you are aware of contrasting prefixes; thus *recession* is not the same as *promulgation*. In (D), most students know that a *rift* means a "split." It is obvious that a *rift* is not a *juncture*. If you were not sure of the word *juncture*, notice how much it is like "junction"—which is what it means. Left with three choices, (A), (C), and (E), you might be one of those fortunate ones who know that the words *garb* and *attire* mean the same thing and are therefore the synonyms you are looking for. But what if you do not know those meanings? There are still one or two clues left to enable you to get closer to the answer. Take another look at choice (C). The prefix *re-*, which means "back" or "again," generally has negative connotations, and the prefix *con-*, which means "with," generally has positive connotations. That might suggest that you are dealing with antonyms. You might also notice that the familiar word "vivacious" seems to have quite a lot in common with the word *conviviality*. Most students know that "vivacious" means "lively"; others might recognize the root *-viv-*, which means "live." The word *conviviality* literally means "festivity." As for *reticence*, you might not only sense negative connotations because of the prefix, but the word itself suggests others, such as "retire," which is certainly not an "up" word. (C) should therefore be eliminated, since *reticence* is not the same as *conviviality*. The word *sobriety* in choice (E) should immediately suggest the word "sober," and similarly, *revelry* should suggest "revel." These words are clearly not the same. So probably you backed into the right answer, which is (A): *garb* is the same as *attire*.

9. (D) This analogy should not have presented a problem. If the word *censure* is unfamiliar, it should at least have reminded you of the more common word "censor," which is someone who removes or prohibits anything that is unsuitable. The negative connotations are easily apparent; the word *censure* means "condemnation." In *acclaim*, most students would recognize the word "claim" and the pre-

fix *ac-*, which is a variation of *ad-* and has positive connotations. The word *acclaim* literally means "shouted approval." What you are dealing with, then, are antonyms. Your sentence is probably: *Censure is not acclaim*. You probably eliminated (A), (B), and (E) immediately. (A) a *slattern* is a *slut;* they both describe women who are careless and sloppy in appearance and character. At any rate, in sound and form the words are highly similar and you might have taken a chance and eliminated this choice. (B) *precedence and anteriority* are both words that deal with the state of being before in time or place; you would have known that from the prefixes *pre-* and *ante-*, which both mean "before." (E) *sequel* and *supplement* are both words that deal with the aftermath or what follows. You are then left with choices (C) and (D), probably because you did not know the meaning of the words (C) *herald* and (D) *obduracy*. You probably had little difficulty with the word *precursor* because of the prefix *pre-*, which again implies something which comes before. You are also probably familiar with the meaning of *concession,* which is a yielding or giving in. Can you think of any word beginning with *ob-* that means "not yielding" or "holding out"? What about the word "obstinacy"? The definition for "obstinacy" is "stubbornness," and that is just what *obduracy* means. The prefix *ob-* is generally a negative prefix and means "against." Think of other words that begin with *ob-*, such as "obstruct" and "obstacle." You might have arrived at the fact that this is the choice you are looking for; *concession* is the opposite of *obduracy*. In (C), the word *herald,* someone who proclaims or foretells, is one of those words you just have to know. Therefore, what you have in (C) are synonyms, and therefore eliminated.

10. (C) This initial analogy should not have presented too much difficulty, although the choices might have been more difficult. The word *color* is simple, and the word *variegation* is just as simple if you study it for a moment. The word that you should recognize in *variegation* is the word "variety"; in fact, variegation means a "variety in appearance." Your sentence is probably some form of the following: *Variegation means many colors*. That would eliminate (A), (B), (D), and (E). (A) a *surveillance* is not many *liberties; surveillance* means "watch" or "observation." (B) a *tirade* is not many *speeches;* a *tirade* is a long speech. (D) a *harangue* is not many *discourses;* a *harangue* is a windy speech. (E) *democracy* is not many *absolutisms;* they are opposed concepts. The correct answer is (C), since a *medley* is many pieces of *music*. Think of the word *medley* for a moment. Certainly you recognize the word "meddle," which means "to mix in," and that is exactly what we call a variety of different musical compositions thrown together—a mix.

READING COMPREHENSION

The basic skill required for college work is reading comprehension. The invention of the printed word is the key device by which our culture has most successfully transmitted its knowledge from generation to generation. There is no avenue of study or course of inquiry that does not depend upon your ability to understand your reading.

For this reason, the largest portion of the Scholastic Aptitude Test is devoted to reading comprehension. The reading-comprehension test *does not* evaluate the candidate's *knowledge* of subjects included in the secondary-school curriculum. It seeks, rather, to *measure the skills* he will need in order to read with understanding, at a college level. These include accurate and precise perception, and the ability to reason and draw inferences, to sense subtle nuances of thought, to weed out irrelevancies, and to make judgments, all within a reasonable framework and close to what may be required for college reading.

It is important that you master the technique of dealing with the reading-comprehension part of the test because it is the section of the test that depends least on your knowledge of vocabulary. What is even more in your favor is that *every answer you need will be built right into the passage*. You are given all the information you need. It is just a question of learning how to read each passage and of familiarizing yourself with the technique of digging out the answer.

Many workbooks spend time teaching various methods of how to read a passage. There is *only one way* to read an SAT passage. The writers of these pieces deliberately set out to do two things: confuse you with their rhetoric and bore you with the dry and tedious nature of the subject matter.

But you are going to learn how to simplify the confusion and overcome the difficulty that most students have: they remember little, if anything, they have read after only four or five lines into the passage. You are going to have the decided advantage of knowing that when you have finished the reading selection you are not only going to understand what you have read, you will know where every answer can be found.

It is important for you to keep in mind that the techniques for the reading comprehension passages involve detailed notation. Unless you master these techniques PRIOR to the Scholastic Aptitude Test, the process may exceed the time allotted for each passage. However, if you practice these techniques faithfully, you will find that when you take the actual SAT, you will automatically read with greater speed and understanding, while minimizing the need to outline your passage in such great detail.

Techniques of Reading Passages

Step 1

Following the sample passage below, there are three questions. Read the *questions* first, *not the answers*, and underline any words that you think will enable you to recall the entire question by just looking at those words. Those words you underline are called "key" words. Try it now.

The decade of the twenties, or more precisely the eight years between the postwar depression of 1920–21 and the stock-market crash in October of 1929, were prosperous ones in the United States. The total output of the economy increased by more than 50 percent. The preceding decades had brought the automobile; now came many more automobiles and also roads on which they could be driven with reasonable reliability and comfort. The downtown section of the mid-continent city—Des Moines, Omaha, Minneapolis—dates to those years. It was then, more likely than not, that what is

still the leading hotel, the tallest office building, and the biggest department store went up.

These years were also remarkable in another respect. For as time passed, it became increasingly evident that the prosperity could not last. Contained within it were the seeds of its own destruction. Herein lies the peculiar fascination of the period for a study in the problem of leadership. For almost no steps were taken during these years to arrest tendencies which were obviously leading, and which did lead, to disaster.

1. The main idea of this passage is best expressed as
   (A) the fabulous twenties
   (B) prosperity and decay in the twenties
   (C) the problem of leadership in the twenties
   (D) the decade of the twenties as a picture of doomed prosperity
   (E) prosperity as reflected in urban development
2. The word "remarkable" as it is used in the first line of the second paragraph most nearly means
   (A) catastrophic
   (B) ironic
   (C) distinctive
   (D) prosperous
   (E) problematic
3. The author implies
   (A) that unchecked prosperity is likely to lead to bad results
   (B) that during this period of prosperity all portents of disaster were ignored
   (C) that the germinating qualities which lead to success eliminate the fear of failure
   (D) that the economy more than doubled during the period which preceded the decade of the twenties
   (E) that the twenties ushered in the beginning of what is now familiarly known as urban renewal

Your questions probably look like this:

1. The main idea of this passage is best expressed as
2. The word "remarkable" as it is used in the first line of the second paragraph most nearly means
3. The author implies

In Question 1, the important thing you want to remember is that you are being asked to find the *main idea* of the paragraph.

In Question 2, you want to keep in mind, if you can, that you are going to be asked to define the word "remarkable," as it is used in the context of the sentence.

In Question 3, you are asked to locate something the author suggests. This is a familiar type of question. Again, the answer will be *stated within the passage*.

Step 2

Go back to the paragraph and read the *first* sentence. Underline any key words. Use the same reasoning process for selecting the words to be underlined as you did with the questions at the end of the passage. Do it now, before reading further.

Your sentence should look something like this:

> The decade of the twenties, or more precisely the eight years between the postwar depression of 1920–21 and the stock-market crash in October 1929, were prosperous ones in the United States.

You have no doubt singled out the above words to underline, since they possess important information; that is, that the "decade" of the twenties, which the writer reduces to an eight-year period, was a prosperous time in the United States.

There is one thing that you should keep in mind: There is no single correct way to underline, so do not be overly concerned about whether or not you are selecting the right words. You are doing something that is far more important. You are

thinking about what you are reading. In addition, you are underlining words that will enable you to recall the meaning of a sentence even after a lapse of many weeks. This is a technique you will use to advantage in college, where you may often have to review hundreds of pages of reading for an exam. Thus you are participating in a threefold process. First, you are thinking about your choice; second, you are making it possible to remember huge chunks of material by a single glance at your underlined words; third, by actually doing the underlining, you are helping to reinforce, in your mind, the ideas you have just read.

There are many students who find themselves completely detached from what they are reading, and since the SAT passages are designed to permit such detachment, this is not surprising. Students complain that the passages are boring and that there is no way they can relate to them. These students have not learned to underline and do not become *active* and *involved* participants. As a result, they constantly report that they can read and reread the same passage without having the slightest idea of what it is about. But you must persist. It is a slow process in the beginning but, in time, you will find yourself in the enviable position of being able to zigzag through the paragraphs with the ease of a slalom racer.

Step 3

Go back to the *first* sentence and *circle* those underlined words which you feel are *more important,* in terms of what the sentence means. Do it now before reading further.

Your sentence probably looks something like this:

> The decade of the twenties, or more precisely the eight years between the postwar depression of 1920–21 and the stock-market crash in October 1929, were prosperous ones in the United States.

If you are like many other students, you will have a tendency to underline many words in a sentence when you read it for the first time, and that is as it should be. There are many key words in a sentence that carry important information. Yet, if we want to crystallize the ideas conveyed in those underlined words, it is often possible to find several that seem to summarize the whole point of the sentence. You probably realize that the main idea the author of this sentence is trying to convey is, simply, that the decade of the twenties was a prosperous period in the United States.

Before we go on to Step 4, go back to the *second sentence* in the paragraph and follow Steps 2 and 3. Underline the key words in the sentence and circle those underlined words which you feel are more important in terms of what the sentence means. Do it now before reading further.

Your second sentence probably looks like this:

> The total output of the economy increased by more than 50 percent.

You may have circled and underlined slightly differently, but the important thing is that although your underlining and circling will not always be the same as our examples, you are following similar processes and for the same reason: to condense the important ideas of each sentence in consecutive stages.

Step 4

This is one of the most important steps in mastering the reading-comprehension technique. Although it is particularly applicable to the SAT passage, proficiency in this area will make you a more discerning reader.

Determine what the second sentence has to do with the first sentence. If it is a *new*

idea or there is a *change in point of view*, put // (double lines) wherever the change occurs. It may happen between sentences or between words in the sentence. Go back to the first and second sentences and decide what the relationship is between the two and whether or not you will use // double lines. Do it now before reading further.

You probably decided against using double lines, and for good reason. The second sentence is an elaboration of the first. It gives support to the original idea that the twenties was a prosperous decade by citing what percent the economy increased.

Step 5

Whenever you come across connecting words or phrases, such as "furthermore," "on the other hand," "in addition," and "but," etc., be sure to circle them *immediately*. These words will indicate whether or not the author is moving on to a completely new idea, or whether he has switched his point of view.

Before we go on to Step 6, go back to the passage and read the third sentence. Follow Steps 2, 3, and 4. Please do it now before reading further.

This is the way your sentence probably looks:

// The preceding decades had brought the automobile; // now came many more automobiles and also roads on which they could be driven with reasonable reliability and comfort.

The reason for the double lines before the sentence is that the author is moving to a new idea: He reaches back to the previous decades to discuss the introduction of the automobile. However, you could just as well have omitted the double lines, deciding that this sentence is another more specific elaboration of the one preceding it. The reason for the double lines between *automobile* and *now* is that the author is comparing the previous decades to the decade of the twenties—whereas there were some automobiles before, the prosperous twenties saw a tremendous increase in their number.

Step 6

If the author lists examples in his paragraph, indicate each one in the following way: Ex. 1, Ex. 2, Ex. 3, etc.

Go back to the paragraph and read the fourth and fifth sentences. Follow steps 2, 3, 4, 5, and 6 wherever possible. Please do it now before reading further.

This is probably the way your third, fourth, and fifth sentences look:

// The preceding decades had brought the automobile; // now came many more automobiles and also roads on which they could be driven with reasonable reliability and comfort. The downtown section of the mid-continent city—Des Moines, Omaha, Minneapolis—dates to these years. It was then, more likely than not, that what is still the leading hotel, the tallest office building, and the biggest department store went up.

When you have finished reading an SAT passage, it is wonderfully reassuring to

look back and see that a major part of the reading is simply a neat succession of examples. It makes it much easier to locate the answers to the questions, because you are secure in your knowledge about the information in at least one section of the passage. The six examples in this paragraph are illustrations to support the idea that the twenties were, indeed, a time of prosperity.

However, not every SAT passage will include statements that you can call examples—some passages will simply read straight through, keeping very close to the subject or using generalities rather than specific examples. It is still worthwhile to try Step 6, though, because when you discover that there *are* no examples, you will automatically have increased your comprehension of the passage and of the way the writer has organized his statements.

Now go back to the passage and read the first sentence of the second paragraph. Follow Steps 2 through 6 wherever possible. Please do it now before reading further.

This is the way your sentence probably looks:

> // These years were also remarkable in another respect, for as time passed, it became increasingly evident that the prosperity could not last.

Although it would appear that this sentence is a continuation of those that precede it, describing another aspect of prosperity, we soon realize that the word *another* takes on the meaning of "different" or "contrasting," hence the double lines before *these*. The point of this sentence is that there was increasing evidence that the prosperity could not last.

Now go back to the passage and read the *second sentence* in the second paragraph. Follow Steps 2 through 6 wherever possible. Please do it now before reading further.

Your sentence probably looks like this:

> Contained within it were the seeds of its own destruction.

This sentence is, obviously, a continuation of the idea that just preceded it. This sentence gives the *reason* why the prosperity could not last: It was somehow unsound and would destroy itself.

Now go back to the passage and read the *third sentence* in the second paragraph. Follow Step 2 through 6 wherever possible. Please do it now before reading further.

Your sentence probably looks like this:

> Herein lies the peculiar fascination of the period for a study in the problem of leadership.

In this sentence the author shifts to a different aspect entirely—that is, the attraction this period has for someone studying the problem of leadership.

Now go back to the passage and read the *last sentence* in the second paragraph. Follow Steps 2 through 6 wherever possible. Please do it now before reading further.

Your sentence probably looks like this:

> For almost no steps were taken during these years to arrest tendencies which were obviously leading, and which did lead, to disaster.

It should be obvious to you that the concluding sentence is a continuation of the idea in the one that preceded it. This sentence indicates what the particular problem of leadership was—that there was none. No one assumed the responsibility of preventing disaster.

Now that you have concluded diagramming the passage, take a long, hard look at what you have outlined. You will not only have little difficulty reassembling the various parts of the passage into a cohesive whole, but you should have a full understanding of what the paragraph means.

Step 7

There is one final technique in reading comprehension. Look over the passage and place an asterisk * before the main idea. There is an easy way to determine what the main idea is: If by reading the sentence you have selected you have a fairly good idea of what the *entire* passage is about, that is probably your best choice.

Go back to the paragraph and place an * before the sentence you think is the main idea. Please do it now, before reading further.

You probably selected the first sentence because of the reason you were asked to keep in mind: The first sentence tells you what the entire paragraph is about; even the content of the second paragraph—about the collapse of prosperity—is foreshadowed by the mention of the stock-market crash at the end of the prosperous period. Let's put it another way: Every sentence in the next two paragraphs relates in some way to the idea that the decade of the twenties was a prosperous period in the United States.

But do not be misled. *The main idea may occur anywhere in the paragraph.*

How to Answer "Main Idea" Questions

Now we are ready to discuss the SAT questions. Follow the directions carefully and soon you will feel confident of your ability to deal with this part of the test.

Most of the SAT paragraphs have, as one of their questions, a title question, which deals with the *main idea* of the paragraph. There are always five choices from which to select your answer, but *the title question has a specific variety of choices which none of the other questions has.* The choices for the title question generally fall into the following categories:

1. *One answer may be too broad.* That will be an answer which is very general, going much farther than the statements in the passage itself.
2. *One answer may be too narrow.* This answer is easy to eliminate because it is a statement that would hold true for only one or two sentences in the passage.
3. *One answer may be irrelevant.* It will have nothing whatever to do with the passage you have just read.
4. *One answer may be out of context.* Actual words are lifted from a sentence in the passage and used as an answer. That is presumably to throw the student off guard. He may remember having seen the statement somewhere in the passage and assume, too readily, that it is the right answer. That is exactly the trap the testers have in mind.
5. *One answer is the correct one.* This is not as obvious as you may think. It must fulfill two requirements. It must *deal with most of the sentences* in the passage and it must *be specifically related* to the subject being discussed. That means if five sentences deal with one idea and six sentences deal with another idea, you must choose that answer which deals with the largest number of sentences.

You may not have one answer of each type—you may have two choices that are too narrow or two choices that are out of context, or any kind of combination. But whatever the combination is, these are the only types that will be used.

You are to work through the choices by drawing a line through them one at a time, eliminating those answers that are *not possible*. Look at the title question; then look over each choice and decide whether it is: 1, too broad; 2, too narrow; 3, irrelevant; 4, out of context; or 5, possibly the right answer. *At any time, if you think the answer is possible or you are not sure, then leave it. But draw a line through all of the other choices as you dismiss them.*

Now go back to the paragraph and read the first question. This is the title question. Follow the instructions on how to narrow down your choices. Until you have internalized your knowledge of the various types that may be included in the title question, refer back to your list. Please do it now, before reading ahead.

Chances are your question looks like this:

1. The main idea of this passage is best expressed as
   (A) the fabulous twenties
   (B) ~~prosperity and decay in the twenties~~
   (C) ~~the problem of leadership in the twenties~~
   (D) the decade of the twenties as a picture of doomed prosperity
   (E) ~~prosperity as reflected in urban development~~

You probably eliminated (B), (C), and (E) without difficulty. (B) is *irrelevant* because there is no direct mention of decay in the passage. (C) is both *out of context* and *too narrow* because it is only mentioned in one sentence. It does seem possible that "leadership in the twenties" would *become* the main topic *if the passage continued,* because the writer has carefully led up to the statement about leadership. But you cannot make assumptions of this kind; *all the answers are somewhere in the passage.* (E) is too narrow because urban development is mentioned in only two sentences; it is used as an example of prosperity but is not the main point. You probably left (A) because as you went through your choices it seemed possible. If one considers the great prosperity of this decade as "fabulous," then this would seem like a reasonable answer. You probably left (D) because the passage does describe the twenties as a picture of prosperity.

If you are left with a choice of two possibilities, chances are that one of these choices is *too broad;* you probably left that answer as a possibility not only because it seemed possible, but because you didn't yet know if any other answer would be more specific. Now go back to choices (A) and (D) and select the answer that more specifically relates to the paragraphs as a whole. Please do it now before reading further.

The answer, of course, is (D), since it is the more *specific* answer of the two. The decade of the twenties may be considered fabulous not only because of its prosperity. But it is the idea of prosperity alone that is discussed throughout this passage, even to accounting for its eventual demise.

How to Answer All Other Questions

For every question you have to answer, *other than the title question,* go back to the paragraph and find the answer. If you cannot verify the choice you have selected somewhere in the passage, it *cannot* be your answer.

Work through the choices on all the questions by a process of elimination. *Do not use* the technique of determining whether some choices are too broad, too narrow, out of context, or irrelevant; this technique is only for the title question. In the other questions, you eliminate answer choices for one reason only: *They are not mentioned in the passage.*

Please go back to Question 2. Read the question and back into the correct answer by drawing a line through those choices that are not possible. Please do it now before reading further.

You are asked to define the word "remarkable" as it suits the context or spirit of the sentence. Rereading the first sentence of that paragraph, it is clear that the word *also* is a clue in defining this word. The first paragraph discusses the special quality of the twenties, namely its prosperity. The word *also* must imply that the author would like to describe *another* special or unique quality of this decade that makes it remarkable. You will be looking for a word synonymous with "special" or "unique." (A) *catastrophic* means "disastrous"; it does not mean "special" or "unique." You should have drawn a line through that answer. (B) *ironic* describes some kind of incongruity, as, for example, when the actual result of something is quite different from the expected result; it does not mean "special" or "unique." You should have drawn a line through that answer. (C) *distinguished* means "separate" or "different"; it is close in meaning to "special" or "unique," and you should have left it as a possibility. (D) *prosperous* would not fit after the word *also* because the passage goes on to mention a decline in prosperity, and you can't have a prosperous decline of prosperity. Besides, *prosperous* does not mean "special" or "unique." You should have drawn a line through that answer. You have backed into the only possible definition for *remarkable*, answer (C).

Now turn back to Question 3. But remember that even though the question asks you to determine something the author *implies*, you must still be able to locate your answer choice within the confines of the passage. Go through the choices one at a time, eliminating those that are not possible. Please do it now before reading further.

You probably eliminated every choice except for (B), because none of the other answer choices was mentioned in the passage. The answer to Question 3 is found in the last line of the second paragraph. The exact words are: "tendencies which were obviously leading to disaster." The word *obviously* means that there were signs people could see; and in the answer, the word *portents* means "things that foreshadow a coming event"—in other words, there were signs that foreshadowed disaster.

PRACTICE EXERCISE 1

Now look at the passage below. Handle it exactly as you did the passage we have just completed. Read the passage, following Steps 1 through 7. Step 1, remember, begins with underlining key words in the *questions* following the passage. Underline the passage just as you will underline the SAT passage on the official examination. (It is perfectly permissible to write on both the verbal and the math sections of the SAT.) When you are finished marking your test and before you answer the questions, compare your diagrammed passage with the one here. Please read the following example now, beginning with the questions and following Steps 1 through 7, before reading further.

1 The adult, even in his most personal and private
2 occupation, even when he is engaged on an inquiry
3 which is incomprehensible to his fellow-beings, thinks
4 socially, has continually in his mind's eye his
5 collaborators or opponents, actual or eventual, at any rate
6 members of his own profession to whom sooner or later he
7 will announce the result of his labors. This mental
8 picture pursues him throughout his task. The task itself

9 is henceforth socialized at almost every stage of its
10 development. Invention eludes this process, but the need for
11 checking and demonstrating calls into being an inner speech
12 addressed throughout to a hypothetical opponent, whom the
13 imagination often pictures as one of flesh and blood.
14 When, therefore, the adult is brought face to face with his
15 fellow-beings, what he announces to them is something already
16 socially elaborated and therefore roughly adapted to his
17 audience, i.e., it is comprehensible. Indeed, the further
18 a man has advanced in his own line of thought the better
19 able is he to see things from the point of view of others and
20 make himself understood by them.

1. The author of this selection feels that men are essentially

(A) isolated from one another
(B) social animals
(C) desirous of being alone
(D) difficult to understand
(E) unable to communicate ideas easily

2. According to the author, the inventive man has a continual dialogue in his mind with

(A) a potential critic
(B) himself
(C) his audience
(D) those who are going to fund him
(E) nobody

3. The author believes that a person is able to communicate his creation to others easily because

(A) he speaks to those in his own field
(B) all men are in constant communication with one another
(C) he has already communicated in his mind as his work progresses
(D) even his opponents are willing to listen
(E) men are primarily social beings

4. The writer considers that the more advanced a man is in his own field

(A) the less he is able to make himself understood
(B) the more he understands how others see things
(C) the more he works with others
(D) the more his opponents criticize him
(E) the more he likes other human beings

5. This passage implies that an essential characteristic of man is

(A) the need for isolation
(B) a vivid imagination
(C) the need to communicate to his fellow man
(D) inventiveness
(E) the need to oppose and criticize inventions

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You probably put an asterisk alongside the first sentence (lines 1–7), since it neatly summarizes what the entire passage is about: Man is a social being and no matter what he does, he thinks about what the reaction of other people will be, particularly those in his own profession. The second sentence (lines 7–8) reiterates and elaborates upon the same idea. You probably put double lines between the third and fourth sentences because the fourth sentence (lines 10–13) switches to a new idea: When an individual first conceives of something new as an "invention," he does *not think* in social terms. The word *eludes* means "escapes from." There are double lines, however, between *process* and *but*, because the writer returns to the point that once the "invention" stage is passed, the social stage takes over. A person begins to think about the comments of other people like himself, those of *flesh and blood*.

There are double lines between the fourth and fifth sentences because the fifth sentence (lines 14–17) deals with the person in actual contact with his audience, rather than just thinking about them. Since he has been thinking about his audience through every stage in the development of his idea, he can now relate to them in a way that makes him thoroughly understood. The last sentence (lines 17–20) reiterates and elaborates on the sentence that precedes it.

Now you are ready to answer the questions. Review the technique and handle them exactly as you handled the questions in the previous passage. Please do them now and compare your answers with ANSWERS AND EXPLANATIONS when you are finished.

ANSWERS AND EXPLANATIONS

1. (B) 2. (A) 3. (C) 4. (B) 5. (C)

1. (B) You could have backed into the correct answer by a process of elimination. The only answer that is mentioned in the passage is (B). It is discussed in lines 3 and 4 in the words *thinks socially;* it is stated strongly in lines 8–10 with the words *the task itself is henceforth socialized at almost every stage of its development.*

2. (A) You probably narrowed your answers down to (A) and (C), since (B), (D), and (E) are never mentioned in the paragraph. But the answer is (A), since the question asks with whom *the inventive man has a continual dialogue in his mind.* You must always go back to the statement where the question is being discussed and search out the answer. In this case the answer at first seems to be in lines 4 and 5. *Opponents* is just another way of saying *potential critics.* That's (A). But *collaborators* confuses the issue a bit. Perhaps (C) *his audience* is the right answer after all, since *audience* could include both opponents and collaborators. But read further, to lines 11 and 12: *an inner speech addressed throughout to a hypothetical opponent.* This is one of the major ways the tester can confuse the test taker; he gives clues that may make you take the wrong answer if you aren't careful. It is your job to find the words in the passage that relate *most directly* to the question. Here it is lines 11 and 12, not lines 4 and 5. Answer (C) is not exactly wrong, since critics are part of the audience, but it is nowhere near as precise as (A): *continued dialogue in his mind with a potential critic* is exactly the same thing as *inner speech addressed throughout to a hypothetical opponent.* So if you chose (C), you have a pretty good idea what the passage is saying—but you haven't been suspicious enough, and you'll be scored wrong.

3. (C) Answer (C) can be found in lines 14–17. The key word is *therefore,* indicating that all of the inner communicating that the person has done with his imaginary opponent throughout the development of his project makes what he has to say to his actual audience *comprehensible.*

4. (B) The clue to this answer is in the question. You were told that one way to locate an answer is to go to the sentence where the question is discussed. That is obviously in the last sentence. You probably noted that answer (B) is simply another way of saying *the better able is he to see things from the point of view of others.* Also, none of the other answers can be found in the passage.

5. (C) Answer (C) is the only answer applicable to the passage. Lines 7 and 8 express it best: *this mental picture pursues him throughout his task.* The word *pursue* indicates a deep need that it is necessary to deal with at all times. Again, however, none of the other answers are remotely possible in answering the question: What *essential characteristic of man* is expressed throughout the passage? Remember, even though the question asks what the author *implies,* the *answer must be found within the passage.*

PRACTICE EXERCISE 2

Now look at the passage below. It is the type of reading passage you are going to have on the SAT in terms of length and the number of questions which follow. It is also comparable in the nature of the subject matter and its level of sophistication. Handle it exactly as you handled passages we have just completed.

Read the questions and the passage, following Steps 1 through 7. Step 1, remember, begins with underlining key words in the questions following the passage. When you are finished and before you answer the questions, compare your diagrammed paragraph with the one here.

One of the values of a college experience can be the exposure it offers to unfamiliar views and attitudes. Throughout elementary and high school years, one meets and studies alongside students from environments generally homogeneous with one's own. Going away to college requires us to confront the fact that some of our most cherished precepts are not shared by intelligent confreres whose background diverges from our own.

We find ourselves in class after class, in a dormitory or social deliberation, forced to question the accustomed axioms of our childhood. In addition, we find that many respected instructors esteem the views of our colleagues above our own, often subjecting us to public humiliation and ridicule. Accustomed for years, in grade after grade, to the respect accorded the oral and written expressions of a school's most eminent students, we now find ourselves in competition with classmates more adroit in their arguments than we, better read, more resourceful.

Or we find ourselves forced to rub elbows with a classmate whose racial or religious background is so dissimilar from our own that he challenges our every premise, forcing our daily life into an almost nonstop debate. We feel harassed and attacked; what began as a love of learning is transformed into a deep hate of a fellow learner.

The simplest way to assuage our frustration is to indulge in the opiate of prejudice to ease our own sense of inadequacy by attributing our shortcomings to presumed characteristics inherent in the background of the hated—his race is crafty, aggressive, arrogant, underhanded . . . And like a snowball, our prejudice attracts the stray twigs and dead leaves of additional justification: He is uncouth, malodorous, a radical, too loud, too studious . . .

If such a transformation is possible in a liberal young person, open to all experience and eager for it, what hope is there for the less intelligent, more provincial citizenry, who never depart their natal insularity?

1. The author believes that
   (A) our childhood experiences are unfortunate because they always cause us to become prejudiced when we go to college
   (B) we become prejudiced simply because we cannot tolerate differences
   (C) we begin to become prejudiced when we try to ease the pain of our own shortcomings by projecting our weaknesses onto others
   (D) prejudice causes immense frustration on a college campus
   (E) none of us is prejudiced when we are in high school

2. According to the author
   (A) a college experience is valuable because it exposes us to students brighter than ourselves
   (B) college causes us to feel harassed and consequently to hate learning
   (C) in college we are forced to become friendly with those of other races and religions
   (D) in college we improve by mingling with those more adroit in arguments than ourselves
   (E) our high school experiences do not prepare us to meet people of equal intelligence who have different backgrounds and values

3. The author is concerned that
   (A) we learn to cope with our frustrations in the easiest way possible.
   (B) if liberal college students can become prejudiced, what about the large number of people who never get beyond their own backyards?

(C) college instructors easily become prejudiced
(D) we become friendly with those of different religions and races while in college
(E) we feel inadequate because we are prejudiced against those of other races

4. Which of the following statements is best supported by the article?

(A) Prejudice is caused by liberal students becoming too conservative when they arrive on campus.
(B) Our feelings of inadequacy are caused by our high school teachers
(C) Those whose background is different from ours often challenge the basic precepts of our thinking.
(D) College instructors often humiliate their colleagues.
(E) Those who are different from us in background always disagree with us on intellectual matters.

5. Which of the following ideas are considered in this passage?

I. the nature of prejudice on a college campus
II. the inadequacy of our high school experience in preparing us to meet those of dissimilar backgrounds
III. the reasons one must become friendly with those of dissimilar backgrounds
IV. the causes of frustration on a college campus

(A) I, II, IV (B) I, II (C) I, IV (D) IV (E) I, II, III

One of the values of a college experience can be the exposure it offers to unfamiliar views and attitudes. // Throughout elementary and high school years, one meets and studies alongside students from environments generally homogeneous with one's own. // *Going away to college requires us to confront the fact that some of our most cherished precepts are not shared by intelligent confreres, whose background diverges from our own.

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EX.1 EX.2
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EX.3
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EX.5 EX.6 EX.7 EX.8
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EX.9
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   (B) we become prejudiced simply because we cannot tolerate differences
   (C) we begin to become prejudiced when we try to ease the pain of our own shortcomings by projecting our weaknesses onto others.
   (D) prejudice causes immense frustration on a college campus
   (E) none of us is prejudiced when we are in high school.

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   (D) in college we improve by mingling with those more adroit in arguments than ourselves
   (E) our high school experiences do not prepare us to meet people of equal intelligence who have different backgrounds and values.

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   (C) college instructors easily become prejudiced
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4. Which of the following statements is best supported by the article?
   (A) Prejudice is caused by liberal students becoming too conservative when they arrive on campus.
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   (E) Those who are different from us in background always disagree with us on intellectual matters.

5. Which of the following ideas are considered in this passage?
   I. the nature of prejudice on a college campus
   II. the inadequacy of our high school experience in preparing us to meet those of dissimilar backgrounds
   III. the reasons one must become friendly with those of dissimilar backgrounds.
   IV. the causes of frustration on a college campus

(A) I, II, IV (B) I, II (C) I, IV (D) IV (E) I, II, III

You probably put an asterisk alongside the third sentence (lines 4–7), since it discusses an idea which is elaborated on, in one way or another, throughout the passage: that the elementary and high school years do not prepare us for one of the most valuable experiences of college, which is meeting people of equal intelligence who have different backgrounds and values. It is possible that you may have put an asterisk alongside the first sentence, because it says the same thing in a more general way, but the better choice would be the third sentence. There are double lines between the first and second sentences because the writer contrasts college with the elementary and high school years. There are double lines between the second and third sentences because the author shifts back to college. Lines 8 through 12 reiterate and elaborate on the idea discussed in the topic sentence. There are double lines between the words *ridicule* and *accustomed* in line 12 because the author again shifts his subject from college to high school, and between the words *students* and *we* in line 14 because he shifts back again. The idea, however, is always the same: In elementary and high school one associates with students of the same background, and the student who excels is accorded the respect due him; however, at college, this same student is confronted with peers of

similar intelligence who do not share his values and beliefs, and he is constantly challenged. There may be some variety of opinion about putting double lines at the beginning of line 16. On the one hand, the writer continues his discussion of what the individual may face in his college experience, but differences in terms of not only values and beliefs but race and religion as well. On the other hand, one may consider the shift to color and religion sufficiently discussed or grand enough in its impact to deserve double lines, indicating a total shift, even though the writer is still talking about what a student is forced to confront on a college campus.

There are double lines between *learner* and *the* on line 21, since the writer shifts his discussion to the way one can relieve or *assuage* his frustration—by prejudice. The key idea inherent in the remainder of this discussion is the fact that one of the characteristics of prejudice is attributing our own shortcomings to someone else—in this case, the person who is dissimilar to us. Finally, there are double lines between *studious* and *if* in line 28 because the author poses a question, the result of his discussion on prejudice: If a liberal young person who has a zest for learning can become a narrow-minded bigot, then what hope is there for those who have none of the positive qualities to begin with?

Now you are ready to answer the questions. Review the techniques and handle the questions exactly as you handled the questions in the previous passages. Please do them now and compare your answers with the ones here.

ANSWERS AND EXPLANATIONS

1. (C) 2. (E) 3. (B) 4. (C) 5. (A)

1. (C) You probably eliminated (A) because it never states in the passage that our childhood experiences are *unfortunate,* and it never states in the passage that our childhood experiences are responsible for our prejudices when we go to college. Choice (B) is never mentioned in the passage. The answer (C) is stated in lines 21–24. Choice (D) is never stated in the passage. The passage does deal with prejudice and it does mention how one might *ease* his frustration, but it never makes the sweeping generalization that *prejudice causes immense frustration on a college campus.* If you think it does, there is always a simple way to test yourself: Find it and read it out loud. If you cannot, it is not the answer. (E) is easily eliminated, since it is never stated in the passage.

2. (E) Choice (A) is easily eliminated, since the passage never states that students are *brighter;* (B) can be eliminated since the passage never says that *college* is what makes students *harassed*—it is rather the contact with students of different races and religions; (C) can be eliminated because nowhere in the passage does it say that a student is *forced to become friendly;* (D) can be eliminated because *improve* has nothing to do with the passage—only negative effects of the college experience are discussed. The answer is (E), since it is clearly stated in the first paragraph.

3. (B) Choice (A) can be eliminated, since the author never expresses any such concern. (B) is a concern of the author's and is expressed in lines 28–31. Again the author changes words around. Since *natal* means "pertaining to birth" and *insularity* means "narrowness" or "the state of being an island," *people who never get beyond their own backyards* can be said to have a *natal insularity;* the forms of expression are quite different, but the meaning is the same. (C) is never mentioned in the passage. (D) is not mentioned either. (E) is tricky; the passage never says that we feel inadequate because we are prejudiced, but rather that we are prejudiced because we feel inadequate (lines 22–27).

4. (C) You should have been able to back into the correct answer for this question, since none of the choices except (C) was mentioned in the passage. (C) can be found in the first paragraph, lines 4–7.

5. (A) The only answer choice not mentioned in the passage is (III); we are never given any such reasons or even told that being friendly with those of dissimilar backgrounds is desirable. The writer might well be able to give such reasons—but in the passage he did not.

Practice Reading Comprehension Passages

Here are some reading comprehension passages for you to practice on. Each passage is followed by questions based on its content. Follow the directions explained above for reading the questions and the passages. Answer all the questions following a passage on the basis of what is stated or implied in the passage. Arrive at your answer by a process of elimination by drawing a line through all those answers that are not possible. Remember that except for the title question, you must be able to locate the answer within the passage.

PASSAGE 1

In our advanced society, the pressures of daily living are heavier, in many ways, than they were for the deprived societies our advances were designed to improve. A hungry man faces one basic problem: finding food. We, whose supermarkets are bulging with endless gustatory options, confront myriad consequent problems: which foods to select, weighing the representations of competitive packagers, proper nutritional balance, economic budgeting, paying bills on time, not to speak of the waste disposal of nonbiodegradable packaging, restive delays at the check-out counter, and the escalating cost of transportation to and from the shopping center.

One of the consequences of this increased pressure is a frustration generated by unfulfilled expectations: More food increases girth, not health; finer homes beget larger mortgage payments, not greater domestic tranquillity; the availability of a wider range of entertainment and vacation possibilities only aggravates the frequency of a family's internecine conflicts.

The growing pressures of such frustration create psychological needs earlier generations never dreamed of. Like a rat who discovers that an anticipated destination is a mere cul-de-sac, we circumgyrate through rounds of endless activity, hoping that the momentum of our movement will mask its undisciplined aimlessness. Psychiatric consultation has become a commonplace of daily living; we pursue entertainment with a fanatic dedication our forebears reserved for religious practices; the blare of radio and television beats a constant counterpoint to our unremitting routine. We are "a nation on the move," as the direction of our self-evident truth has drifted from "the pursuit of happiness" to the happiness of pursuit. We seem mortally afraid that if the frenetic beat slackens, we will be forced to face ourselves in silent vacuity.

1. The author suggests that one of the main reasons for the pressures of daily living being heavier now than in former times is
   (A) technological advances leading to greater options and choices
   (B) the satisfaction of the constant hunger which plagued deprived societies
   (C) the fact that more people seek and are able to receive psychological help
   (D) the fact that we are able more easily to satisfy our everyday frustrations
   (E) that we pursue happiness at too frantic a pace

2. Which of the following statements, according to the author, are true?

   I. families are closer together than ever before
   II. satisfaction of frustration is easier because we have more goods and services
   III. we have psychological needs today which our grandparents did not feel
   IV. we pursue entertainment to the same degree that our ancestors sought out religious experiences
   V. shopping centers have eased the nation's hunger problem

(A III, V (B) I, III, IV (C) III, IV (D) IV, V (E) II, III, IV

3. The author implies that one of the main reasons for our present psychological distress is:

   (A) having to make so many choices in supermarkets, entertainment, etc.
   (B) a constant running around in search of answers
   (C) the expenses brought about by our technological advances
   (D) the influence of radio and television
   (E) our expectation that more and better material goods will lead to more happiness

4. Which of the following best reflects what the author feels is the direction we ought to go in to alleviate the psychological stress of modern life?

   (A) Make fewer alternatives available to ourselves.
   (B) Know what our real values are and act in accordance with them.
   (C) Watch less TV and vacation less frequently.
   (D) Pursue happiness with more determination.
   (E) Eliminate family quarrels by limiting choices.

5. The author concludes that we pursue happiness frantically because

   (A) we have too many material goods
   (B) we have too many choices to make and cannot cope with making decisions
   (C) too many pressures create frustration
   (D) we are afraid that if we stop the frantic pursuit, we may find ourselves facing the problems of what are our real values

PASSAGE 2

Logotherapy is neither teaching nor preaching. It is as far removed from logical reasoning as it is from moral exhortation. To put it figuratively, the role played by a logotherapist is rather that of an eye specialist than of a painter. A painter tries to convey to us a picture of the world as he sees it, an ophthalmologist tries to enable us to see the world as it really is. The logotherapist's role consists in widening and broadening the visual field of the patient so that the whole spectrum of meaning and values becomes conscious and visible to him. Logotherapy does not need to impose any judgments on the patient; for, actually, truth imposes itself and needs no intervention.

By declaring that man is a responsible creature and must actualize the potential meaning of his life, I wish to stress that the true meaning of life is to be found in the world rather than within man or his own psyche, as though it were a closed system. By the same token, the real aim of human existence cannot be found in what is called self-actualization. Human existence is essentially self-transcendence rather than self-actualization. Self-actualization is not a possible aim at all; for the simple reason that the more a man would strive for it, the more he would miss it. For only to the extent to which man commits himself to the fulfillment of his life's meaning, to this extent he also actualizes himself. In other words, self-actualization cannot be attained if it is made an end in itself, but only as a side-effect of self-transcendence.

1. The author believes that the therapist's main role is to

   (A) help the patient become cognizant of the true values that life holds.
   (B) help the patient to become self-actualized
   (C) help the patient see with the eye of the painter
   (D) help the patient make correct judgments
   (E) help the patient to reason more logically

2. The author implies that truth
   (A) is in the mind of the person
   (B) can only be reached through the intervention of the therapist
   (C) exists in the world outside of man's mind
   (D) is relative: dependent on the emotional state of the person trying to perceive it
   (E) has no real meaning

3. According to the author, self-actualization
   (A) is the proper goal of man
   (B) gives the only real meaning to human existence
   (C) cannot be realized without the intervention of the therapist
   (D) can only be attained as a by-product of a search for his life's meaning
   (E) is the basis of human existence

4. According to this selection, which of the following statements are true?
   I. Therapy is necessary to find meaning in life.
   II. Human existence finds its meaning in the world outside of man's psyche.
   III. One cannot find complete self-actualization by directly striving for it.
   IV. The logotherapist tries to help the patient see the world as he sees it.
   V. Logotherapy does not impose values on the patient.

   (A) II, III, IV (B) I, III, V (C) III, V (D) II, III, V (E) I, II, III

5. The author suggests that man is a responsible person who must
   (A) transcend his psyche and find meaning in life outside himself
   (B) actualize his inner potential only
   (C) learn how to make value judgments
   (D) find the true meaning of his life within himself
   (E) strive for a commitment to values that have meaning for him alone

PASSAGE 3

When you look at transportation in general there is no suggestion that our improving technology is likely to work miracles, as it sometimes seems to do in some other fields—electronics, for instance. It is easy to understand what has been happening over the past century and a half in terms of straightforward principles. The first great transportation system, the railroad, was a linear system. It consisted in connecting a comparatively few points, the big cities, by more or less straight lines. It was essentially a one-dimensional system. It lost out in a large measure to the automobile because our road system is essentially two-dimensional. There are so many more roads than there are rail lines that we can travel by car from everywhere to everywhere else. It is this gain of an extra dimension that gives road transportation the edge over rail transportation, and not at all the efficiency over any particular link. From central New York to central Chicago it is still more efficient to go by rail than by turnpike. But if you want to go from a small town in upstate New York to the wilds of Wisconsin the situation is different. The two-dimensional character of our roads wins over the railroad; getting from upstate New York to central New York is where the railroad loses. And of course if you add a third dimension, if you go by air, you do better still. The essential features of transportation lie in geometry. A two-dimensional system is better than a one-dimensional system for most purposes and a three-dimensional system is better than two dimensions.

1. According to the author, improvements in our transportation system
   (A) are due to the miracles of technology
   (B) increase in geometrical progression
   (C) can be expressed in linear terms
   (D) are due in each case to the gaining of a dimension
   (E) are due to the efficiency of the automobile as compared to the railroad

2. The passage is concerned mainly with

(A) how to make our transportation system more efficient
(B) the structure of our railroad system
(C) how to get from New York to Chicago in the most efficient way
(D) analyzing the essential features of our transportation system over the last century and a half
(E) analyzing the reasons our railroads are the least used of all our methods of transportation

3. Select the statement which, according to the author, is true.

(A) The automobile is always more efficient than the railroad.
(B) The airplane is always the better way to travel because it is three-dimensional.
(C) Improving technology will cause miraculous improvement in transportation in the future.
(D) What makes one mode of transportation better than another in a given situation is determined by geometry.
(E) The railroad connects more large cities than the automobile.

4. According to this selection, automobile transportation gains over rail transportation because

(A) railroads can only go in straight lines
(B) roads enable us to get to many more places
(C) there are more automobiles than railroad cars
(D) the railroad cannot get you from upstate New York to central New York
(E) our road system connects all three modes of transportation: automobiles, trains, airplanes

5. The author implies that

(A) we can expect more efficient transportation systems in the future
(B) improved technology will eventually make railroads obsolete
(C) air travel will eventually replace all other modes of transportation
(D) efficiency is the key to what makes a good transportation system
(E) the most important feature of any transportation system is the ability to get people where they want to go

PASSAGE 4

Something wild has been going on in our century—a rhythmic revolution, led by the formidable Stravinsky, who in this great monument to rhythm, *The Rite of Spring,* unleashed forces that have all but annihilated the comfortable symmetries of yesteryear. How had Stravinsky arrived at rhythms of such jaggedness and irregularity in the few decades that separated him from César Franck? Let's see if we can track it down. It's perfectly true that unequal meters and unequal rhythmic patterns are also to be found in the great music of the past. The only difference is that such instances of asymmetry in the eighteenth and nineteenth centuries are notable precisely because they are exceptional. They represent moments of inspired freshness, or madness, if you will, in the minds of composers for whom symmetry was always the normal state of affairs; whereas in our time asymmetry has almost become the norm.

1. According to the author, the essential difference between Stravinsky's music and that of composers of the eighteenth and nineteenth centuries is to be found in

(A) the nature of the composition
(B) unequal meters
(C) the irregularity of the composition
(D) the asymmetry of the rhythm
(E) his inspiration

2. According to this paragraph, how does the author feel about Stravinsky's music?

(A) He does not like it.
(B) He does not tell us.
(C) He likes it.
(D) He is concerned about it.
(E) He prefers César Franck's music.

3. The asymmetry of eighteenth-century music is different from Stravinsky's because

   (A) it was the exception
   (B) it was inspired
   (C) it was the norm
   (D) it did not occur in the eighteenth century
   (E) it was considered madness in the eighteenth century

4. According to the author, today asymmetry in rhythm is considered

   (A) inspiration
   (B) madness
   (C) unusual
   (D) irregular
   (E) almost the norm

5. Stravinsky was considered a revolutionary because

   (A) he changed the rhythmic patterns of music
   (B) he was inspired
   (C) he broke away from the music of César Franck
   (D) he despised the music of the eighteenth and nineteenth centuries
   (E) he was different from most other musicians

PASSAGE 5

Of all parts of the sea, the continental shelves are perhaps most directly important to man as a source of material things. The great fisheries of the world, with only a few exceptions, are confined to the relatively shallow waters over the continental shelves. Seaweeds are gathered from their submerged plains to make scores of substances used in foods, drugs, and articles of commerce. As the petroleum reserves left on continental areas by ancient seas become depleted, petroleum geologists look more and more to the oil that may lie, as yet unmapped and unexploited, under these borderline lands of the sea.

The shelves begin at the tidelines and extend seaward as gently sloping plains. The 100-fathom contour used to be taken as the boundary between the continental shelf and the slope; now it is customary to place the division wherever the gentle declivity of the shelf changes abruptly to a steeper descent toward abyssal depths. The world over, the average depth at which this change occurs is about 72 fathoms; the greatest depth of any shelf is probably 200 to 300 fathoms.

Nowhere off the Pacific coast of the United States is the continental shelf much more than 20 miles wide—a narrowness characteristic of coasts bordered by young mountains perhaps still in the process of formation. On the American east coast, however, north of Cape Hatteras the shelf is as much as 150 miles wide. But at Hatteras and off southern Florida it is merely the narrowest of thresholds to the sea. Here its scant development seems to be related to the press of that great and rapidly flowing river-in-the-sea, the Gulf Stream, which at these places swings close inshore.

1. According to the article, the great fishing grounds of the world seas are

   (A) all confined to the continental shelves
   (B) found only where there is seaweed
   (C) the most important resource of the continental shelf
   (D) found mostly in the shallow waters of the continental shelves
   (E) becoming scarcer as we dig more and more for oil.

2. According to the author, the dividing line between the continental shelf and the slope to the ocean is

   (A) the 100-fathom contour
   (B) twenty miles from the low-tide mark
   (C) where the shelf begins to descend abruptly
   (D) at about 150 miles out to sea on the east coast of the United States
   (E) at about 300 fathoms

3. The reason given for the narrowness of the continental shelf on the Pacific coast is that

   (A) the Gulf Stream comes close in shore there
   (B) the depth goes beyond 100 fathoms
   (C) the slope declines gradually rather than abruptly
   (D) this is common in areas where mountain ranges are probably still being formed

(E) the shelf begins at the tidelines and extends seaward with a gentle slope

4. The article implies that the continental shelf on the east coast, north of Cape Hatteras, probably

(A) is more valuable to man than the continental shelf south of Cape Hatteras
(B) will not offer the possibility of much oil
(C) is 300 fathoms deep
(D) is influenced quite a bit by the nearness of the Gulf Stream
(E) is caused by the continental formation of new mountains

5. According to the article, which of the following are true?

I. The continental shelf of the United States is in some places as much as 150 miles wide.
II. Geologists think the continental shelf may yield oil.
III. The narrowest part of the continental shelf is along the Pacific Ocean.
IV. Products made from seaweed are part of our economy.
V. The Gulf Stream comes close inshore north of Cape Hatteras.

(A) I, II, III (B) II, IV, V (C) I, II, IV (D) II, IV (E) I, IV

PASSAGE 6

In 1932 Aldous Huxley's *Brave New World* was an immediate success. So enduring was this success that it is even more timely forty years later. Huxley missed on several points: there is no mention of atomic bombs or nuclear energy, "sleep teaching" is described as a potent force in shaping his society, and the timescale of his predictions errs mightily. In *Brave New World,* the Central London Hatchery and Conditioning Centre is decanting its mixed bag of planned citizens in the year A.F. (After Ford) 632. By 1946, when he revised the introduction to the book, Huxley had reduced the time to his vision of the future to only 100 years, but even that estimate was far too conservative. The genetic engineering he feared is no longer fiction.

A speaker at a recent genetics conference suggested that his colleagues reread *Brave New World,* which they would find far less revolutionary than it had seemed in the thirties. As a matter of fact, parts of the book sound more like a report of what is happening on the genetics front than a cautionary tale about a controlled society. Indeed, about all that needs doing to make it a factual report is to change the fictional names Huxley used to those of the biologists who are actually doing these things in genetics laboratories.

The first prescriptions for "test-tube babies" were written centuries ago when straight-faced "scientists" set down recipes for creating "homunculi" from sperm cultivated in a jar. The idea has continued to nag at us since then, first on the fictional fringes, then in scientific papers, and finally in the laboratory where the factual forerunners of Huxley's decanted Alphas and Betas are being hatched.

1. The theme of this passage is:

(A) the revolutionary nature of Huxley's writings
(B) the closeness to today's reality of Huxley's predictions in the field of genetics
(C) the enduring success of Huxley's writings
(D) the reasons we should read *Brave New World*
(E) genetic engineering

2. Huxley, in *Brave New World,* correctly predicted

I. genetic engineering
II. the atomic bomb
III. sleep teaching
IV. "test-tube babies"
V. conditioning centers

(A) I, II (B) I, IV (C) III, IV (D) I, III (E) III, IV

3. The author implies that Huxley

(A) was a skilled scientist
(B) feared the results of genetic engineering
(C) was an uncanny predictor of human behavior
(D) purposely disguised facts in novel form
(E) was the first to predict "test-tube babies"

4. According to the author, when Huxley revised his introduction to *Brave New World,* the future he was predicting would arrive about

(A) 1946
(B) A.F. 632
(C) 2046
(D) 2077
(E) 1956

5. The author indicates that the society predicted by Huxley in *Brave New World* was

(A) liberal
(B) conservative
(C) violent
(D) creative
(E) controlled

PASSAGE 7

Investigators in many laboratories have been finding that blood pressures in animals can be made to rise and fall predictably; that squirrel monkeys with induced hypertension can be trained by conditioning techniques to lower their blood pressures; and that by similar procedures human subjects can be trained to raise and lower pressure.

For thousands of years, mystics and yogis have claimed remarkable control over mind and body, including the ability even to drop the pulse to the vanishing point. Westerners generally have scoffed at this. They have based their scoffing on the fact that there are two nervous systems—that, supposedly, while one, the voluntary nervous system, is under the control of the conscious mind, the other, the autonomic or involuntary system, is not—and it is this autonomic system which controls the activities of the heart, blood vessels, and visceral organs.

But now, through a technique called biofeedback, investigators have been able to show that the autonomic nervous system is not so autonomic after all; that man can learn to control his internal organs much as he does his arms, legs, and other parts.

The principle behind biofeedback is basically simple. We learn to do many things by virtue of visual and neuromuscular feedback cues. Biofeedback cues can be obtained through laboratory instruments that pick up and amplify blood-pressure changes and heart-rate changes. The instruments are not new—only this application of them is. They are hooked up to produce sound or light signals, and the individual, through the signals, can get cues for internal changes.

1. The theme of this article is

(A) the invention of the biofeedback machine
(B) the development of techniques for controlling the autonomic nervous system
(C) the remarkable predictions of eastern mystics and yogis
(D) the ability of squirrel monkeys to lower their blood pressure
(E) high blood pressure and its results

2. The autonomic nervous system controls, among other things,

(A) the conscious mind
(B) the unconscious mind
(C) the movement of arms and legs
(D) blood pressure
(E) the ability to feel pain

3. The author implies that for thousands of years eastern mystics:

(A) have claimed to know techniques to lower blood pressure unknown to western scientists
(B) scoffed at westerners
(C) invented the biofeedback machine
(D) have always had more control over the unconscious
(E) have not used scientific techniques

4. According to the author, the technique of biofeedback is based on

(A) electricity
(B) control of the unconscious mind
(C) control of internal changes in the body
(D) neuromuscular feedback cues
(E) control of the voluntary nervous system

5. The author states that biofeedback instruments

(A) were developed by eastern mystics
(B) were recently developed
(C) were developed as a result of this new technique
(D) are not used very much in western applications of the biofeedback principle
(E) are not new

PASSAGE 8

It now appears that the long-mystifying mechanism of action for aspirin may lie in the fact that aspirin counters the activities of a prostaglandin that, under some circumstances, produces fever, headaches, and inflammation.

Some researchers believe that prostaglandins are part of a vast monitoring system of the body, and that they may encourage or inhibit the activities of the better-known hormones. The fascinating mechanism may be this: the better-known hormones are really messengers transmitting messages that are to be carried out within body cells. The carrying out is the responsibility of a remarkable substance called cyclic AMP, a versatile "second messenger." But a critical factor for the production of cyclic AMP is an enzyme in the cell wall. Prostaglandins may control this enzyme and thus control cyclic AMP production and the ability of the cell to carry out hormonal instructions. Put another way, prostaglandins appear to be the only known agents capable of raising the level of cyclic AMP in some cells and decreasing it in others.

Dr. Lee believes that a deficiency of certain prostaglandins naturally present in the kidneys may explain many if not most cases of hypertension. Actually, more than a decade ago, while at Harvard, Dr. Lee found that kidney extracts injected into test animals produced a marked blood-pressure drop. He has since found that three active compounds in kidney tissue are three different prostaglandins. It appears that the sole job of one of them is to lower blood pressure. Animal experiments suggest that it does so by relaxing small blood vessels, thus easing blood flow.

1. What does the author feel is the job of cyclic AMP?
   (A) the stimulation of cell enzymes
   (B) the carrying out of messages transmitted to the cells by hormones
   (C) control of the prostaglandins
   (D) production of hormones
   (E) lowering of blood pressure

2. Dr. Lee believes that hypertension may be caused by
   (A) an overabundance of cyclic AMP in the kidneys
   (B) the influence of hormones on prostaglandins
   (C) absence of prostaglandins in the kidneys
   (D) deficient amounts of hormones in the kidneys
   (E) overproduction of enzymes in the kidneys

3. Some people believe that aspirin works by
   (A) stimulating the prostaglandins in the kidney
   (B) inhibiting the action of a prostaglandin that produces headaches
   (C) stimulating the production of cyclic AMP
   (D) stimulating the production of kidney enzymes
   (E) encouraging the activity of the body's hormones

4. Which of the following are true?
   I. Prostaglandins may affect the activity of hormones.
   II. Aspirin encourages the activity of prostaglandins.
   III. The kidney has three prostaglandins that control blood pressure.
   IV. Cyclic AMP is influenced by a certain enzyme.

   (A) I, II (B) I, IV (C) II, IV (D) I (E) III

5. According to this article, kidney prostaglandin lowers blood pressure by
   (A) encouraging the manufacture of hormones
   (B) inhibiting the manufacture of hormones
   (C) inhibiting the action of certain enzymes
   (D) transmitting messages to body cells
   (E) encouraging the flow of blood in certain blood vessels

PASSAGE 9

The Greeks have a story that when God made the world he put the earth through a sieve and used the soil to make the different countries; but the stones that were left in the sieve he threw over his shoulder and they became Greece. The land is so mountainous, precipitous, and stony that out of the 50,100 square miles that constitute the area of Greece, only 25 percent are usable as fields, orchards, and pas-

tures. Yet on this land, out of a population of 7,335,675, according to 1940 figures, 60 percent are directly engaged in agriculture and another 15 percent cater to the needs of the farmers. This condition, as well as a growing population, is basic to the need for technical change in Greece. Through more than 2,500 years, Greece solved the problem of increasing population on limited land by sending out her people as colonists or immigrants. In the first two decades of this century, emigration to the United States provided the main solution. Early in the 1920s, two things took place which combined to make the population problem acute: the United States passed a law under which a quota was applied to immigrants, which limited drastically the number of Greeks who could thereafter enter the country; and the exchange of populations with Turkey, which increased the population of Greece suddenly by more than a fourth. Since then, the population figures from 1928 to 1940 indicate an annual increase of 1.5 percent. The traditional ways of farming and the traditional crops cannot support this growing population. Any solution to the problem must affect the very roots of the economy, striking at the relationship of man to the land and of man to his work, which is a close and meaningful one to the Greeks. How to do this without destroying the meaning of life is the question facing those entrusted with the introduction of change into this culture.

1. According to this article, the Greek economy
   (A) is dependent on its mountainous state
   (B) revolves around farming
   (C) is changing because of the change in Greek politics
   (D) concerns the necessity for colonists to return to Greece
   (E) is destroying the Greek way of life

2. According to the author, one of the basic problems faced by those who are responsible for making changes in the Greek economy is
   (A) how to encourage emigration
   (B) how to encourage a reduction in the population
   (C) the immigration laws of the United States
   (D) how to introduce change without destroying the meaning and values of the Greek culture
   (E) how to keep Turkey from repatriating more people of Greek descent

3. The author states that the need for technical change in the economy of Greece results from
   (A) a shortage of arable land and a growing population
   (B) the need to house new immigrants
   (C) the need to preserve the traditions of Greek culture
   (D) the change in the immigration laws in the United States
   (E) the loss of Greece's colonial Empire

4. According to this article, in ancient times the Greeks solved their economic problems by
   (A) warring with Turkey
   (B) developing technological change
   (C) developing traditional crops and traditional ways of farming
   (D) exporting excess population to other countries
   (E) living in the mountainous parts of the country

5. Which of the following statements, according to this article, are true?
   I. From 1900 to 1940, the population of Greece increased 1.5 percent a year.
   II. In the 1920s, the United States established emigration quotas.
   III. In the 1920s, Greece and Turkey exchanged populations.
   IV. Greece is more than 2,500 years old.

   (A) II, III, IV (B) I, III, IV (C) III, IV (D) II, III (E) II, IV

ANSWERS AND EXPLANATIONS, PASSAGES 1–9

Passage 1

1. (A) 2. (C) 3. (E) 4. (B) 5. (D)

Passage 2
1. (A) 2. (C) 3. (D) 4. (D) 5. (A)
Passage 3
1. (D) 2. (D) 3. (D) 4. (B) 5. (E)
Passage 4
1. (D) 2. (B) 3. (A) 4. (E) 5. (A)
Passage 5
1. (D) 2. (C) 3. (D) 4. (A) 5. (C)
Passage 6
1. (B) 2. (B) 3. (B) 4. (C) 5. (E)
Passage 7
1. (B) 2. (D) 3. (A) 4. (C) 5. (E)
Passage 8
1. (B) 2. (C) 3. (B) 4. (B) 5. (E)
Passage 9
1. (B) 2. (D) 3. (A) 4. (D) 5. (C)

Passage 1

1. (A) The question is asking for the *reasons* why the pressures of daily living are heavier now than in former times. (A) can be found in the first two paragraphs, which discuss the great options and choices that are now open to all of us. Answer choice (B) is incorrect because the author only discusses the dissatisfaction. (D) is the opposite of what is being discussed and does not answer the question. The main reason for the pressures of daily living is not that we are able to *satisfy* our everyday frustrations. (C) is incorrect because the fact that people seek psychological help is a result of, not a reason for, the pressure of daily living. (E) is incorrect because the author mentions a frantic pursuit of *entertainment*, not happiness. In fact, he says just the opposite; we no longer pursue happiness but are happiest when we pursue.

2. (C) Statement (I) is never stated; just the opposite is true. The only reference to the family is when the author comments on the *family's internecine conflicts. Internecine* means "mutually destructive." Statement (II) is dead wrong; the entire passage supports the idea that just the *opposite* is true. Nowhere does the author state that satisfaction of frustration is easier. Statement (III) is made in the first sentence of the third paragraph, and statement (IV) near the end of the passage. Statement (V) is never mentioned in the passage.

3. (E) Choice (A) is incorrect because the author never states that making choices in supermarkets, etc., is one of the main reasons for our psychological distress. Choices (B), (C), and (D) are never mentioned in the paragraph. The answer is in the second paragraph: the idea that expectations cannot be fulfilled and that "more" does not lead to happiness but only to frustration. The answer is (E).

4. (B) The question asks what the author feels is the direction we ought to go in to alleviate the psychological stress of modern life. You could have easily backed into the correct answer because (A) and (C) are never mentioned in the passage. (D) is never mentioned either, but would be just the opposite if it were. (E), eliminating family quarrels, is not going to alleviate the psychological stress of modern life. The answer is found in the last paragraph of the passage in the words *undisciplined aimlessness*, which is the opposite of knowing our real values and acting in accordance with them.

5. (D) Choices (A), (B), and (C) are never stated in the passage and are not why

the author feels we are frantically pursuing happiness. Answer (D) can be found in the last sentence in the passage.

Passage 2

1. (A) The correct answer can be found in the first sentence of the second paragraph. (B) is incorrect because the author states in the middle of the second paragraph of the passage that self-actualization is not a possible aim at all. (C) is incorrect because the author says in the third sentence that *the role played by a logotherapist is rather that of an eye specialist than a painter.* (D) is incorrect because in the last sentence of the first paragraph the author says that *logotherapy does not need to impose any judgments on the patient.* (E) is incorrect since in the second sentence of the passage, the author states that logotherapy is *far removed from logical reasoning.*

2. (C) Choice (A) is incorrect since the author says just the opposite in the first sentence of the second paragraph: *the true meaning of life is to be found in the world rather than within man.* (B) is incorrect because he says just the opposite in the last line of the first paragraph:—*truth imposes itself and needs no intervention.* (C) is the right answer. It can be found in the first sentence of the second paragraph. (D) and (E) are incorrect because they are not mentioned in the passage.

3. (D) Choice (A) is incorrect because the author states just the opposite in the second paragraph: *the real aim of human existence cannot be found in what is called self-actualization.* (B) is incorrect because the author never says that; in fact, he says the opposite. (C) is incorrect because the author never says the therapist is necessary for anything, and in the second paragraph he says *self-actualization is not a possible aim at all*—so it cannot be realized either with or without a therapist. (D) is correct because the author presents this idea in the next-to-last sentence in the passage: *only to the extent to which man commits himself to the fulfillment of his life's meaning, to this extent he also actualizes himself.* (E) is incorrect for the same reason as (A)—because the author states just the opposite in the second paragraph.

4. (D) Statement (I) is incorrect because the author never states the idea that therapy is necessary to find meaning in life. He says in the first line of the second paragraph that *the true meaning of life is to be found in the world.* Statement (II) is explicitly made in the first line of the second paragraph. Statement (III) can be found in the second paragraph: *for the simple reason that the more a man would strive for it, the more he would miss it.* Statement (IV) is incorrect because in the first paragraph, the author states that contrary to the painter, who encourages us to share his vision of the world, the logotherapist is more like the eye specialist, or ophthalmologist, whose aim is to enable the patient to see the world as it really is. Statement (V) is made in the last sentence of the first paragraph.

5. (A) Choice (A) is stated clearly in the first sentence of the second paragraph. Choices (B), (D), and (E) are all dead wrong—the entire passage, especially the second paragraph, stresses that it is the outside world, not the inner world of the individual, that is important. Choice (C) is irrelevant; *judgments* are mentioned in the last sentence of the first paragraph, but only as something that need not be imposed on the patient.

PASSAGE 3

1. (D) Choice (A) is incorrect because the author never states this idea. In the first sentence of the passage, in fact, he suggests that improving technology will *not* result in improvements in transportation. (B) is incorrect because the author

never says that the improvements in our transportation system *increase in geometrical progression*. In the next-to-last sentence of the passage, the author says only that *the essential features of transportation lie in geometry*. (C) is incorrect because the author never says that the *improvements in our transportation system can be expressed in linear terms;* the only statement he makes that has to do with *linear* is that *the first great transportation system, the railroad, was a linear system*. (D) can be found in more than one place: *it is this gain of an extra dimension that gives road transportation the edge over rail transportation*, etc. (E) is incorrect because the author never says this; he compares roads and railroads, not automobiles and trains.

2. (D) This question is another form of the "title" question. You are looking for an answer that deals with most of the lines in the passage. Choice (A) is too narrow; the idea of efficiency is mentioned, but the passage contains no advice for improving efficiency. (B) is also too narrow; not just the railroad system but also roads and air travel are discussed. (C) is not only too narrow, it is directly out of context. The passage does say that *from central New York to central Chicago it is still more efficient to go by rail than by turnpike*, but this is only an example he is giving, not the main concern of the passage. (D) is correct because almost every sentence in this passage deals with some feature of our transportation system over the last century and a half. (E) is irrelevant. The writer does not even claim that the railroads *are* the least used of all our methods of transportation; he is discussing the capabilities of different transportation methods, not how frequently they are used.

3. (D) This question is telling you that four of the five choices are *not true*. Only one is. (A) is untrue; the passage states that between some points the railroad is more efficient. (B) is untrue because the writer never states that the airplane is *always* the better way to travel. (C) is contradicted by the first sentence of the passage. (D) is clearly stated in the last two sentences of the passage. (E) is incorrect because *we can travel by car from everywhere to everywhere else;* the author does say that the railroads connect the big cities, but he doesn't say that they connect *more large cities than the automobile*.

4. (B) The question asks why automobile transportation gains over rail transportation. Choice (A) is incorrect because the passage states that the railroad *consisted in connecting a comparatively few points . . . by more or less straight lines*, not that *railroads can only go in straight lines*, which is obviously untrue. (B) is clearly stated: *there are so many more roads than there are rail lines that we can travel by car from everywhere to everywhere else*. (C) is irrelevant; it is never stated in the passage. (D) is incorrect, but may fool some people because the author does say *getting from upstate New York to central New York is where the railroad loses*. This does not mean that you cannot travel that way; obviously you can, although you may have to use some other form of transportation to get to the upstate railroad station. (E) is irrelevant; it is never stated in the passage.

5. (E) Remember that even though the question asks what the author *implies*, you must still be able to locate the answer in the passage. (A) is never stated in the passage; the first sentence suggests that the author does not believe there will be automatic improvement as technology advances. (B) is irrelevant; the idea that railroads will one day be obsolete is never discussed. (C) is also irrelevant. The idea that air travel will eventually replace all other modes of transportation is never discussed. (D) is also irrelevant. The author never says that efficiency is the key to what makes a good transportation system. (E) is the answer; not only does the entire passage evaluate modes of transportation in terms of the mobility each

provides, but there is specific support for this answer when the author gives the reason why automobile transportation gained over railroad transportation: *it lost out in a large measure to the automobile because our road system is essentially two-dimensional. There are so many more roads than there are rail lines that we can travel by car from everywhere to everywhere else.*

Passage 4

1. (D) Choice (A) is irrelevant; the author never discusses the nature of the composition. (B) is not correct because it is too narrow; this passage is talking about *asymmetry,* and it is clear that this means not only *unequal meters* but *unequal rhythmic patterns.* (C) This answer is not correct because it is too broad; the author uses the word *irregularity* in connection with *rhythms* but not in connection with the entire *composition.* (D) is the correct answer and is clearly stated in the first and last sentences of the passage: *all but annihilated the comfortable symmetries of yesteryear . . . assymetry has almost become the norm.* (E) is irrelevant; the passage does not attempt to compare the *inspiration* of Stravinsky and earlier composers. The word *inspired* is used, but in fact it refers to the earlier composers, not Stravinsky.

2. (B) The author never tells us what his opinion of Stravinsky's music is, so (A), (C), (D), and (E) are obviously wrong.

3. (A) Choice (A) is correct because it is explicitly stated in the next-to-last sentence: *instances of asymmetry in the eighteenth and nineteenth centuries are notable precisely because they are exceptional.* (B) is irrelevant; the author does not say whether Stravinsky was inspired or not. (C) is contradicted by the same statement that supports (A). (D) is incorrect because the author says asymmetry did occur in the eighteenth century, though it was *exceptional.* (E) has a certain attraction because the author does say that *instances of asymmetry* in eighteenth-century music *represent moments of inspired freshness, or madness.* But he does not go so far as to say that asymmetry *was considered madness*—in fact, it is the author, rather than listeners of the eighteenth century, who applies the term *madness.*

4. (E) The answer to this question is almost stated word for word in the last line: *asymmetry has almost become the norm.*

5. (A) The answer is presented in the first sentence of the passage. It was a *rhythmic revolution*; therefore, Stravinsky was considered a revolutionary because of his rhythms. (B) is incorrect because the passage never says Stravinsky was *inspired*; the word is used only in reference to earlier music. (C) is incorrect because it assumes things that are not in the passage; Franck is merely used as an example of an earlier composer. (D) is incorrect because the passage never says that Stravinsky despised the music of the eighteenth and nineteenth centuries. (E) may seem appealing, but no such broad statement is made.

Passage 5

1. (D) Choice (A) is incorrect because the passage does not say that *all* fishing grounds are confined to the continental shelves; in the second sentence it says there are *a few exceptions.* (B) is incorrect because though the passage does mention both fishing grounds and seaweed, it does not say that there is any connection between the two. (C) is incorrect because the author merely gives three examples of the importance of continental shelves—fisheries, seaweed, and oil—but does

not say which is the most important. (D) This answer is explicitly stated in lines 5–8. (E) It is not the fishing grounds that are becoming scarcer as we dig for oil but the "petroleum reserves" in the third sentence.

2. (C) Choice (A) is incorrect because in the second sentence of the second paragraph, the author says that the 100-fathom contour *used to be taken as the boundary*, not now. (B) and (E) are clearly wrong because in the second and third paragraphs the writer points out at some length that both the width and the depth of the continental shelves vary from place to place. (D) at first looks as if it could be true, but the actual words in the passage are *on the American east coast . . . north of Cape Hatteras the shelf is as much as 150 miles wide,* and farther south the shelf is much narrower. (C) is correct; the words in the passage are *wherever the gentle declivity of the shelf changes abruptly to a steeper descent*—which is simply another way of saying *where the shelf begins to descend abruptly.*

3. (D) The discussion concerning the narrowness of the shelves is in the third paragraph. Choice (A) is incorrect because the Gulf Stream has nothing to do with the Pacific Coast, which is what the question is asking about. (B) is incorrect because the depth and width of the shelves are never related; they are discussed in separate paragraphs. (C) is similarly incorrect; the first sentence of the second paragraph implies that all shelves decline gradually, not just narrow ones. (D) is stated in the first sentence of the third paragraph. (E) is incorrect for the same reason as (C); no connection is ever made between the slope of the continental shelves and their width.

4. (A) You could have backed into choice (A) by a process of elimination. However, the answer is found in the first and third paragraphs. In the first paragraph the passage points out the importance of the continental shelves, and in the third paragraph it states that the shelf north of Cape Hatteras is very wide and the shelf south of Cape Hatteras very narrow. It seems safe to assume that the wide shelf is more important than the narrow one. (B) is just the opposite, since the author states in the first paragraph that petroleum geologists look to the oil that may lie under the continental shelves and the shelf north of Cape Hatteras is a particularly wide one. (C) is irrelevant, since the depth of the continental shelf north of Cape Hatteras is never related to its width. (D) is incorrect because the mention of the Gulf Stream in the third paragraph does not apply to the continental shelf on the east coast north of Cape Hatteras, but to the area off southern Florida. (E) is incorrect for a similar reason; the mention of *young mountains* applies to the *Pacific Coast,* not the continental shelf on the east coast north of Cape Hatteras.

5. (C) Statement (I) is made in the third paragraph. Statement (II) is made in the first paragraph; *petroleum geologists look more and more to the oil that may lie under* the continental shelves. Statement (III) is contradicted in the last paragraph: *at Hatteras and off southern Florida it is merely the narrowest of thresholds to the sea*—or narrower than along the Pacific coast. Statement (IV) can be found in the first paragraph: *seaweeds are gathered from their submerged plains to make scores of substances used in foods, drugs, and articles of commerce.* Statement (V) would be correct if only it read *south* of Cape Hatteras instead of *north;* if you were not alert you probably were fooled into choice (B).

Passage 6

1. (B) Choice (A) is too narrow. The idea that Huxley's writing may be considered revolutionary is mentioned only incidentally in the second paragraph. (B) is

the correct answer, since nearly every sentence in the passage relates to this idea. (C) is too narrow, since the success of the book is mentioned only in the first sentence. (D) is irrelevant; the author does not give us reasons why we should read *Brave New World*. (E) is too broad; the author may be leading up to a discussion of genetic engineering—in fact that is very likely—but the passage itself depends very heavily on Huxley's predictions, not just genetic engineering in general.

2. (B) Statement (I) is clearly made in the second paragraph. Statement (II) is contradicted in the first paragraph; the author states explicitly that Huxley *missed* predicting the atomic bomb. Statement (III) is wrong for the same reason. Statement (IV) can be found in the last sentence of the last paragraph. Statement (V) cannot be found in the passage.

3. (B) Choice (A) is irrelevant; Huxley's standing as a scientist is never mentioned in the passage. (B) is clearly stated in the last sentence of the first paragraph. (C) is irrelevant; the passage never mentions Huxley's predictions of human behavior. (D) is also irrelevant; the passage never says he *purposely disguised facts in novel form*. (E) is contradicted in the first sentence of the final paragraph: *the first prescriptions for "test-tubes babies" were written centuries ago*.

4. (C) The specific answer to this question can be found in the first paragraph, the next-to-last sentence: *by 1946, when he revised the introduction to the book, Huxley had reduced the time to his vision of the future to only 100 years*. That would make the predicted date 2046.

5. (E) The answer to this question is in the second sentence of the second paragraph: *As a matter of fact, parts of the book sound more like a report of what is happening on the genetics front than a cautionary tale about a controlled society*.

Passage 7

1. (B) Choice (A) is incorrect because the author never discusses the *invention* of the biofeedback machine. (B) is the correct answer, since nearly every sentence in the paragraph deals with ways to control the autonomic nervous system. (C) is too narrow; mystics and yogis are mentioned only in the first sentence of the second paragraph. (D) is also too narrow, since squirrel monkeys are mentioned only briefly as an example. (E) is irrelevant; there is no mention of the *results* of high blood pressure. Also, the passage deals with the entire autonomic nervous system, not just blood pressure.

2. (D) You could have backed into the right answer, since the other choices are not given in the passage.

3. (A) The answer to this question is in the first sentence of the second paragraph. The other choices are irrelevant.

4. (C) Choice (A) is irrelevant. The author never says that the technique of biofeedback is based on electricity. (B) is incorrect because the unconscious mind is never mentioned—only the *conscious mind*, and how the supposedly *involuntary system* can be brought under its control. (C) can be found in the third and fourth paragraphs: *man can learn to control his internal organs* and *the individual, through the signals, can get cues for internal changes*. (D) looks good at first, but note that in the second sentence of the last paragraph the words are *visual and neuromuscular feedback cues*—not just *neuromuscular feedback cues*. So (D) is too narrow. (E) This answer is incorrect because the technique of biofeedback is

based on control of the *involuntary* nervous system. This answer can be found in the first sentence of the third paragraph.

5. (E) The correct answer is explicitly stated six lines from the end of the passage: *the instruments are not new.*

Passage 8

1. (B) Choice (A) is incorrect because the author never says that the job of cyclic AMP is *the stimulation of cell enzymes.* The only mention of enzymes is in the fourth sentence of the second paragraph: *a critical factor for the production of cyclic AMP is an enzyme in the cell wall.* (B) is clearly stated in the second paragraph: hormones transmit messages, and cyclic AMP carries them out. (C) is incorrect since the author says not that the job of cyclic AMP is *control of the prostaglandins,* but rather the opposite. He states in the fourth line of the second paragraph: *a critical factor for the production of cyclic AMP is an enzyme in the cell wall. Prostaglandins may control this enzyme and thus control cyclic AMP production.* (D) is irrelevant; nowhere does the author mention the *production of hormones*—just their function. (E) is incorrect; in the last paragraph, it is stated that a *prostaglandin,* not cyclic AMP, lowers blood pressure.

2. (C) The correct answer, (C), is stated in the first sentence of the third paragraph: *Dr. Lee believes that a deficiency of certain prostaglandins naturally present in the kidneys may explain many if not most cases of hypertension.*

3. (B) Choice (A) is irrelevant; aspirin is mentioned only in the first paragraph—it *counters the activities of a prostaglandin,* and the prostaglandin is not said to be in the kidney. (B) is clearly stated in the first sentence of the passage: *aspirin counters the activities of a prostaglandin that . . . produces . . . headaches.* Choices (C), (D), and (E) are never stated in the passage. Remember that aspirin is discussed only in the first paragraph, before cyclic AMP, enzymes, and hormones are mentioned.

4. (B) Statement (I) is clearly made in the first sentence of the second paragraph. Statement (II) is contradicted in the first sentence of the passage: *aspirin counters the activity of a prostaglandin.* Statement (III) seems good at first, because Dr. Lee is said to have found *three different prostaglandins* in the kidney, but the next sentence is *it appears that the sole job of one of them is to lower blood pressure*—and nothing is said about the functions of the other two. Statement (IV) is made in the second paragraph: *a critical factor for the production of cyclic AMP is an enzyme in the cell wall.*

5. (E) The correct answer is clearly stated in the last two sentences of the passage; the prostaglandin acts *by relaxing small blood vessels, thus easing blood flow.*

Passage 9

1. (B) The correct answer is found in the third sentence of the passage: *60 percent are directly engaged in agriculture and another 15 percent cater to the needs of the farmers.*

2. (D) The correct answer is found in the last three sentences of the passage: *The traditional ways of farming and the traditional crops cannot support this growing population. Any solution to the problem must affect the very roots of the economy, striking at the relationship of man to the land and of man to his work, which is a close*

and meaningful one to the Greeks. How to do this without destroying the meaning of life is the question facing those entrusted with the introduction of change into the culture.

3. (A) The answer to this question can be found early in the passage: *The land is so mountainous, precipitous and stony that out of the 50,100 square miles that constitute the area of Greece, only 25 percent are usable as fields, orchards and pastures. . . . This condition, as well as a growing population, is basic to the need for technical change in Greece.* Choice (D) may fool some, but this answer is much too narrow; U.S. immigration laws just intensify the population problem. All the other choices are not supported at all by the passage.

4. (D) The answer is clearly stated in the passage: *through more than 2,500 years, Greece solved the problem of increasing population on limited land by sending out her people as colonists or immigrants.*

5. (C) Statement (I) is incorrect because the increase of 1.5 percent a year is not from 1900 to 1940 but from 1928 to 1940. Statement (II) is tricky; in the 1920s the United States established *immigration* quotas, not *emigration* quotas. An *emigration* quota would determine the number of people that could *leave* the United States, not *enter* it. Statement (III) is clearly supported by the passage: *early in the 1920s, two things took place,* one of which was *the exchange of populations with Turkey.* Statement (IV) is not made directly, but the passage discusses Greece's method of handling its population problem *through more than 2,500 years,* so the statement is correct.

ANTONYMS

All of the types of questions on the verbal SAT depend to some extent on vocabulary. However, the questions we here call "vocabulary questions" are *completely* dependent on vocabulary, because there are no clues from context as there are in other types of questions. You are simply given a word and asked to find its opposite, or antonym, among the five words that follow.

The simplicity of the vocabulary questions frightens some students, because they think that if they don't know the meaning of the words, there is no way they can arrive at the right answer. Yet as you shall see, some of the techniques for "backing into" the right answer that you have learned for sentence-completion and word-relationship questions can help you here too.

Remember, however, that what you are learning in this section is only a *technique*—a technique of taking a test. The technique will improve your score and help you avoid wrong answers, but it will *not* automatically increase your vocabulary. A later section of this book, "Building Your Vocabulary," is devoted to improving your vocabulary and your ability to deduce the meaning, or at least the approximate meaning, of unfamiliar words.

If You Know the Word

Look at the following example:

> ORAL: (A) whisper (B) printed (C) voluble (D) verbal
> (E) demonstrated

You know that the word *oral* has something to do with speech. Therefore, you must determine what *opposite word* you are looking for. The opposite of "speech" will be a word having something to do with "non-speech" or "silence." Back into the answer by going through the choices one at a time, drawing a line through those words that are not possible. Leave words you do not know. Go back to the question and do it now without reading ahead.

Chances are you eliminated (A) and (D) because *whisper* and *verbal* both have to do with speech. You may have left (C) because you did not know the meaning of *voluble*. But if you check the first few letters of *voluble*, you will see the beginning of a word you do know, "volume," which has to do with amount or size. A more familiar use of the word has to do with intensity of sound. Chances are you would eliminate (C), since *voluble* does seem related to *oral* and, in fact, it does mean "talkative." Left with (B) and (E), you should select the word that is more *specific*. You probably selected (B), since *oral* has to do with speech and *printed* has to do with writing. Something could be *demonstrated* either by speech or writing, so (E) is not an opposite.

Look at the next example:

> UNMARRED: (A) bachelor (B) scared (C) fought (D) ruined
> (E) irritated

If you look closely at the word *unmarred*, you probably recognize the word

"mar," which means "to spoil." Since "marred" means "spoiled" and the prefix *un-* added to the word changes the meaning to "unspoiled," you are looking for a word which means "spoiled." Go through the choices one at a time, drawing a line through those words which are *not possible*. Leave words you do not know. Please do it now before reading ahead.

You probably eliminated (A), (C), and (E) since (A) *bachelor* does not mean "spoiled"—the testers are probably counting on some students to read *unmarred* as "unmarried"; (C) *fought* does not mean "spoiled"; and (E) *irritated* does not mean "spoiled." If you left (B) you probably thought the word was formed from the word "scar," but that word would be spelled "scarred"; *scared* is from the word "scare." Therefore (D) is the right answer.

You may not think it is necessary to go through these steps since (D) *ruined* is such an obvious answer. But even if the word does seem to leap out at you as the obvious choice (and chances are it will not on the SAT), do *back into the correct answer* by eliminating those that are not possible. The Educational Testing Service (ETS) has constructed the test choices so that several may vary by shades of difference. Answers are often so apparently similar that they seem arguable. In order to ensure your best performance on these questions, approach each one systematically. Firmly eliminating choices will sharpen your perception in making fine, and final, decisions.

So when you know the word, (1) determine what the opposite meaning is; (2) go through the answer choices one at a time, drawing a line through those that are not possible and leaving words you do not know; and (3) if you are left with a choice between two words, choose the one that is more specific.

There is one special problem that many students—no matter how bright—have with vocabulary questions. Almost always, one or more of the answer choices *means the same* as the capitalized word. It's easy to forget that you're supposed to find the word of *opposite* meaning, because the synonym looks so "right," especially if you are quite sure of the meanings of both words! Don't be caught by this pathetic mistake.

If You Do Not Know the Word

Look at the following example:

> PITHY: (a) verbose (B) united (C) irritable (D) limited
> (E) callous

The first step in dealing with a word you do not know is to *circle the prefix, root, and suffix* if you can. (A later section in this book, "Building Your Vocabulary," will be of great help in learning to do this.) If you cannot identify any parts of the word, then circle either any word that you recognize (which may be the root), or any group of letters that may suggest a word that you know. Please go back to the question and do it now.

Most students will be able to circle the word "pit" and others will recognize the word "pith": PITH Y; PIT HY. Both suggest the meaning of something being central or in the center. Therefore, you are looking for a word that means "away from the center," "spread out," or "decentralized." Please go back to the question. Back into the right answer by eliminating all those words that are *not possible*. Please do it now before reading further. Your question probably looks like this:

> PITHY: (A) verbose (B) ~~united~~ (C) ~~irritable~~ (D) ~~limited~~
> (E) ~~callous~~

You probably eliminated (B) and (D) immediately because they both have to do with the idea of self-contained units—an idea similar to that of *pith*. Neither (C) nor (E) has anything to do with moving away from a fixed or central point. If you did not know the meaning of the word *callous*, think of any context in which you can place the word. How about the callus a person acquires on his hands and feet as a result of hard work? What is a callus? It is a layer of tough skin, and *callous* means "having calluses," or hardened or tough. So *callous* is not the word you are looking for. The answer is (A) *verbose*. Notice in it the word "verb," which is a shortened version of the word "verbal." *Verbose* means "wordy" or "talkative," which has the right connotation of "spread out." Actually, *pithy* means "concise" or "to the point," so the words are indeed opposites.

Look at the next example:

> DISSIDENCE: (A) ambivalence (B) hostility (C) amicability
> (D) irascibility (E) antipathy

Answer the question by following the procedure just outlined for you: (1) If you do not know the meaning of the word *dissidence*, circle the prefix, the root, and the suffix if you can; (2) If you cannot identify any parts of the word, circle any words you recognize or any group of letters that may suggest a word that you know.

An additional technique: If you cannot arrive at an approximate definition of the word being tested, *determine whether or not the word is positive or negative* on the basis of either the prefix, the root, or words suggested by the first few letters. If you are working with a negative word, your answer will be positive, so eliminate all the negative choices. If you are working with a positive word, your answer will be negative, so eliminate all the positive choices.

Please go back to the question and back into the correct answer by eliminating all those words that are not possible. You probably recognized the prefix *dis-* and circled it, along with the suffix *-ence*. If you know that the prefix *dis-* means "away," "off," "down," or "opposing," you realize you are dealing with a negative word. If you do not know what the prefix means, you may have thought of words beginning with those letters, such as "disassemble" and "disappoint." You know that you are looking for a positive word. The first step, then, is to go through the question eliminating all the negative choices. Your question probably looks like this:

> DISSIDENCE: (a) ambivalence (B) ~~hostility~~ (C) amicability
> (D) irascibility (E) ~~antipathy~~

You probably eliminated (B) and (E) immediately because they are clearly negative words. The next step should have been to circle the prefix, root, and suffix of the remaining words. The words should look like this: (A) ambi val ence (C) amic ability (D) ir asc ibility.

Although many students do not know that the prefix *ambi-* means both, most students are familiar with the word "ambidextrous," which means "skillful with both hands," so the meaning of "both" might be arrived at in that way. The word "ambiguous" might also have come to mind, a word which suggests something that can be taken in two ways or is unclear. The word *ambivalence* will therefore pose a problem, since "ambidextrous" suggests certain positive qualities and "ambiguous" certain negative ones. You probably left it and went on to the word *amicability*. Since most students are required to take a language in high school, there should be a certain degree of familiarity with the root of this word, *amic-*, which means "love" or "friend." Words derived from this root are "amiable" and "amity"; it is obviously a positive word and should be left as a possibility. The word *irascibility* should be easy to eliminate; it sounds negative. But more important, the prefix *ir-* means "not," or "opposing." There are also many words beginning with *ir-*, such as "irregular" and "irritating." These should only rein-

force your feeling that *irascibility* is a negative word. Left with (A) and (C), you probably decided on the correct answer, which is *amicability*, the more positive of the two choices.

PRACTICE EXERCISE

Here are some antonym questions for you to practice on. Follow the steps outlined for you. The answers and explanations will follow.

Each question below consists of a word in capital letters followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

1. ABHOR: (A) surrender (B) attach (C) adore (D) regret (E) detest
2. VIVACIOUS: (A) clever (B) active (C) versatile (D) phlegmatic (E) animated
3. DISCORD: (A) noise (B) reserve (C) amity (D) misconduct (E) loathing
4. INDIFFERENT: (A) curious (B) uninquisitive (C) rigid (D) similar (E) flexible
5. OBSTREPEROUS: (A) servile (B) fawning (C) foolish (D) naive (E) tractable
6. CIRCUITOUS: (A) peripheral (B) direct (C) cautious (D) amusing (E) radial
7. MOLLIFY: (A) vindicate (B) adorn (C) intensify (D) sweeten (E) appease
8. QUERULOUS: (A) indisposed (B) critical (C) accepting (D) curious (E) complaining
9. CAPTIOUS: (A) fault-finding (B) carping (C) complimentary (D) caustic (E) critical
10. REPREHENSIBLE: (A) idealistic (B) cowardly (C) praiseworthy (D) stubborn (E) offensive

ANSWERS AND EXPLANATIONS

1. (C)	2. (D)	3. (C)	4. (A)	5. (E)
6. (B)	7. (C)	8. (C)	9. (C)	10. (C)

1. (C) If you did not know that the word *abhor* means "detest" or "loathe," you either recognized the prefix *ab-*, which means "from" or "down," or thought about words beginning with those first few letters, such as "absent" or "abdicate," and realized you were dealing with a negative word. You probably eliminated (D) and (E) because they are both negative words and you are looking for a word that is positive. If you carefully consider choices (A), (B), and (C), you realize that the only word with positive connotations is (C). (A) *surrender* is a word with negative associations; and (B) *attach* is simply an action which can be either positive or negative.

2. (D) Most high school students will probably be familiar with the prefix *viv-*. In French, *vivre* means "to live," and from history texts we have the much-heralded expression *vive la France* or "long live France." Students who are not familiar

with this root might be able to think of words beginning with the first few letters of the word, such as "vital," which means "full of life." In either case, you are looking for a word which means the opposite, or "not full of life." You were probably able to eliminate (A), (B), (C), and (E) because none of these choices means that. You probably left (D) because you did not know the definition of *phlegmatic*, which is "impassive" or "apathetic," and you would have backed into the correct choice.

Another technique is to decide whether *vivacious* is a positive or negative word. Depending upon how much information you were able to deduce, you probably decided that it is a positive word; therefore, you would be looking for a negative answer. You would probably have eliminated (A), (B), (C), and (E) because they are positive words. The only negative-sounding word is *phlegmatic*. In fact, you can even recognize the word "phlegm" from which this word is built, and "phlegm" is rarely found in a positive context.

3. (C) If you did not know that *discord* means "disagreement," it should have been easy to fix on the negative connotation of the word because of the prefix *dis-*, which means "away" or "off," and negative words associated with *dis-* like "discard" or "disregard." You were probably able to back into the correct answer by eliminating all the negative words and those that are neither negative or positive.

4. (A) This question should have presented few problems. Most students know that *indifferent* means "marked by lack of interest or concern." Therefore, you would be looking for a word which means "interested" or "concerned." You should have been able to eliminate (B), (C), (D), and (E) since none of the four choices means that. The right answer is *curious*, which is certainly not a difficult word.

5. (E) *Obstreperous* is a word that is frequently found on tests like the SAT. If you were not certain about the prefix—and *ob-* is a fairly difficult prefix, with several possible meanings—you might have been able to think of words beginning with the prefix, such as "obstinate" and "obstruct," both negative words. You are therefore probably looking for a positive word for your answer. The first thing you should have done is eliminate all the negative words you can. You probably eliminated (A) because you recognized the letters *serv-*, which begin words such as "servant" and "serve," which are not positive words, even if you do not know that *servile* means "befitting a slave." You probably also eliminated (C) and (D) because you recognize these words as negative. You may have left (B) and (E) if you do not know that *fawning* is groveling like a dog or shrinking in fear before someone in a superior position and that *tractable* means "manageable." Left with these two choices, and knowing that you are looking for a positive word, you probably selected the right one, *tractable*, since the suffix, *-able*, is frequently found in positive words.

6. (B) This word should have presented you with little difficulty. You were probably able to ascertain the prefix as *circum-*, which means "around," and knew that you would be looking for a word which means "not around" or (B) *direct*.

7. (C) If you did not know the meaning of this word, it would have helped to look at the choices. You may have noticed that (B), (D), and (E) are obviously positive words, and if you know that *vindicate* means "to free from blame," you would have seen that of the five choices, four are positive. Now examine *mollify*. Does it sound positive or negative? Again, that will be a personal feeling, but it does seem to have a soft sound, like some of the choices. You may not have been

able to think of words beginning with the first few letters, but there is "emollient," which has become a fashionable household word. The family members who handle the weekly laundry may know that "emollient" means "softening." The best answer would be (C) *intensify*.

8. (C) You should have little difficulty in determining that *querulous* is a negative word, even without knowing the definition, which is "whining" or "complaining." *Querulous* sounds like the word "quarrel." You probably eliminated (A), (B), and (E) immediately, since they are all negative words. Left with (C) and (D), it should have been apparent that (C) is the more positive of the two.

9. (C) If you did not know that *captious* means "fault-finding," the word would probably present some difficulty. Knowing that the prefix *cap-* means "take" or "seize" would be some help, since it is definitely not a positive word. Therefore, you would eliminate all of the negative choices. You probably drew a line through (A), (B), (D), and (E), backing into the correct answer. You might also have determined that *captious* was a negative word by first examining the choices, four out of five of which are negative.

10. (C) The word *reprehensible* should have been easy to define. You probably associated the first few letters of the word with a word with which you are familiar, "reprimand," which means "to express disapproval" or "to censure." The only positive choices are (A) *idealistic* and (C) *praiseworthy*, and (C) is obviously more nearly an opposite.

SAMPLE TESTS

We have now covered all the types of questions that occur on the verbal part of the SAT.

At the beginning of this book, you were asked to take Verbal Aptitude Test 1. Now that you have learned various techniques for the test, go back and take the test again, using the second answer sheet provided on page 13. Use all the techniques of circling and underlining that you have learned. Do it now.

Now determine your scores for your first and second tries. The answers are on page 125. If you have learned the test-taking techniques thoroughly, your score should be higher on your second try.

Now you are ready for Verbal Aptitude Tests 2–8. If you find you are having trouble with any particular type of question, go back to the section of this book dealing with that type. If you have general problems with vocabulary, you might study the section "Building Your Vocabulary" before you take all the tests. You can also improve your vocabulary simply by looking up any words that confused you in questions you got wrong.

As on the actual SAT, you will use an answer sheet like the one you used for Verbal Aptitude Test 1 for the practice tests that follow.

Verbal Aptitude Test 2

Time—30 Minutes
45 Questions

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole.

1. The hard-hearted Pharaoh caused the Hebrew slaves to serve with _____.

   (A) rigor (B) revulsion (C) retraction
   (D) restitution (E) respite

2. The _____ odor from the bottle of cooking oil convinced Bernice that the oil was no longer usable.

   (A) rancid (B) rabid (C) quizzical
   (D) querulous (E) pulmonary

3. On the _____ and narrow mountain road, the automobile almost fell over many a time.

   (A) sinuous (B) sinewy (C) simian
   (D) serrated (E) sedulous

4. From the new moon to the full moon, Sam's covetous obsession worsened. "It seems," remarked a disconsolate store owner, "that his _____ increases with the _____ moon."

   (A) paranoia . . waning
   (B) kleptomania . . waxing
   (C) hysteria . . postprandial
   (D) afflatus . . apocalyptic
   (E) delusion . . anomalous

5. To _____ violence because of poverty will in the long run not solve poverty but only _____ violence.

   (A) laud . . diminish
   (B) appease . . abate
   (C) applaud . . destroy
   (D) teach . . immolate
   (E) condone . . exacerbate

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

6. INTIMATE: (A) prove (B) establish
   (C) hint (D) suggest (E) frighten

7. CULPABLE: (A) noble (B) innocent
   (C) obdurate (D) reprehensible (E) disgraceful

8. CHICANERY: (A) candor (B) sophistry
   (C) prudence (D) duplicity (E) subterfuge

9. CONTEMPT: (A) approbation (B) scorn
   (C) insult (D) interest (E) antagonism

10. ARTFUL: (A) ingenuous (B) crafty
    (C) discriminating (D) knowing (E) keen

11. TACITURN: (A) reticent (B) reserved
    (C) voluble (D) uncommunicative (E) mute

12. BUOYANT: (A) sanguine (B) levity
    (C) resilient (D) oppressive (E) light

13. CEDE: (A) yield (B) surrender (C) plant
    (D) seize (E) relinquish

14. CACOPHONY: (A) incompatibility
    (B) irregular (C) harmony (D) strife (E) regret

15. TURGID: (A) tumid (B) swollen (C) corpulent
    (D) clear (E) redundance

16. EQUITABLE: (A) monotonous (B) biased
    (C) constant (D) uneven (E) reasonable

17. COGENT: (A) compelling (B) valid
    (C) untenable (D) relevant (E) persuasive

18. PENURIOUS: (A) opulent (B) desolate (C) greedy (D) ignoble (E) mean

19. BULWARK: (A) restraint (B) impediment (C) breach (D) prohibition (E) legion

20. ARCHETYPE: (A) original (B) ideal (C) copy (D) prototype (E) standard

Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Select the lettered pair that *best* expresses a relationship similar to that expressed in the original pair.

21. CHEESE:EDIBLE:: (A) water:potable (B) meat:cattle (C) milk:dairy (D) solid:liquid (E) rotundity:solvency

22. LEMON:CITRUS:: (A) cow:bovine (B) bee:avian (C) man:simian (D) cat:canine (E) dog:ursine

23. NEEDLE:EMBROIDERY:: (A) hammer:fascism (B) automobile:aviation (C) mushroom:mycologist (D) lobster:cuisine (E) scalpel:surgery

24. QUAY:SIDEWALK:: (A) ship:truck (B) airplane:ship (C) plaster:wood (D) propulsion:dispersion (E) car:canoe

25. VANDAL:PROPERTY:: (A) erudition:knowledge (B) excision:operation (C) fanfare:trumpet (D) fracas:brawl (E) termite:wood

26. OVOID:EGG:: (A) conical:square (B) spherical:ball (C) tridentate:box (D) elliptical:circle (E) pyramidal:cylinder

27. SCRAWL:CALLIGRAPHY:: (A) light:diffraction (B) infirmity:debility (C) incongruity:neptitude (D) infraction:law (E) splatter:painting

28. POVERTY:PENURIOUS:: (A) lassitude:vivacious (B) celibacy:lecherous (C) levity:lethal (D) austerity:lascivious (E) hairiness:hirsute

29. FILM:IMAGE:: (A) palette:solvent (B) canvas:painting (C) smock:easel (D) filter:light (E) darkness:effulgence

30. INCLEMENT:HALCYON:: (A) equivocal:ambiguous (B) feigned:insincere (C) unkempt:neglected (D) unyielding:acquiescent (E) venial:trivial

Select the word or set of words that *best* completes each of the following sentences.

31. Offensive jokes about national origin are both _____ and _____.

(A) admirable . . advantageous
(B) propitious . . pejorative
(C) destructive . .salutary
(D) derogatory . . disparaging
(E) beatific . . unpleasant

32. The manufacturer's plan was designed to _____ energy wastefulness without appreciably _____ profits.

(A) curtail . . diminishing
(B) augment . . increasing
(C) amplify . . dilating
(D) animate . . degenerating
(E) enrich . . benefiting

33. Children in poverty-stricken areas have a tendency to be _____.

(A) overfed (B) opulent (C) ostentatious
(D) emaciated (E) pugnacious

34. This plant is so rare that it is _____ only to one section of a single Pacific island.

(A) absent (B) indigenous (C) splendid
(D) terminal (E) tropical

35. David never tries to conceal anything. He is _____ and to the point.

(A) surreptitious (B) explicit (C) endemic
(D) elegiacal (E) covert

The passages below are followed by questions based on their content. Answer all questions following a passage on the basis of what is *stated* or *implied* in that passage.

There is no evidence of the manufacture of weapons as instruments of offense or defense against other men until the Neolithic. When tribes began to cultivate herds of livestock, raids began to occur. In order to protect property against marauders, defenses were erected, and new and more weapons manufactured. Sometimes a marauder was captured, leading to the discovery that instead of

killing the enemy one could enslave him. Thus was slavery born, in the Neolithic of what is now the Middle East.

A Neolithic raid may not have been comparable to a Second World War, but it was war nonetheless. After the horse was introduced into America, raiding for horses became a common practice among the Plains Indians, but the raid was usually made without any desire to harm the owners of the horses. Often as many as a hundred horses would be taken by the raiding parties, as well as mules and cattle. Not infrequently these raiding parties would result in bloody conflicts.

In the Neolithic the increase in the number of weapons found—battle-axes, rapiers, and swords of bronze or iron—suggests that either individuals or groups or both had made the sorry discovery that by acquiring the property of one's neighbor, one could increase one's own wealth, that war, in fact, was an economically productive activity—a totally erroneous belief which bedeviled man down to the present day. It was in the Neolithic that man started off on the wrong foot with the discovery that the acquisition of large amounts of property leads to power.

Nonliterate peoples like the Australian aborigines and the Eskimos do not engage in warlike activities at all. The raiding of the American Indians is to be regarded as a form of warfare, but the head-hunting activities of New Guinea natives, and until recently the Dayaks of Borneo, are not properly so regarded. The motivation in such cases is not really to inflict injury upon another tribe in order to obtain its property, but rather to obtain for ritual purposes a magical increase in strength.

Since wood is perishable, we know very little concerning its uses as material for weapons, but from the bone and stone points which have been recovered by the thousands, it is clear that in many cases they were associated with a wooden handle or shaft. Slingstones, used as ammunition, found in Iran date back to about 4500 B.C. Maceheads have been found at Merimde, and battle-axes became abundant during the Bronze Age (1500 B.C.).

36. The writer implies that

(A) only highly literate people like the Eskimos engage in warlike activities
(B) man has been deluding himself since the Neolithic Age concerning the positive consequences of war
(C) the raiding practices of the American Indian were retaliatory in nature
(D) there is no evidence that weapons were manufactured as instruments of offense or defense
(E) the head-hunting activities of New Guinea natives, although carried out for ritual purposes, can properly be regarded as a form of warfare

37. The author's attitude toward man's belief that war is economically productive can be described as

(A) approving (B) impartial (C) deploring
(D) genial (E) uncertain

38. The author provides information to answer which of the following questions?

(A) What is the corresponding relationship between illiteracy and pacifism?
(B) How was the horse introduced into America?
(C) Why was the "Neolithic" Age so called?
(D) What was the origin of the "raid"?
(E) Why can we not consider war an economically productive activity?

39. The author suggests that

(A) the Neolithic period paved the way for free enterprise
(B) self-aggrandizement in any period leads to bloody conflicts
(C) the phenomenon of slavery was born out of a need to defend one's property against marauders
(D) the Plains Indians adopted raiding practices as retaliatory rituals
(E) warlike activities can only be labeled as such when there is a clear motivation to inflict injury on another tribe.

40. Which is *not* true of slingstones?

(A) They were used as ammunition.
(B) They became abundant during the Bronze Age.
(C) They date later than 5000 B.C.
(D) They were found in Iran.
(E) They were recovered by the thousands.

The annelid (ringed or segmented) worms include several classes, one of which, the Polychaeta (many bristles) includes most marine annelids. Many of the polychaetes, or bristle worms, are active swimmers that make their living as predators; others are more or less sedentary, building tubes of various

sorts in which they live, either feeding on detritus in sand or mud or on plankton which they strain from the water. Some of these worms are among the most beautiful creatures of the sea, their bodies shining with iridescent splendor, or adorned with feathery crowns of tentacles in soft and beautiful colors.

In their structure they represent a great advance over lower forms. Most of them possess a circulatory system (although the blood worm, Glycera, much used as bait, has no blood vessels but a blood-filled cavity between the skin and the alimentary canal) and so are able to dispense with the thinness of body of the flatworms, for the blood flowing through vessels transports food and oxygen to all parts of the body. The blood is red in some, green in others. The body consists of a series of segments, several of the anterior ones being fused to form the head. Each segment bears a pair of unbranched, unsegmented paddlelike appendages for crawling or swimming.

Bristle worms include many forms. The nereids, or clam worms, often used for bait, spend most of their lives in crude burrows among stones on the sea bottom but emerge to hunt or, in swarms, to spawn. The sluggish scale worms live under rocks in muddy burrows or among the holdfasts of seaweed. The serpulid worms build variously shaped limy tubes from which only their heads emerge; other worms, like the beautifully plumed Amphitrite, form mucous tubes under rocks or crusts of coralline algae or on muddy bottoms.

41. According to the writer, clam worms

(A) spawn in crude burrows on the sea bottom
(B) form mucous tubes under rocks or crusts of coralline algae
(C) emerge from sea burrows to forage
(D) burrow into limy tubes from which only their heads emerge
(E) have no blood vessels

42. The word "sedentary" as applied to bristle worms in the first paragraph most probably means

(A) stationary
(B) noninterfering
(C) defective
(D) diligent
(E) suggestive

43. This passage describes

(A) the living habits of the annelid
(B) the structure and variety of the Polychaeta
(C) plankton as a primary food source
(D) the circulatory system of the Amphitrite
(E) the advance in structure of the annelid as compared with lower forms

44. One can conclude from the passage that the author's attitude toward marine annelids is

(A) admiring (B) condescending (C) tolerant
(D) caustic (E) facetious

45. The major advance in the structure of the Polychaeta over lower forms is

(A) the feathery crown of tentacles
(B) the segmented paddlelike appendages
(C) the circulatory system
(D) their predatory nature
(E) their iridescent coloration

Answer Sheet—Verbal Aptitude Test 2

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
25	☐	☐	☐	☐	☐
26	☐	☐	☐	☐	☐
27	☐	☐	☐	☐	☐
28	☐	☐	☐	☐	☐
29	☐	☐	☐	☐	☐
30	☐	☐	☐	☐	☐
31	☐	☐	☐	☐	☐
32	☐	☐	☐	☐	☐
33	☐	☐	☐	☐	☐
34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐
36	☐	☐	☐	☐	☐
37	☐	☐	☐	☐	☐
38	☐	☐	☐	☐	☐
39	☐	☐	☐	☐	☐
40	☐	☐	☐	☐	☐
41	☐	☐	☐	☐	☐
42	☐	☐	☐	☐	☐
43	☐	☐	☐	☐	☐
44	☐	☐	☐	☐	☐
45	☐	☐	☐	☐	☐

Verbal Aptitude Test 3

Time—30 Minutes
45 Questions

The passages below are followed by questions based on their content. Answer all questions following a passage on the basis of what is *stated* or *implied* in that passage.

The Church from the first took account of the influence of color as well as music upon the emotions. From the earliest times it employed mosaics and paintings to enforce its dogmas and relate its legends, not merely because these were the only means of reaching people who could neither read nor write, but also because they instructed them in a way which, far from leading to critical inquiry, was peculiarly capable of being used as an indirect stimulus to moods of devotion and contrition. Next to the finest mosaics of the first centuries, the early works of Giovanni Bellini, the greatest Venetian master of the fifteenth century, best fulfill this religious intention. Painting had in his lifetime reached a point where the difficulties of technique no longer stood in the way of the expression of profound emotion. No one can look at Bellini's pictures of the Dead Christ upheld by the Virgin or angels without being put into a mood of deep contrition, nor at his earlier Madonnas without a thrill of awe and reverence. And Giovanni Bellini does not stand alone. His contemporaries, Gentile Bellini, the Vivarini, Crivelli, and Cima de Conegliano all began by painting in the same spirit, and produced almost the same effect.

The Church, however, thus having educated people to understand painting as a language and to look to it for the expression of their sincerest feelings, could not hope to keep it always confined to the channel of religious emotion. People began to feel the need of painting as something that entered into their everyday lives almost as much as we nowadays feel the need of the newspapers; nor was this unnatural considering that, until the invention of printing, painting was the only way, apart from direct speech, of conveying ideas to the masses. At about the time when Bellini and his contemporaries were attaining maturity, the Renaissance had ceased to be a movement carried on by scholars and poets alone. It had become sufficiently widespread to seek popular as well as literary utterance, and thus, toward the end of the fifteenth century, it naturally turned to painting, a vehicle of expression which the Church, after a thousand years of use, had made familiar and beloved.

1. The main idea of this selection is best expressed as
   (A) the political influence of the church
   (B) Bellini's religious influence
   (C) the Renaissance spirit and its influence through art
   (D) art as a method of communicating with the masses
   (E) the importance of color and music upon emotion

2. The word "contrition" in the second sentence most nearly means
   (A) humility (B) critical inquiry (C) remorse
   (D) fidelity (E) loyalty

3. According to the author,
   (A) the Church used great art to manipulate the will of the masses
   (B) when the Church recognized the emotional effects of color and music, it assumed, correctly, that painting would evoke the same response.
   (C) the Church perceived the value of art in teaching its precepts to the illiterate masses
   (D) the Church encouraged the critical inquiry which great art engendered
   (E) Bellini and his contemporaries were unique in the fact that they achieved basically the same effect through widely different approaches

4. From the information given it can be concluded that
   (A) as compared with the printed word, art is a vastly more primitive method of communication
   (B) great religious art put people in a religious frame of mind

(C) since the Church had educated people to understand painting as a language, as an expression of their sincerest feelings, it has come to be considered an artificial medium
(D) the original impetus of the Renaissance was Church-inspired
(E) Bellini struggled with the difficulties of techniques throughout most of his life

5. The author implies that painting

(A) was the only effective way to stir the emotions of the masses
(B) was a less than effective way of relating by the Church
(C) was a less than effective way of relating legends and ideas
(D) in the first centuries rivaled mosaics for communicating church dogma
(E) could never hope to assume the place in people's lives that the newspaper has come to assume in our own

Critics of Marx have frequently argued that the logic of Marx's theory is not impeccable. As a necessitarian, so it is said, he leaves no room for freedom, and thus deprives ideological exhortation itself of any point. In this matter, his critics have failed to see that the term "inevitable," as used by Marx, is primarily a term of encouragement for some and of doom for others. It belongs not to the language of scientific description, but to that of ideological conflict. Its function is the prophetic one of awakening the workingmen of the world to a sense of their own historic mission as deliverers of mankind.

One further implication of Marx's interpretation of history remains to be observed. For Marx, developments in religious, philosophical, or political thought are essentially by-products of modifications in the modes of material production and organization. What he proposed, in effect, was to invert Hegel's idealistic interpretation of history so that the ideational "superstructure" could be correctly understood for the first time as an effect rather than as a determining cause of basic changes in the social system. The consequence of this inversion is that Marx construed progress, not in terms of spiritual self-development or "freedom," but, rather, in terms of the improvement of the underlying economic conditions of social life. The amelioration of man's lot, for Marx, thus begins, even if it does not end, with a solution to the problem of poverty. And the salvation of man, the cure for his loneliness as well as for his misery, lies, at bottom, in a collective attack upon this problem. "The anatomy of civil society," said Marx, "is to be sought in political economy." So, also, he might have added, with the anatomy of human progress.

6. The word "impeccable" in the first sentence most nearly means

(A) pragmatic
(B) effete
(C) perfect
(D) effective
(E) ideological

7. The critics of Marx cite as his failure

(I) the admission of a basic underlying inevitability by which man's actions are determined
(II) his concept of inevitability, which lacks scientific description
(III) his definition of progress as improvement of economic conditions
(IV) a closed ideological theory that admits no freedom
(V) the fact that he viewed material production as a by-product of political thought

(A) I only (B) II only (C) II and III only
(D) I and IV only (E) III, IV, and V.

8. According to the author,

(A) Hegel considered the ideational "superstructure" an effect rather than a determining cause of basic changes in the social system
(B) Marx believed that the improvement of man's condition begins with an understanding of the anatomy of human progress
(C) the solution to the problem of man's misery rests in a collective attack upon the social system
(D) the mass of men treat the idea of inevitability as a term of doom
(E) Marx considered the solution to the problem of poverty the first step toward the salvation of man

9. According to the author, which of the following observations are true concerning Marx's interpretation of history?

(A) Ideological conflict is the basis for sound political theory.
(B) Workingmen will inevitably awaken to a sense of their own historic mission as deliverers of mankind.

(C) Changes in man's thinking result from changes in modes of material production and organization.
(D) The composition of civil society lies in an understanding of the conditions of the social environment.
(E) Determining the cause of basic changes in the social system will lead to the amelioration of man's lot.

10. The author's development of the passage suggests that, if continued, the passage would next
(A) discuss Marx's views on collectivism
(B) further develop Hegel's idealistic interpretation of history
(C) further discuss arguments propounded by the critics of Marx
(D) defend the workingman as prophet and deliverer
(E) discuss the logic of Marx's theory

Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Select the lettered pair that *best* expresses a relationship similar to that expressed in the original pair.

11. PHYSICIAN:X-RAY::
(A) chronometer:timekeeper
(B) whistle:referee (C) actuary:statistics
(D) quilt:breadbox (E) compose:sonata

12. IRRIGATION:GROWTH:: (A) weeds:grass
(B) trees:birds (C) sunlight:photosynthesis
(D) approval:approbation (E) cream:whey

13. JOURNALIST:ARTICLE:: (A) satyr:myth
(B) cloud:sky (C) galaxy:star (D) awl:augur
(E) farmer:grain

14. TRICKLE:DELUGE:: (A) breeze:gale
(B) storm:drizzle (C) precipitation:rain
(D) eclipse:lumination (E) gelid:frozen

15. PIG:MAMMAL:: (A) climate:weather
(B) orange:citron (C) peony:flower
(D) book:journal (E) painting:poster

16. PECCADILLO:FELONY::
(A) quilt:bedspread (B) import:significance
(C) immolation:sacrifice (D) puddle:sea
(E) imprecation:curse

17. PERVADE:PERMEATE:: (A) rejuvenate:age
(B) replenish:deplete (C) repudiate:recognize
(D) retrieve:lose (E) distend:inflate

18. POSTPONE:DEFER:: (A) exude:absorb
(B) deflect:attract (C) profane:sanctify
(D) solicit:refuse (E) fluctuate:oscillate

19. CADAVEROUS:BUXOM::
(A) churlish:courteous (B) fraught:laden
(C) defective:lacking
(D) methodical:systematic
(E) slovenly:unkempt

20. EROSION:GULLY:: (A) elevator:weight
(B) embryo:emendation (C) smoke:pollution
(D) ergot:wheat (E) paralysis:ether

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole.

21. The experts wanted to determine whether the painting was _____ or merely a(n) _____ imitation.
(A) radical . . reactionary
(B) authentic . . bogus
(C) lustrous . . incendiary
(D) supernumerary . . essential
(E) sleazy . . flawless

22. His _____ habits result in his _____ work.
(A) indolent . . inferior
(B) impeccable . . objectionable
(C) slothful . . praiseworthy
(D) pleasant . . offensive
(E) concise . . shameful

23. Government agents are not likely to be _____ about _____ operations.
(A) happy . . charitable
(B) olfactory . . onerous
(C) militant . . mutinous
(D) candid . . clandestine
(E) intuitive . . irrevocable

24. Their noble reputation was _____ because they associated with the lower classes.
(A) boisterous (B) blithe (C) blighted
(D) bland (E) bilious

25. That sly little rabble-rouser is as _____ as a fox.
(A) acrid (B) acquiescent (C) acetic
(D) winsome (E) wily

26. The most _____ feature of his struggle was his overcoming _____ practices.

(A) pleasant . . constructive
(B) disconsolate . . bovine
(C) parasitic . . carnivorous
(D) factious . . liberal
(E) arduous . . discriminatory

27. The acrobat's _____ maneuvers, if performed by an untrained amateur, could prove _____.

(A) sinful . . blissful
(B) daring . . disastrous
(C) engaging . . annoying
(D) chaotic . . edifying
(E) dramatic . . callous

28. When the storm had ended, the choppy waters _____ and the tide _____.

(A) stirred . . rose
(B) increased . . ceased
(C) devalued . . decreased
(D) abated . . subsided
(E) clamored . . sank

29. The hypocrite loves to _____ others for wrongdoing even though he is as guilty as they.

(A) conciliate (B) commandeer (C) cover
(D) covet (E) chide

30. Another name for a druggist or pharmacist is _____.

(A) apothecary (B) apex (C) antipathy
(D) annuity (E) amnesia

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

31. OVERT: (A) manifest (B) hidden
(C) surmountable (D) indigent (E) under

32. INDEFATIGABLE: (A) tireless
(B) flagging (C) exuberant (D) competent
(E) self-complacent

33. DECOROUS: (A) dainty (B) rude
(C) cowardly (D) heroic (E) pusillanimous

34. VAPID: (A) insipid (B) unsavory
(C) imaginative (D) prosaic (E) jejune

35. VILIFY: (A) sympathize (B) menace
(C) praise (D) defy (E) intimidate

36. PHLEGMATIC: (A) vital (B) apathetic
(C) insensitive (D) serene (E) indolent

37. COGNIZANCE: (A) anecdote
(B) ignorance (C) sensibility (D) competence
(E) adhesiveness

38. RELEVANT: (A) necessary (B) consistent
(C) extraneous (D) indigenous (E) impious

39. NETTLE: (A) ruffle (B) provoke (C) vex
(D) comfort (E) irritate

40. ESCHEW: (A) masticate (B) joust
(C) pursue (D) reprimand (E) denounce

41. RESOLUTE: (A) vacillating
(B) tough-minded (C) persuasive (D) boastful
(E) valiant

42. PAUCITY: (A) abundance (B) wasting
(C) indifference (D) impressiveness
(E) insignificance

43. PROWESS: (A) mettle (B) timidity
(C) imaginative (D) forthright (E) intrepidity

44. HOODWINK: (A) delude (B) hoax
(C) enlighten (D) condemn (E) sorcery

45. TYRO: (A) autocrat (B) dilettante
(C) plutocrat (D) virtuoso (E) philanthropist

Answer Sheet—Verbal Aptitude Test 3

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
25	☐	☐	☐	☐	☐
26	☐	☐	☐	☐	☐
27	☐	☐	☐	☐	☐
28	☐	☐	☐	☐	☐
29	☐	☐	☐	☐	☐
30	☐	☐	☐	☐	☐
31	☐	☐	☐	☐	☐
32	☐	☐	☐	☐	☐
33	☐	☐	☐	☐	☐
34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐
36	☐	☐	☐	☐	☐
37	☐	☐	☐	☐	☐
38	☐	☐	☐	☐	☐
39	☐	☐	☐	☐	☐
40	☐	☐	☐	☐	☐
41	☐	☐	☐	☐	☐
42	☐	☐	☐	☐	☐
43	☐	☐	☐	☐	☐
44	☐	☐	☐	☐	☐
45	☐	☐	☐	☐	☐

Verbal Aptitude Test 4

Time—30 Minutes
45 Questions

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole.

1. By _____ the problem, Charles found a way to _____ its ramifications.

   (A) circumventing . . avoid
   (B) demonstrating . . decay
   (C) deceiving . . elude
   (D) modifying . . earn
   (E) evading . . affront

2. The physician prescribed the _____ medicament for its _____ effect.

   (A) narcotic . . detrimental
   (B) palliative . . soothing
   (C) toxic . . deleterious
   (D) carcinogenic . . pernicious
   (E) proscribed . . baneful

3. His _____ patriotism was reflected in his _____ love of his native language.

   (A) obscure . . pleasant
   (B) zealous . . ardent
   (C) logical . . irrational
   (D) pernicious . . magnanimous
   (E) evident . . convert

4. "How repugnant! How abject! How _____!"

   (A) pleasant (B) emollient (C) reverent
   (D) grotesque (E) celestial

5. Unlike the petty king, who is petulant and _____, the prince is both easy-going and _____.

   (A) cretinous . . sluggish
   (B) theocratic . . slothful
   (C) retrograde . . repugnant
   (D) propitious . . oblique
   (E) vindictive . . magnanimous

6. Although Tom usually exhibits _____, he was so excited by the last remark that he retorted in complete and unbridled _____.

   (A) participation . . exposure
   (B) happiness . . instigation
   (C) complacency . . complaisance
   (D) resurgence . . resilience
   (E) composure . . fury

7. It is unlikely that a(n) _____ person can serve as an impartial judge.

   (A) urbane (B) querulous (C) biased
   (D) gruff (E) hedonistic

8. Everyone is entitled to _____ remuneration for his work.

   (A) charitable (B) pitiful (C) bucolic
   (D) equitable (E) elusive

9. While the authorities of the disease control center were once confident that this type of bacteria is _____, now they are afraid that it may not only be disease-carrying but even _____.

   (A) innocuous . . virulent
   (B) contagious . . communicable
   (C) pathological . . harmless
   (D) fatal . . suspicious
   (E) rampant . . quiescent

10. When it is essential to conserve precious resources, we discover that many of our "needs" are really _____ and unnecessary.

    (A) extraneous (B) imperative (C) incumbent
    (D) indispensible (E) mandatory

Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Select the lettered pair that *best* expresses a relationship similar to that expressed in the original pair.

11. SCULPTOR:CHISEL:: (A) director:play (B) vendor:merchandise (C) confectioner:candy (D) seamstress:needle (E) priest:belfry

12. FURRIER:STOLE::
(A) biographer:life history
(B) prelate:cataclysm
(C) orator:satchel
(D) chiropractor:electrolysis
(E) podiatrist:shoes

13. WARMTH:GERMINATION::
(A) paper:wood (B) ink:water
(C) grass:spirits (D) soaking:saturation
(E) paint:chips

14. MOISTURE:MILDEW::
(A) prejudice:discrimination
(B) evangelism:revelry (C) expiation:sin
(D) faction:party (E) feuilleton:literature

15. BIG:COLOSSAL:: (A) discerning:astute (B) distraught:upset (C) thin:emaciated (D) encomiastic:eulogistic (E) errant:wandering

16. MATCH:CONFLAGRATION::
(A) gloss:explanation (B) grapnel:hook
(C) grotto:cave (D) gunwale:boat
(E) acorn:oak

17. BRUISE:WOUND:: (A) error:crime (B) moisture:humidity (C) hubbub:tumult (D) howdah:elephant (E) imbroglio:entanglement

18. CACOPHONY:DISSONANCE::
(A) aphasia:loquacity
(B) avocation:profession
(C) spouse:paramour (D) tendon:sinew
(E) benison:affliction

19. PUGNACIOUS:BELLICOSE::
(A) pulmonary:cardiac (B) prurient:platonic
(C) passive:serene
(D) rampant:sedated
(E) rancid:fresh

20. PERFECT:MANGLED::
(A) altered:modified (B) persistent:enduring
(C) unyielding:adamant
(D) transmutable:convertible
(E) quintessential:impure

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

21. SPORADIC: (A) hypnotic (B) occurring at regular intervals (C) indiscreet (D) relating to plants (E) flimsy

22. ICONOCLAST: (A) worshipper (B) attacker of tradition (C) rebel (D) patrician (E) agnostic

23. IMPECUNIOUS: (A) plausible (B) indigent (C) opulent (D) altruistic (E) heathen

24. INCEPTION: (A) discord (B) culmination (C) hostility (D) withdrawal (E) prelude

25. RECREANT: (A) faithful (B) craven (C) annoying (D) hedonistic (E) combatant

26. REPEL: (A) allure (B) exhibit (C) coax (D) hinder (E) invalidate

27. GARRULOUS: (A) taciturn (B) boundless (C) cheap (D) wildly excited (E) raucous

28. TORPID: (A) motionless (B) active (C) muddy (D) swollen (E) dull

29. APOCRYPHAL: (A) nefarious (B) authentic (C) zealous (D) imprecise (E) satisfying

30. SANGUINE: (A) bloody (B) discouraged (C) auspicious (D) roseate (E) confident

31. EXTANT: (A) no longer in existence (B) valueless (C) still in existence (D) superfluous (E) indigenous

32. DELETERIOUS: (A) prejudicial (B) pernicious (C) salutary (D) eliminating (E) tasty

33. PERNICIOUS: (A) noxious (B) salutary (C) faulty (D) offensive (E) immaculate

34. PERFUNCTORY: (A) superficial (B) assiduous (C) frivolous (D) mechanical (E) crude

35. OPULENCE: (A) plethora (B) lucre (C) penury (D) affluence (E) superabundance

The passages below are followed by questions based on their content. Answer all questions following a passage on the basis of what is *stated* or *implied* in that passage.

However mean your life is, meet it and live it; do not shun it and call it hard names. It is not so bad as you are. It looks poorest when you are richest. The fault-finder will find faults even in paradise. Love your life, poor as it is. You may perhaps have some pleasant, thrilling, glorious hours, even in a poorhouse. The setting sun is reflected from the windows of the alms-house as brightly as from the rich man's abode; the snow melts before its door as easily in the spring. I do not see but a quiet mind may live as contentedly there and have as cheering thoughts, as in a palace. The town's poor seem to me often to live the most independent lives of any. Maybe they are simply great enough to receive without misgiving. Most think that they are above being supported by the town; but it oftener happens that they are not above supporting themselves by dishonest means, which should be more disreputable. Cultivate poverty like a garden herb, like sage. Do not trouble yourself much to get new things, whether clothes or friends. Turn the old; return to them. Things do not change; we change. Sell your clothes and keep your thoughts. God will see that you do not want society.

If I were confined to a corner of a garret all my days, like a spider, the world would be just as large to me while I had my thoughts about me. The philosopher said: "From an army of three divisions one can take away its general, and put it in disorder; from the man the most abject and vulgar one cannot take away his thought."

Moreover, if you cannot buy books and newspapers, you are but confined to the most significant and vital experiences; you are compelled to deal with the material which yields the most sugar and the most starch. It is life near the bone where it is sweetest. You are defended from being a trifler. No man loses ever on a lower level by magnanimity on a higher. Superfluous wealth can buy superfluities only. Money is not required to buy one necessary of the soul.

36. The main idea of this selection is best expressed as

(A) adjusting to life
(B) the acquisition of things
(C) the relationship of material goods to individual happiness
(D) spiritual transcendence
(E) poverty and freedom

37. From this selection one would assume that the author's attitude toward business success would be one of

(A) geniality
(B) severe condemnation
(C) pessimism
(D) regret
(E) suspicion

38. According to the passage, which of the following statements is true?

(A) Discontentment is a primary condition of poverty.
(B) Poverty is a condition to be nurtured.
(C) The way toward progress is to set aside the old and familiar.
(D) Change in one's inner self is best reflected in one's outward appearance.
(E) Wealth is the only means whereby one can acquire the real necessities of life.

39. The philosopher's comment in the second paragraph can best be interpreted as meaning that

(A) military efficiency and organization are highly dependent on external controls
(B) deprivation and disorder are conditions which result from the excessive exercise of authority
(C) the abject and vulgar are limited in their freedom of expression
(D) any individual is capable of extending himself beyond his physical environment, since that which is within him can never be taken away
(E) man's reach cannot exceed his grasp

40. According to the writer,

(A) newspapers and books are the source of life's most meaningful experiences
(B) triumph over adversity is but a question of attitude
(C) the individual whose propensity has been to complain about his abject conditions if

faced with a better situation will respond in a more pleasing and sanguine manner
(D) man views life as rich in proportion to the amount of his acquisitions
(E) man's unquestionable achievement is his capacity to support himself with whatever means are at his disposal

The household as an economic unit was supplemented by small business enterprise—by the merchant who was in command of his own capital, who employed his own servants and agents, and who assumed personally the risks of buying and selling household products. Household manufacture itself gave way to the relatively small factory. Its owner or his immediate agent directed the labor force, identified himself with the product, assumed the risks of the business, and took the profits. He was a simple and comprehensible figure, and he had a straightforward role to perform. It would have been surprising had he not also become the object of social nostalgia, and he has. He is the small businessman.

There is no more distinctive feature of the modern industrial society than the great corporation. These have no single owner; management, direction, and achievement are identified not with any individual but with an organization. Perhaps these corporations suffer from a kind of social elephantiasis. Size brings rewards in executive prestige, and growth *qua* growth is the most obvious measure of executive achievement. The distinction of heading the largest corporation far exceeds that of heading the most profitable corporation. Yet we also owe the more distinctive achievements of modern capitalism to the large corporation. Certainly we are indebted to it for most of the goods by which we set such store. Automobiles, their gasoline, the electronic marvels, washing machines, refrigerators, the food that goes into them, bathtubs and the steel, copper, nickel and aluminum from which or by which they are fabricated, all come from the vast organization. The reason is simple. Instead of genius, the large corporation makes use of the combined efforts of many men of specialized but not remarkable ability. It substitutes organization for exceptional individual qualifications. This is highly efficient and something the small firm, by definition, cannot do. In addition the large corporation can command capital, can minimize risk, and is a good vehicle for routine consumers' goods innovation.

41. The main idea of this section is best expressed as

(A) the debt we owe the large corporation
(B) modern capitalism and the profit motive
(C) modern industrial society
(D) characteristics of the large corporation
(E) nostalgia and the small businessman

42. Which one of the following statements is not true?

(A) Corporations suffer from the social stigmata of hugeness.
(B) Achievement on the executive level is measured in growth.
(C) Heading a corporation which is distinctive in terms of its profits is less advantageous than heading a corporation which is distinctive in terms of its size.
(D) The large corporations capitalize on the combined mediocrity of many rather than the exceptionally qualified few.
(E) One of the primary differences between the small business and the large corporation is that achievement in the former is identified with an individual and achievement in the latter is identified with an organization.

43. In his discussion of the large corporation the author's tone can be described as

(A) tolerant
(B) caustic
(C) sympathetic
(D) ironic
(E) sarcastic

44. In this passage, the writer implies

(A) that one could do worse than operating a small business
(B) that the benefits derived from modern capitalism are largely produced by the giant corporations
(C) that the large corporation suffers from inefficiency
(D) that the original economic unit was the small business enterprise
(E) that the large corporation is an excellent vehicle for capital risk

45. The word "elephantiasis" in the second paragraph most probably means

(A) toughness
(B) animalism
(C) hugeness
(D) disease
(E) clumsiness

Answer Sheet—Verbal Aptitude Test 4

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	10	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	28	☐	☐	☐	☐	☐	37	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	29	☐	☐	☐	☐	☐	38	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐	30	☐	☐	☐	☐	☐	39	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐	31	☐	☐	☐	☐	☐	40	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐	32	☐	☐	☐	☐	☐	41	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐	33	☐	☐	☐	☐	☐	42	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐	34	☐	☐	☐	☐	☐	43	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	26	☐	☐	☐	☐	☐	35	☐	☐	☐	☐	☐	44	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	27	☐	☐	☐	☐	☐	36	☐	☐	☐	☐	☐	45	☐	☐	☐	☐	☐

Verbal Aptitude Test 5

Time—30 Minutes
45 Questions

Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Select the lettered pair that *best* expresses a relationship similar to that expressed in the original pair.

1. FREQUENT:INCESSANT::
(A) colossal:gigantic (B) stagnant:stationary
(C) flexible:limp (D) harangue:diatribe
(E) simultaneous:contemporaneous

2. PERPETUITY:PERMANENCE::
(A) prolepsis:anticipation
(B) multiplication:division (C) animosity:love
(D) lemon:pineapple (E) precision:ambiguity

3. FISH:SCHOOL:: (A) chair:furniture
(B) lamp:floor (C) pen:ink (D) cream:whey
(E) terrace:verandah

4. MEAL:REPAST:: (A) scourge:salve
(B) youth:senility (C) mercury:quicksilver
(D) stamina:weakness (E) stricture:tact

5. CHAOTIC:PEACEFUL::
(A) nascent:incipient (B) chimerical:realistic
(C) terminal:conclusive
(D) sequential:consecutive
(E) clustered:bundled

6. KEY:LOCK:: (A) treasure:coffer
(B) vault:rubies (C) tea:kettle
(D) combination:vault (E) cry:help

7. ANTHROPOLOGY:MAN::
(A) mushrooms:mycology
(B) words:etymology (C) ethnogeny:race
(D) bacteriology:arthropods
(E) werewolves:lycanthropy

8. MURKY:WATER:: (A) edible:viand
(B) musky:fragrance (C) hazy:atmosphere
(D) piscatorial:natation (E) obdurate:volition

9. PARRY:BLOW:: (A) fly:bird
(B) evade:arrest (C) exude:digest
(D) demonstrate:antipathy
(E) extinguish:ashes

10. MEDICINE:ILLNESS:: (A) ink:paper
(B) blotter:water (C) hydrophobia:dog
(D) money:insolvency
(E) serum:immunization

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole.

11. That _____ speech was full of _____ remarks!

(A) caustic . . complimentary
(B) acrimonious . . mordant
(C) stinging . . laudatory
(D) carping . . edifying
(E) callow . . mature

12. A(n) _____ description is valuable because it is so _____ .

(A) lengthy . . condensed
(B) prolix . . laconic
(C) intricate . . simple
(D) labyrinthine . . incomprehensible
(E) summary . . concise

13. Her constant flow of words characterized Wendy as _____.

(A) waggish
(B) vulnerable
(C) voracious
(D) voluble
(E) volatile

14. In spreading a _____ like that, he is perpetrating libel and slander.

(A) canard (B) cameo (C) charisma
(D) catechism (E) candor

15. Like the _____ creatures of Greek mythology, he loved roaming through the forest.

(A) tenuous (B) sylvan (C) tautological
(D) tactile (E) surly

16. "Now that you've insulted Mrs. Anderson, I suggest you go to her house and make an apology to _____ her, unless you want her to take action against you."

(A) palpitate (B) mulct (C) molt
(D) mollify (E) languish

17. The judge gave me three angry, deafening strokes of the _____. "Will your contemptible behavior force me to _____ this hearing?" he shouted at the top of his lungs.

(A) hammer . . discontinue
(B) gavel . . adjourn
(C) bat . . assume
(D) curtain . . perform
(E) whip . . disqualify

18. While the dictionary is quite clear about the _____ of the word, the ambiguity of current usage makes its precise _____ somewhat uncertain.

(A) devolution . . evolution
(B) expurgation . . inclusion
(C) history . . progeny
(D) denotation . . connotation
(E) compendium . . breviary

19. While they may choose to _____ their political obligations in the name of expediency, we can be sure that it will hardly _____ their guilt in the eyes of other countries.

(A) redeem . . coalesce
(B) plagiarize . . indemnify
(C) retract . . harm
(D) renounce . . mitigate
(E) ignore . . upset

20. I hope their sincere and _____ efforts will be justly rewarded.

(A) senile (B) sedulous (C) sartorial
(D) ribald (E) refractory

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

21. BOGUS: (A) frank (B) fake (C) kindly
(D) assertive (E) genuine

22. LIMPID: (A) opaque (B) lively (C) lucid
(D) stiff (E) pliable

23. ELICIT: (A) instill (B) legal
(C) draw forth (D) authorized (E) extricate

24. EGREGIOUS: (A) excellent (B) wanton
(C) outstanding (D) entering (E) aggressive

25. NEBULOUS: (A) earthy (B) vague
(C) starry (D) clear (E) distant

26. VASSAL: (A) tyro (B) master (C) boat
(D) sycophant (E) slave

27. AUGUST: (A) February (B) grand
(C) Roman (D) paltry (E) sublime

28. SOMATIC: (A) mental (B) sprightly
(C) sanguine (D) physical (E) watchful

29. OBEISANCE: (A) submission
(B) slenderness (C) resistance (D) acuteness
(E) gratitude

30. JOCOSE: (A) humorous (B) sedate
(C) lugubrious (D) waggish (E) athletic

31. EUPHONY: (A) sadness (B) symphony
(C) homophony (D) cacophony (E) harmony

32. NADIR: (A) pinnacle (B) basis
(C) consumer (D) foreigner (E) mindful

33. MALIGN: (A) vilify (B) laud
(C) calumniate (D) benefit (E) construct

34. BLITHE: (A) cheerful (B) balmy
(C) spiritless (D) bland (E) canine

35. HALCYON: (A) turbulent (B) ignominy
(C) placid (D) obtuse (E) tranquil

The passages below are followed by questions based on their content. Answer all questions following a passage on the basis of what is *stated* or *implied* in that passage.

Any significant change in the life of an individual tends to introduce some degree of instability or dis-

harmony in the way his life activities, and his beliefs and attitudes, are organized. Such instability can be described psychologically as emotional tension.

A significant change results in tension either because old behavior is found to be inadequate or by creating new situations for which new behavior must be acquired. The old responses—the way a man handled a tool, or led a work party, or called in a shaman to cure a sick child—have usually been an essential part of the individual's sense of his "self"; now their inadequacy or uselessness may be felt as a threat to the whole hitherto prized way of life. Even if the individual is willing to give up his old responses for new ones, he will be in a state of tension while he unlearns the old responses. Here he will benefit from support given him, which can take a variety of forms: accounts of the difficulties which other people in similar situations have encountered; group work in which individuals learning new habits can encourage each other by sharing accounts of difficulties and making suggestions for new habits; interpretations from experienced workers that the tensions experienced are to be expected.

Although we may expect that the existence of such tensions will be accompanied by changes in the individual's behavior which will tend to reduce the original tension, the dissipation of such tensions may take a considerable time or may not be successful. If the dissipation of the tension is not successful, the individual remains in a state of maladjustment or frustration.

36. Which of the following statements is not true?

    (A) Tension is a human response to change.
    (B) An individual who willingly relinquishes his old responses will avoid experiencing tension.
    (C) Shared experiences concerning difficulties make the transition easier.
    (D) Inexperienced workers should be reassured by seasoned workers about the predictable difficulties they will encounter.
    (E) The tension engendered during the changeover has much to do with his feelings of inadequacy.

37. The main idea of this section is best expressed as

    (A) attitude shifts and instability
    (B) the dynamics of change
    (C) the effect of change on an individual's state of mind
    (D) tension and human behavior
    (E) unlearning old responses

38. The word "shaman" in the second paragraph probably means a

    (A) charlatan
    (B) physician
    (C) tribal chief
    (D) midwife
    (E) medicine man

39. Help is available to the individual experiencing tension in all of the following ways except

    (A) group support in a variety of forms
    (B) unlearning old responses
    (C) sharing accounts of the difficulties others have experienced
    (D) group suggestions for new habits
    (E) testimony from the group that the feelings being experienced are normal

40. The author infers that the distress an individual feels over changes in his life is primarily due to the fact that

    (A) the individual feels a loss of his sense of self
    (B) his beliefs and attitudes are disorganized
    (C) he is psychologically unstable
    (D) maladjustment and frustration inevitably accompany any major change in an individual's life
    (E) the individual is unwilling to give up his old responses for new ones

Because of man's great capacity for adaptability and his remarkable ingenuity, he can improve in a great variety of ways upon the manner in which the lower animals meet their needs. Within every society there are particular ways in which needs are met. The origins of these ways are usually "lost in the mists of antiquity" or, as the Australian aborigines say, they "belong in the dream time." In fact, one of the hardest things in anthropology is to trace the origin of a custom. There is usually no one old enough to remember its origin because as a rule it came into being a very long time ago, long before written history. Culture represents man's response to his basic needs. Culture is man's way of making himself comfortable in the world. It is the behavior he has learned as a member of society. We may define culture as the way of life of a people, the environment which a group of human beings occupying a common territory have created in the form of ideas, institutions, pots and pans, language, tools, services, and sentiments.

It is this man-made environment, culture, that all human societies impose upon the physical environ-

ment and in which all human beings are trained. Culture becomes so identified with life itself that it may be fairly said that it is not so much superimposed on life as it is a substitute for it. Just as a tool substitutes for and extends the capacities of the hand, so culture substitutes for and extends the capacities of life.

The criteria by which culture may be recognized are: (1) it must be invented; (2) it must be transmitted from generation to generation; and (3) it must be perpetuated in its original or in modified form. While some lower animals are capable of very limited cultural behavior, man alone seems to be virtually unlimited in his capacity for culture. The process of creating, transmitting, and maintaining the past in the present is culture—the capacity which the American semanticist Alfred Korzybski called "time-binding." Plants bind chemicals, animals bind space, but man alone binds time.

41. The main idea of this passage is best expressed as

    (A) man's capacity for adaptability
    (B) the origin of custom
    (C) culture and the physical environment
    (D) criteria of culture
    (E) why culture develops and what it is

42. The author implies that

    (A) within every society there are standardized ways to meet the needs of its members
    (B) the main business of anthropology is to trace the origin of custom
    (C) every human being is subjected to the teachings of his culture
    (D) because of man's creative mind, he is superior to the lower animals in meeting his needs
    (E) life is a substitute for culture

43. The criteria by which culture may be recognized include all the following except that

    (A) it must be maintained in its original form or a modified form
    (B) it must have a highly developed capacity for symbolic thought
    (C) it must be invented
    (D) it must be passed on through succeeding generations
    (E) it may be perpetuated in some form

44. The author suggests that a characteristic which distinguishes man from some animals is that

    (A) a lower animal can extend his capacity for life to a greater extent than man
    (B) some lower animals are capable of extended cultural behavior
    (C) man is unlimited in his capacity for culture
    (D) man is able to interact with his environment
    (E) man is able to bind space

45. Man's "time-binding" ability permits him to

    (A) substitute a tool for his hand
    (B) recognize and define culture
    (C) modify his culture
    (D) preserve the past
    (E) discover the origins of culture

Answer Sheet—Verbal Aptitude Test 5

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	10	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	28	☐	☐	☐	☐	☐	37	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	29	☐	☐	☐	☐	☐	38	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐	30	☐	☐	☐	☐	☐	39	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐	31	☐	☐	☐	☐	☐	40	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐	32	☐	☐	☐	☐	☐	41	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐	33	☐	☐	☐	☐	☐	42	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐	34	☐	☐	☐	☐	☐	43	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	26	☐	☐	☐	☐	☐	35	☐	☐	☐	☐	☐	44	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	27	☐	☐	☐	☐	☐	36	☐	☐	☐	☐	☐	45	☐	☐	☐	☐	☐

Verbal Aptitude Test 6

Time—30 Minutes
45 Questions

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole.

1. Words are of value only when written with _____ and precision; poetic or not, they are of little worth if they bask in _____ .

   (A) clarity . . obscurity
   (B) bombast . . plenitude
   (C) luster . . munificence
   (D) dissipation . . conciseness
   (E) apostasy . . aplomb

2. "It was not through _____ that America was made great," claimed the industrialist, "but through constant, relentless, and persevering _____ ."

   (A) culmination . . conviviality
   (B) caricature . . carping
   (C) diffidence . . aggressiveness
   (D) arrogance : . imperturbability
   (E) blasphemy . . sacrilege

3. Augustus was so mournful all the time that his friends called him " _____ Gus."

   (A) maudlin (B) lurid (C) lugubrious
   (D) lucrative (E) loquacious

4. This certificate of divorce represents the legal _____ of their nuptial vows.

   (A) abridgement (B) abrogation
   (C) absolution (D) abstinence (E) accession

5. Diane, who is generally a very _____ person, has unfortunately become rather _____ lately.

   (A) culinary . . congenial
   (B) affable . . unsociable
   (C) vicious . . kind
   (D) meretricious . . lovable
   (E) chauvinistic . . crusading

6. "For once," shouted the indignant Senator, "let us have a modicum of _____ in contradistinction to our usual _____."

   (A) striation . . marking
   (B) malleability . . ductility
   (C) insularity . . isolation
   (D) concurrence . . discord
   (E) sobriety . . probity

7. In those critical days, his _____ in the matter was greatly appreciated.

   (A) acquiescence (B) acquittal (C) acridity
   (D) acerbity (E) acme

8. Man is of a striking duality: on the one hand he strives for the _____ ; on the other hand he is frequently engaged in _____ activities.

   (A) disingenuous . . naive
   (B) taciturn . . silent
   (C) sublime . . disreputable
   (D) demoniac . . baleful
   (E) corrosive . . deleterious

9. Since he lives in such an opulent mansion, I do not doubt that he is _____ .

   (A) adverse (B) esthetic (C) affable
   (D) affirmative (E) affluent

10. We must not let him _____ her with perfumes and jewels.

    (A) aggravate (B) agitate (C) allay
    (D) allude (E) allure

Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Select the lettered pair that *best* expresses a relationship similar to that expressed in the original pair.

11. PERDITION:WICKED::

    (A) purgatory:elect (B) paradise:righteous
    (C) army:soldier (D) pulpit:ecclesiastic
    (E) synagogue:rabbi

12. HYPERBOLE:TRUTH::
(A) onomatopoeia:alliteration
(B) premonition:forecast
(C) exaggeration:reality (D) syllogism:logic
(E) symmetry:rectangle

13. PENCHANT:DESIRE::
(A) stupefaction:shock (B) penury:vulgarity
(C) manifestation:illusion (D) inclination:wish
(E) mettle:cowardice

14. DEFECTION:COUNTRY::
(A) desertion:army (B) excavation:dig
(C) restitution:compensation
(D) indigence:poverty (E) recuperation:illness

15. COUNTRYSIDE:RURAL::
(A) rain:gelid (B) firmament:terrestrial
(C) sextant:astrological (D) bear:ursine
(E) covenant:antique

16. HILLOCK:MOUNTAIN::
(A) pretension:quackery
(B) touchstone:criterion (C) mark:sign
(D) tenacity:firmness (E) pond:ocean

17. CREAM:FATTY:: (A) parody:grave
(B) party:boisterous (C) verbosity:laconic
(D) infamy:laudable (E) originality:banal

18. COMPLICITY:PARTICIPATE::
(A) perfection:bungle (B) destruction:erect
(C) encouragement:abet (D) coruscation:fade
(E) exacerbation:improve

19. EDUCATION:AWARENESS::
(A) forester:ranger (B) fratricide:mother
(C) frequency:similitude (D) frieze:cornice
(E) stimulus:response

20. AUDIBLE:STRIDENT::
(A) benign:harmless (B) beatific:blissful
(C) balmy:fragrant (D) bright:visible
(E) fancy:rococo

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

21. ASCETIC: (A) self-indulgent (B) sweet
(C) austere (D) caustic (E) cunning

22. VIRILE: (A) effeminate (B) peaceful
(C) docile (D) honest (E) masculine

23. TENUOUS: (A) slender (B) shaky
(C) voluble (D) firm (E) wan

24. GAUCHE: (A) unfashionable (B) clumsy
(C) sinister (D) forlorn (E) adroit

25. SPURIOUS: (A) genuine (B) placating
(C) false (D) scattered (E) indigent

26. COGITATE: (A) excogitate (B) reflect
(C) ignore (D) compute (E) teach

27. SAGACITY: (A) cowardice (B) stupidity
(C) indigence (D) wisdom (E) composure

28. GAUNT: (A) gauntlet (B) bony
(C) graceful (D) obese (E) polite

29. MALIGN: (A) vilify (B) laud (C) calumniate
(D) benefit (E) construct

30. HACKNEYED: (A) smooth
(B) commonplace (C) choppy (D) unusual
(E) easy

31. CHASTISE: (A) rape (B) laud (C) punish
(D) procrastinate (E) buttress

32. FATUOUS: (A) silly (B) allusive
(C) lively (D) slender (E) serious

33. ZEALOUS: (A) disinclined (B) angry
(C) enthusiastic (D) happy (E) jealous

34. COALESCE: (A) disintegrate (B) purify
(C) disembody (D) melt (E) condense

35. ESTRANGE: (A) alienate (B) common
(C) befriend (D) exotic (E) originate

The passages below are followed by questions based on their content. Answer all questions following a passage on the basis of what is *stated* or *implied* in that passage.

The edge of the sea is a strange and beautiful place. All through the long history of Earth it has been an area of unrest where waves have broken heavily against the land, where the tides have pressed forward over the continents, receded, and then returned. For no two successive days is the shoreline precisely the same. Not only do the tides advance

and retreat in their eternal rhythms, but the level of the sea itself is never at rest. It rises or falls as the glaciers melt or grow, as the floor of the deep ocean basins shifts under its increasing load of sediments, or as the earth's crust along the continental margins warps up or down in adjustment to strain and tension. Today a little more land may belong to the sea, tomorrow a little less. Always the edge of the sea remains an elusive and indefinable boundary.

The shore has a dual nature, changing with the swing of the tides, belonging now to the land, now to the sea. On the ebb tide it knows the harsh extremes of the land world, being exposed to heat and cold, to wind, to rain and drying sun. On the flood tide it is a water world, returning briefly to the relative stability of the open sea.

Only the most hardy and adaptable can survive in a region so mutable, yet the area between the tidelines is crowded with plants and animals. In this difficult world of the shore, life displays its enormous toughness and vitality by occupying almost every conceivable niche. Visibly, it carpets the intertidal rocks; or half-hidden, it descends into fissures and crevices, or hides under boulders, or lurks in the wet gloom of sea caves. Invisibly, where the casual observer would say there is no life, it lies deep in the sand, in burrows and tubes and passageways. It tunnels into solid rock and bores into peat and clay. It encrusts weeds or drifting spars on the hard, chitinous shell of a lobster.

The shore is an ancient world, for as long as there has been an earth and sea there has been this place of the meeting of land and water. Yet it is a world that keeps alive the sense of continuing creation and of the relentless drive of life.

36. The main idea of this section is best expressed as

    (A) the edge of the sea
    (B) the changing shoreline and its life forms
    (C) the rhythm of the tide
    (D) the difficult world of the shore
    (E) the ancient world that is the shore

37. The word which best describes the author's tone is

    (A) reasonable
    (B) confident
    (C) fascinated
    (D) dejected
    (E) cynical

38. The word "mutable" in the third paragraph most probably means

    (A) indefinable
    (B) changeable
    (C) vital
    (D) eternal
    (E) relentless

39. The fact that the shoreline harbors such an extensive range of animal and plant life illustrates

    (A) the burrowing quality of many sea animals
    (B) the effect of the floor of the ocean basins' shifts under its load of sediment
    (C) life's great vigor
    (D) the warp of the earth's crust as it adjusts to the strain and tension.
    (E) the relative stability of the open sea

40. All of the following are true except that

    (A) the sea's edge has always been considered an area of turmoil
    (B) the shoreline has a perceptible and distinguishable boundary
    (C) glaciers have a direct effect on the changing shoreline
    (D) the shore has two characters which depend on the shifts of the tide
    (E) the shore is a place that testifies to the vitality of life

The more we become aware of the underlying mechanism that powers man and nature, the more we discover the pragmatic profundity of ancient insights and folk wisdom. The cultures that regarded bee pollen as ambrosia, the food of the gods, were not aware that pollen is rich in lecithin, protein, and vitamins. The medieval knights were not conscious that the presence of allantoin in the comfrey plant made it a potent mender of broken bones and battle wounds; but, without this analytical information, the medicine men and restorers of healing in past generations accumulated a considerable body of experience that served to ease human suffering.

So, too, with human personality. It was assumed that the shape of a human destiny was determined by the constellations dominant at the hour of birth, over which neither the individual nor his parents exercised control.

Reflecting his modern, scientific view, Shakespeare put into the mouth of Cassius the anachronistic notion "The fault, dear Brutus, is not in our stars but in ourselves." Our current scientific studies, however, have revealed that elements of our personality, as of our physical structure, are determined by a genetic code which is established by the genes of random sperm and egg cells that unite to

become each unique individual. The elements that combine to shape our character are, therefore, as far beyond our personal control as though they had been determined by the constellations; for character is the key to destiny.

41. The author's attitude toward the folk wisdom of earlier centuries can best be expressed as

    (A) denigrating
    (B) full of praise
    (C) captious
    (D) sarcastic
    (E) understanding

42. The disagreed with view of human destiny that Cassius expressed when he said "The fault . . . is not in our stars but in ourselves" was based on

    (A) anachronism
    (B) pragmatism
    (C) the constellations
    (D) the genetic code
    (E) the presence of allantoin

43. According to the author, the shape of man's destiny

    (A) is determined by the constellations dominant at the hour of birth
    (B) is determined by vigilant parental control
    (C) is determined by our character, which is determined by our genes
    (D) is under current scientific investigation
    (E) is a vague mechanism which we do not fully understand

44. The author makes all of the following statements about earlier cultures except that

    (A) medieval knights, without a conscious awareness of what they were doing, used a medicinal preparation which modern science acknowledges as a healer of wounds
    (B) what was regarded as "food for the gods" is today considered a source of nutritional supplements
    (C) we owe a tremendous debt to the medicine men of the past for the storehouse of information about matters of health which they passed on to the scientist of today
    (D) what we once regarded as simple folk wisdom about healing has turned out to be valid information
    (E) without sophisticated analytical information about matters relating to illness, man nevertheless developed effective medicines

45. The author implies that Shakespeare

    (A) took anachronistic views of life
    (B) did not believe in astrology
    (C) forced his characters to illustrate unpopular ideas
    (D) recognized that man's future was shaped by forces beyond his control
    (E) was aware of the underlying mechanism that powers man

Answer Sheet—Verbal Aptitude Test 6

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	10	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	28	☐	☐	☐	☐	☐	37	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	29	☐	☐	☐	☐	☐	38	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐	30	☐	☐	☐	☐	☐	39	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐	31	☐	☐	☐	☐	☐	40	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐	32	☐	☐	☐	☐	☐	41	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐	33	☐	☐	☐	☐	☐	42	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐	34	☐	☐	☐	☐	☐	43	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	26	☐	☐	☐	☐	☐	35	☐	☐	☐	☐	☐	44	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	27	☐	☐	☐	☐	☐	36	☐	☐	☐	☐	☐	45	☐	☐	☐	☐	☐

Verbal Aptitude Test 7

Time—30 Minutes
45 Questions

The passages below are followed by questions based on their content. Answer all questions following a passage on the basis of what is *stated* or *implied* in that passage.

The skill of a great master lies in his ability to mask his consummate craftsmanship behind a childlike simplicity and naiveté. Marc Chagall, with eighty years of experience behind him, most of which has been spent in France, still peoples his canvases, glass, or stone with symbols of his Vitebsk childhood. Peasants, clowns, ramshackle huts, violins, and all kinds of village animals populate his paintings. His father was a herring merchant, and ubiquitous fish dart in and out of his imagery. Like a child, his proportions reflect the relative importance of his subjects; as in a child's delineations, there is no wall between fantasy and reality and all his animals sport human eyes.

But Chagall is no child. He does not dull his hand by forcing your attention to his meticulous draftsmanship, but if you analyze what appear on the surface to be clumsy, juvenile portrayals, you will discover a careful design, deliberately contrasting textures, a masterful use of chiarascuro and color—all suffused with a sense of humor and a *joie de vivre,* remarkably sustained for over ninety years, during which he has inevitably confronted a considerable proportion of the pains reserved for every human creature.

Of course, Chagall does not execute all of this consciously; he paints what comes naturally. It comes out the way it does because he is a genius.

1. The title that best expresses the idea of this passage is

   (A) The Genius of Marc Chagall
   (B) The Paintings of a Great Master
   (C) Chagall, the Child and the Genius
   (D) The Influence of Chagall's Early Childhood on His Paintings
   (E) Great Art Appears Simple

2. A great master must have

   (A) an exciting childhood
   (B) the ability to create in many media
   (C) no wall between fantasy and reality
   (D) the ability to conceal his skill
   (E) years of suffering

3. Chagall's paintings appear to be clumsy because

   (A) they show his sense of humor
   (B) he uses childlike techniques
   (C) a man of ninety years is less meticulous
   (D) their textures contrast
   (E) he has lived most of his life in France

4. Because Chagall is a genius

   (A) he is able to paint as he does
   (B) every painting shows a conscious design
   (C) he has never forgotten Vitebsk
   (D) his proportions reflect the relative importance of his subjects
   (E) all his animals sport human eyes

5. The writer of the passage implies that a great artist is not a child because

   (A) his art includes sophisticated elements
   (B) his use of color is masterful
   (C) his textures are unconscious
   (D) his memories are too deep
   (E) his craftsmanship is meticulous

Self-appointed literary pundits have begun an outcry against the numerous best sellers based on tape-recorded interviews. One acerbic columnist categorized them as "honks," suggesting that they constitute a third category, to be distinguished from both fiction and nonfiction.

How many trees have had to fall in order to provide pulp for the endless accounts of love affairs with prominent people and the diaries of those made prominent by the uncovering of their sordid affairs? Without the availability of the tape recorder, the forests and the public would have been spared these invasions pretending to be books. Oral history is not literature.

Isn't it? When the camera came into common use, there were those who blamed the new instru-

ment for what they believed was the corruption of art. Today, museums, display the work of skilled and gifted photographers, and of the making and selling of art there is still no end. What was once the prerogative of the titled and the opulent is now the possession of the masses, as original works of art and their reproductions are displayed in the land.

The basic question remains, as it did before the advent of the camera and the tape recorder—is it good? Boswell's classic *Life of Johnson* was based on interviews. So was Daniel Defoe's *A Journal of the Plague Year*. It is not the instrument used to collect the information that determines its quality, but what the author makes of the material. And some of the current "as told to" volumes are not only more accurate but more literate than many books of the pre-tape-recorder era. They are destined to endure for quite a while, perhaps considerably longer than the essays of some of the jaundiced reviewers, whose Sunday editions also consume forests of wood pulp.

6. The author's main point is that

   (A) the number of books published today is destroying our forests
   (B) the form of a work is no criterion of its value
   (C) there is no end to the reproduction of art works
   (D) love affairs with prominent people are increasingly popular best sellers
   (E) basing a book on interviews corrupts the art of bookmaking

7. Many reviewers are against books derived from recorded interviews because they believe

   (A) a tape recorder is a mechanical, not a creative, device
   (B) Boswell did not use a tape recorder
   (C) too many of these depend for their success upon sensationalism
   (D) it makes it possible for anyone to write a book, even if he is no author
   (E) books of the pre-tape-recorder era were more accurate

8. The author implies that art that endures

   (A) is found only in museums
   (B) is either fiction or nonfiction
   (C) is never in the possession of the masses
   (D) is never found in the pages of a diary
   (E) is the result of the artist's skill, not his instrument

9. Some reviewers condemn tape-recorded interviews because they believe

   (A) they are an invasion of privacy
   (B) most of the material is sordid
   (C) the writing is undistinguished
   (D) a book should begin in the author's head
   (E) oral history is not literature

10. The author believes that oral history can be literature

    (A) if it is written without a tape recorder
    (B) if the interviewer is well trained
    (C) if a camera is used as well as a tape recorder
    (D) if basic questions are answered
    (E) if the writer is truly skilled

Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Select the lettered pair that *best* expresses a relationship similar to that expressed in the original pair.

11. FRICTION:HEAT:: (A) courier:court (B) denotation:connotation (C) duchess:duke (D) earnest:pledge (E) death:grief

12. COLLISION:DEFORMATION::
    (A) cochlea:ear (B) flagellation:soreness
    (C) respiration:defamation
    (D) expedition:explorer (E) paper:pulp

13. COLD:GELID:: (A) active:frenetic
    (B) fortuitous:accidental
    (C) forward:obstinate (D) germane:pertinent
    (E) gnarled:twisted

14. BOTHER:PERSECUTE::
    (A) extricate:disentangle (B) eschew:avoid
    (C) extol:glorify (D) flaccid:flabby
    (E) esteem:worship

15. DIFFERENT:ANTITHETICAL::
    (A) small:minuscule (B) canny:skillful
    (C) crestfallen:dejected
    (D) despicable:contemptible
    (E) demure:serious

16. DISSEMINATION:KNOWLEDGE::
    (A) gazette:newspaper
    (B) gemination:gemmation
    (C) knee:genuflection (D) gland:organ
    (E) circulation:magazine

17. SHEEP:FLOCK:: (A) crop:produce (B) student:fraternity (C) fruit:vegetable (D) punishment:pillory (E) remark:platitude

18. PODIATRIST:PHYSICIAN::
(A) barrister:lawyer (B) communist:democrat (C) socialist:reactionary (D) carpenter:painter (E) fisherman:diver

19. WANTING:DEFICIENT::
(A) tousled:disheveled (B) lovable:repugnant (C) vapid:tasty (D) noxious:innocuous (E) aquamarine:vermilion

20. JUNGLE:DENSE:: (A) languor:lateral (B) feather:pinnate (C) junta:jovial (D) lassitude:lecherous (E) maelstrom:stagnant

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole.

21. "Has this work received the _____ of the critics? Of course not! It has been viewed with the greatest and most laudatory _____."

(A) expropriation . . denunciation
(B) buttress . . bulwark
(C) acclaim . . apprehension
(D) altercation . . animosity
(E) censure . . commendation

22. "When you send your delegate to the conference," suggested the adviser, "make sure he is not a mere _____ of your district but the very _____ of all you stand for."

(A) obelisk . . obitury
(B) representative . . epitome
(C) mentor . . consummation
(D) libertine . . composition
(E) factotum . . fledgling

23. Whereas Jack often presents the facts with the hardly honorable intention of _____, his amoral cohort Stephen dispenses with the partial deception and _____ outright.

(A) equivocation . . lies
(B) parlance . . placates
(C) litigation . . prosecutes
(D) probity . . reiterates
(E) rhetoric . . panders

24. A work which is readily comprehensible by the general public is _____.

(A) esoteric (B) exoteric (C) prodigious (D) singular (E) overwhelming

25. Only when properly tuned can a piano produce _____ sound.

(A) redolent (B) effulgent (C) euphonious (D) discordant (E) religious

26. I find it difficult to understand how Hermione, who speaks with such _____ in public lectures, can show such _____ at home.

(A) consequence . . harmony
(B) grace . . elegance
(C) zeal . . apathy
(D) rapidity . . swiftness
(E) hysteria . . euphoria

27. That long-winded writer does not realize that his audience would much prefer _____ to flowery, bombastic _____.

(A) succinctness . . verbosity
(B) temerity . . impetuosity
(C) refection . . perfection
(D) obduracy . . opalescence
(E) metamorphosis . . modulation

28. This _____, hard-working student will go far; his _____ brother, plagued by indecision, may not.

(A) sebaceous . . secular
(B) persevering . . irresolute
(C) immutable . . mordant
(D) penurious . . sacrilegious
(E) inimical . . insatiable

29. The meaning of the law was in no way _____; it was clear, precise, and _____.

(A) juridical . . legal
(B) compatible . . amenable
(C) temporary . . modern
(D) concise . . terse
(E) ambiguous . . definitive

30. "The reasoned approach would be _____," explained the mediator. "But it is all too apparent that with typical irrationality, the other side will choose _____."

(A) chagrin . . emulation
(B) castigation . . clemency

(C) circumspection . . prudence
(D) conciliation . . defiance
(E) deference . . submission

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

31. MORDANT: (A) biting (B) ornamental (C) mild (D) supreme (E) mixed

32. PANEGYRIC: (A) spherical (B) bureaucratic (C) praise (D) encomium (E) admonition

33. LICENTIOUS: (A) lustful (B) wealthy (C) chaste (D) debauched (E) lackadaisical

34. GERMANE: (A) relevant (B) benign (C) healthy (D) pertinent (E) inapplicable

35. JAUNTY: (A) jocose (B) buoyant (C) showy (D) lethargic (E) modest

36. FURTIVE: (A) clandestine (B) frugal (C) open (D) inhabited (E) benevolent

37. HUMID: (A) damp (B) sticky (C) hot (D) cold (E) dry

38. LUCRATIVE: (A) unprofitable (B) remunerative (C) thick (D) opaque (E) redundant

39. EXPLICIT: (A) irrevelant (B) clear (C) probable (D) vague (E) indigent

40. FEIGN: (A) not pretend (B) discern (C) deceive (D) vague (E) fluctuate

41. MOTLEY: (A) chic (B) mixed (C) homely (D) homogeneous (E) forbidden

42. PREVARICATE: (A) speak the truth (B) fluctuate (C) exaggerate (D) vary (E) postulate

43. NEOPHYTE: (A) bryophyte (B) expert (C) embryophyte (D) pteridophyte (E) novice

44. RAZE: (A) raise (B) level (C) dull (D) abrasive (E) disprove

45. UPBRAID: (A) condemn (B) unravel (C) praise (D) soothe (E) wane

Answer Sheet—Verbal Aptitude Test 7

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
25	☐	☐	☐	☐	☐
26	☐	☐	☐	☐	☐
27	☐	☐	☐	☐	☐
28	☐	☐	☐	☐	☐
29	☐	☐	☐	☐	☐
30	☐	☐	☐	☐	☐
31	☐	☐	☐	☐	☐
32	☐	☐	☐	☐	☐
33	☐	☐	☐	☐	☐
34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐
36	☐	☐	☐	☐	☐
37	☐	☐	☐	☐	☐
38	☐	☐	☐	☐	☐
39	☐	☐	☐	☐	☐
40	☐	☐	☐	☐	☐
41	☐	☐	☐	☐	☐
42	☐	☐	☐	☐	☐
43	☐	☐	☐	☐	☐
44	☐	☐	☐	☐	☐
45	☐	☐	☐	☐	☐

Verbal Aptitude Test 8

Time—30 Minutes
45 Questions

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words that *best* fits the meaning of the sentence as a whole.

1. The magnificence of man's achievements in the past is cause for _____ in facing the future; but the future, unknown and imponderable, still _____ us.

   (A) agony . . castigates
   (B) consolation . . perplexes
   (C) uncertainty . . assures
   (D) mourning . . enlightens
   (E) austerity . . ingratiates

2. The minister was wont to _____ and _____ sinful conduct.

   (A) approve . . encourage
   (B) deprecate . . decry
   (C) examine . . laud
   (D) admonish . . vindicate
   (E) vilify . . compliment

3. The human psyche is given to conflicting moods; sometimes we are _____ and active, sometimes we are reticent and _____.

   (A) captious . . carping
   (B) avid . . disinclined
   (C) checkered . . vicissitudinous
   (D) atheistic . . agnostic
   (E) canny . . sapient

4. "It is through persistent _____ and not carefree _____ that success is achieved," remarked the English professor.

   (A) covetousness . . culpability
   (B) assiduity . . indolence
   (C) decadence . . defamation
   (D) torpor . . blandishment
   (E) insolence . . contumely

5. Unlike the _____ and magnificent heroes of the old legends, we felt _____ in the face of danger.

   (A) intrepid . . fearful
   (B) cowardly . . fearless
   (C) aromatic . . redolent
   (D) antiseptic . . bacterial
   (E) arduous . . ardent

6. Do the _____ really envy what their _____ brothers indulge in?

   (A) sluggish . . torpid
   (B) abstinent . . gluttonous
   (C) lecherous . . celibate
   (D) salutary . . sanguine
   (E) sententious . . aphoristic

7. Simon is quite silly. The other day he had the _____ to dig for clams in the Sahara Desert.

   (A) levity (B) lethargy (C) wrath
   (D) fatuousness (E) sinecure

8. If you cannot settle the dispute, we will submit it to _____, with full legal procedures to be followed exactly and equitably.

   (A) addendum (B) addiction (C) adjudication
   (D) admonition (E) adulation

9. I warned him not to drive too fast, but knowing how _____ he is, he must have.

   (A) temperate (B) temerarious (C) gluttonous
   (D) sober (E) indulgent

10. "The _____ poverty of my subjects," mused the regretful king, "makes me ashamed of my noble and _____ domain."

    (A) sportive . . stringent
    (B) retrospective . . introspective
    (C) abject . . exalted
    (D) rampant . . ignominious
    (E) inconsequential . . incontrovertible

Each question below consists of a related pair of words or phrases, followed by five lettered pairs of words or phrases. Select the lettered pair that *best* expresses a relationship similar to that expressed in the original pair.

11. PUGILIST:GLOVE:: (A) boat:fisherman (B) artist:palette (C) weaver:type (D) hunter:game (E) contractor:building

12. LITHOGRAPHER:ENGRAVING:: (A) deity:prayer (B) poet:poem (C) threnody:cloth (D) sycophant:phrase (E) artist:tryst

13. DRAFTSMAN:COMPASS:: (A) writer:book (B) angler:rod (C) sand:glazier (D) hammer:carpenter (E) planter:produce

14. SOLDIER:ARMY:: (A) pilot:airplane (B) messenger:courier (C) shield:lampshade (D) nail:hook (E) cow:herd

15. GENESIS:INCEPTION:: (A) isolation:communication (B) dock:cargo (C) procession:cortege (D) gasoline:reservoir (E) tachycardia:heart

16. DIFFIDENT:TIMOROUS:: (A) beholden:independent (B) brackish:desalinated (C) callow:ripe (D) colloquial:formal (E) perspicacious:discerning

17. EUPHORIA:ECSTASY:: (A) whit:megaton (B) sadness:despair (C) zephyr:hurricane (D) advent:departure (E) eon:moment

18. ENERVATING:DEBILITATING:: (A) petulant:tolerant (B) futile:nugatory (C) pious:sacrilegious (D) carping:laudatory (E) vicious:domesticated

19. VACILLATING:STEADFAST:: (A) sebaceous:oily (B) sinuous:winding (C) sleazy:flimsy (D) sonorous:resonant (E) presumptuous:unassuming

20. EAGER:RELUCTANT:: (A) pacific:tranquil (B) mediatory:intercessory (C) property:landed (D) ripe:mature (E) bogus:authentic

Each question below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

21. OBSEQUIOUS: (A) orderly (B) fawning (C) arrogant (D) sycophantic (E) routine

22. TRITE: (A) turgid (B) exotic (C) wanton (D) commonplace (E) conscious

23. MANDATORY: (A) manufactured (B) imperative (C) requisite (D) prerequisite (E) unnecessary

24. PERSPICACIOUS: (A) obtuse (B) dry (C) discerning (D) conspicuous (E) forbearing

25. ALTERCATION: (A) settlement (B) constant (C) quarrel (D) alteration (E) difference

26. WANE: (A) blush (B) quarrel (C) decrease (D) wax (E) attenuate

27. CRASS: (A) indifferent (B) erudite (C) superficial (D) generous (E) perceptive

28. DOGMATIC: (A) categorical (B) open-minded (C) insistent (D) bigoted (E) inaccessible

29. URBAN: (A) discourteous (B) cosmopolitan (C) rural (D) metropolitan (E) naive

30. BIBLIOPHOBE: (A) library (B) bibliophile (C) illiterate (D) dabbler (E) writer

31. VOCIFEROUS: (A) clamorous (B) blatant (C) subdued (D) steady (E) uncouth

32. DESCRY: (A) not see (B) espy (C) discern (D) laugh (E) redeem

33. BALEFUL: (A) meager (B) beneficial (C) nocuous (D) sinister (E) banal

34. INEBRIETY: (A) sobriety (B) intemperance (C) bibacity (D) lassitude (E) willingness

35. CURSORY: (A) thorough (B) pure (C) hasty (D) precipitate (E) slang

The passages below are followed by questions based on their content. Answer all questions following a passage on the basis of what is *stated* or *implied* in that passage.

Dozens of Eastern cult movements have developed in America during the past generation. Young people seeking brotherhood, truth, and simplification have been swept up in this neo-oriental religious revival in numbers that have no precedent in American religious history. Everywhere you go, in large urban centers, one is confronted by invitations to meditate, to practice martial arts, to purchase books or tracts, to join in chants, to intone mantras, to share in feasts, to listen to lectures on the mysteries of devotion, or to participate in dancing, stretching the torso, or deep-breathing exercises.

Why have so many American young people in their late teens or early twenties turned eastward, eschewing the appeals of personal advancement, politics, and money that motivate most of their peers? These exotic movements offer small, supportive communities and a personal encounter with God or nature. Instead of words about it, they provide a return to innocence in place of advanced technology and a voice of authority that offers certainty to refugees from a society of dissolving moral codes and eroding traditional values. Those suffering from the fatigue of too much freedom find that following a guru, whose wisdom and charisma provide all the answers, eliminates the exhausting need for making choices.

But the world the seekers hunger for does not exist. Asia, itself, recognizing its own multifarious problems in the modern world, is in the process of turning West, hungering after the science, technology, cultural forms, and political systems that the Occident has developed. For maturity means learning to live in a world of complex shades of gray, where choices must be made. The issues of work, competition, and politics do not disappear if we turn our backs on them. What our young people need is not to be relieved of the discomfort of making decisions, but support in developing confidence in their ability to choose wisely.

36. According to the writer, a goal many youths seek is

    (A) simplification
    (B) personal advancement
    (C) maturity
    (D) devotion
    (E) camaraderie

37. According to the author, one of the major attractions of the Eastern movements is

    (A) the opportunity to meditate
    (B) help in coping with the problems of the modern world
    (C) avoidance of the need to make decisions
    (D) escaping one's parents
    (E) a chance to learn yoga

38. According to the author,

    (A) a modern adult cannot avoid exercising options
    (B) Asia is spiritually more advanced than the West
    (C) young people need an authoritative voice
    (D) devotion, dancing, and deep breathing lead to a fuller life
    (E) charisma provides all the answers

39. Large urban centers, the writer states,

    (A) offer more advanced technology
    (B) require more work and political knowhow
    (C) help a young person to develop self-confidence
    (D) present many opportunities to join Eastern cult movements
    (E) will be the downfall of the West.

40. The author claims that one reason cults have been very successful in America is that

    (A) the American emphasis on sports has exhausted the youth
    (B) they offer a personal encounter with nature
    (C) the wearing away of our moral values forces each individual to establisn his own code of behavior
    (D) the opportunities they offer are so exotic
    (E) American youth are not as narrow-minded as their parents

The biographer's prose is without grace and endlessly repetitive, sometimes so confused as to be virtually incomprehensible. The most sympathetic reader is forced to battle the writer's underbrush of disorganized expression in order to win his way to the meaning of a page.

Yet, his dedication has unearthed such an accumulation of research that through the jumbled clouds of prose, there flashes an occasional clear gleam of insight. For these sparks of fresh illumination, any devotee of Jones must read this new biography as a supplement to existing works, which have tantalized us for so many years with their unanswered questions.

With such a clumsy instrument, we may come closer to a more understanding diagnosis of one of the most brilliant and bizarre authors in the English language. His novels and historical romances retell vividly and inexhaustibly the eccentric story of his own life—a would-be priest, twice expelled from

seminaries, a painter, photographer, and author, ending his life as a gondolier in Venice. His works sparkle with unforgettable aphorisms and glowing images, even as they reek of fulminating vituperations.

41. The author characterizes the biography as

(A) contemptuous
(B) reticent
(C) crude
(D) confused
(E) sparkling

42. Jones' life, as reflected in his writings, was

(A) conventional
(B) inspiring
(C) incomprehensible
(D) dedicated
(E) queer

43. The author suggests that a useful biographer

(A) tantalizes his readers with reticence
(B) engages in endless research
(C) repeats his facts to drive them home
(D) must know his subject personally
(E) adds to our understanding

44. This biography of Jones

(A) reflects his brilliance
(B) elucidates his eccentricities
(C) describes his career in Venice
(D) explains things that others have not
(E) sparkles with aphorism

45. The author admires Jones for

(A) his rich and varied life
(B) the accuracy of his historical romances
(C) the beauty and vividness of his stories
(D) his understanding diagnosis of his characters
(E) his skillful vituperation

Answer Sheet—Verbal Aptitude Test 8

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	10	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	28	☐	☐	☐	☐	☐	37	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	29	☐	☐	☐	☐	☐	38	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐	30	☐	☐	☐	☐	☐	39	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐	31	☐	☐	☐	☐	☐	40	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐	32	☐	☐	☐	☐	☐	41	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐	33	☐	☐	☐	☐	☐	42	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐	34	☐	☐	☐	☐	☐	43	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	26	☐	☐	☐	☐	☐	35	☐	☐	☐	☐	☐	44	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	27	☐	☐	☐	☐	☐	36	☐	☐	☐	☐	☐	45	☐	☐	☐	☐	☐

ANSWERS, VERBAL APTITUDE TESTS 1–8

Test 1	Test 2	Test 3
1. (C)	1. (A)	1. (D)
2. (A)	2. (A)	2. (C)
3. (B)	3. (A)	3. (C)
4. (C)	4. (B)	4. (B)
5. (C)	5. (E)	5. (B)
6. (A)	6. (A)	6. (C)
7. (B)	7. (B)	7. (D)
8. (E)	8. (E)	8. (E)
9. (B)	9. (A)	9. (C)
10. (E)	10. (A)	10. (E)
11. (A)	11. (C)	11. (C)
12. (A)	12. (D)	12. (C)
13. (A)	13. (D)	13. (E)
14. (C)	14. (C)	14. (A)
15. (B)	15. (D)	15. (B)
16. (C)	16. (B)	16. (D)
17. (C)	17. (C)	17. (E)
18. (C)	18. (A)	18. (E)
19. (C)	19. (C)	19. (A)
20. (C)	20. (C)	20. (C)
21. (C)	21. (A)	21. (B)
22. (C)	22. (A)	22. (A)
23. (A)	23. (E)	23. (D)
24. (A)	24. (A)	24. (C)
25. (C)	25. (E)	25. (E)
26. (C)	26. (B)	26. (E)
27. (A)	27. (E)	27. (B)
28. (C)	28. (E)	28. (D)
29. (C)	29. (B)	29. (E)
30. (A)	30. (D)	30. (A)
31. (E)	31. (D)	31. (B)
32. (A)	32. (A)	32. (B)
33. (A)	33. (D)	33. (B)
34. (A)	34. (B)	34. (C)
35. (C)	35. (B)	35. (C)
36. (B)	36. (B)	36. (A)
37. (C)	37. (C)	37. (B)
38. (C)	38. (D)	38. (C)
39. (C)	39. (E)	39. (D)
40. (B)	40. (B)	40. (C)
41. (C)	41. (C)	41. (A)
42. (C)	42. (A)	42. (A)
43. (B)	43. (B)	43. (B)
44. (C)	44. (A)	44. (C)
45. (C)	45. (C)	45. (D)

Test 4	Test 5	Test 6
1. (A)	1. (C)	1. (A)
2. (B)	2. (A)	2. (C)
3. (B)	3. (A)	3. (C)
4. (D)	4. (C)	4. (B)
5. (E)	5. (B)	5. (B)
6. (E)	6. (D)	6. (D)
7. (C)	7. (C)	7. (A)
8. (D)	8. (C)	8. (C)
9. (A)	9. (B)	9. (E)
10. (A)	10. (D)	10. (E)
11. (D)	11. (B)	11. (B)
12. (A)	12. (E)	12. (C)
13. (D)	13. (D)	13. (D)
14. (A)	14. (A)	14. (A)
15. (C)	15. (B)	15. (D)
16. (E)	16. (D)	16. (E)
17. (A)	17. (B)	17. (B)
18. (D)	18. (D)	18. (C)
19. (C)	19. (D)	19. (E)
20. (E)	20. (B)	20. (E)
21. (B)	21. (E)	21. (A)
22. (A)	22. (A)	22. (A)
23. (C)	23. (A)	23. (D)
24. (B)	24. (A)	24. (E)
25. (A)	25. (D)	25. (A)
26. (C)	26. (B)	26. (C)
27. (A)	27. (D)	27. (B)
28. (B)	28. (A)	28. (D)
29. (B)	29. (C)	29. (B)
30. (B)	30. (C)	30. (D)
31. (A)	31. (D)	31. (B)
32. (C)	32. (A)	32. (E)
33. (B)	33. (B)	33. (A)
34. (B)	34. (C)	34. (A)
35. (C)	35. (A)	35. (C)
36. (C)	36. (B)	36. (B)
37. (E)	37. (C)	37. (C)
38. (B)	38. (E)	38. (B)
39. (D)	39. (B)	39. (C)
40. (B)	40. (A)	40. (B)
41. (D)	41. (E)	41. (B)
42. (A)	42. (C)	42. (C)
43. (C)	43. (B)	43. (C)
44. (B)	44. (C)	44. (C)
45. (C)	45. (C)	45. (B)

Test 7

1. (D)
2. (D)
3. (B)
4. (A)
5. (A)
6. (B)
7. (D)
8. (E)
9. (E)
10. (E)
11. (E)
12. (B)
13. (A)
14. (E)
15. (A)
16. (E)
17. (B)
18. (A)
19. (A)
20. (B)
21. (E)
22. (B)
23. (A)
24. (B)
25. (C)
26. (C)
27. (A)
28. (B)
29. (E)
30. (D)
31. (C)
32. (E)
33. (C)
34. (E)
35. (D)
36. (C)
37. (E)
38. (A)
39. (D)
40. (A)
41. (D)
42. (A)
43. (B)
44. (A)
45. (C)

Test 8

1. (B)
2. (B)
3. (B)
4. (B)
5. (A)
6. (B)
7. (D)
8. (C)
9. (B)
10. (C)
11. (B)
12. (B)
13. (B)
14. (E)
15. (C)
16. (E)
17. (B)
18. (B)
19. (E)
20. (E)
21. (C)
22. (B)
23. (E)
24. (A)
25. (A)
26. (D)
27. (E)
28. (B)
29. (C)
30. (B)
31. (C)
32. (A)
33. (B)
34. (A)
35. (A)
36. (A)
37. (C)
38. (A)
39. (D)
40. (B)
41. (C)
42. (E)
43. (E)
44. (D)
45. (C)

BUILDING YOUR VOCABULARY

There is no substitute for having a substantial vocabulary. For that reason we have compiled several important lists for you to learn. The lists include words that are often mistaken for similar-looking words, and prefixes, roots, and suffixes that frequently appear in the English language. The longest list is composed of words that appear especially frequently in college entrance examinations.

If you really want to score well on the SAT, you can use the lists to make "flash cards." A good method is to cut up ordinary file cards; one 3 × 5 file card will make five 1 × 3 flash cards. On each flash card, write one root, prefix, suffix, or vocabulary word. Then write the definition on the reverse side. Making the flash cards may seem tedious, but it is not wasted time. On the contrary, writing each word or word element and its definition will go a long way toward fixing the information in your memory. Shuffle these cards so that they are in random order and put them together with a rubber band or paper clip. Set yourself a reasonable number of cards to learn each week. A reasonable number would be anywhere from 25 to 40. As you go through each stack, put the ones you get right aside, but put the ones you get wrong on the bottom of the stack. Eventually you will get them all right. Then go back through the whole stack again, to make sure you really know them all. Remember, there may be many you already know. Carry these cards with you and use any spare moments you have to study them. You may want to keep the complete words in a separate stack and learn all the prefixes, roots, and suffixes first, because these will make it possible for you to figure out, for yourself, the definitions of many thousands of words.

If you do not want to make flash cards, you can merely study the lists in the book, covering the definitions with a file card or small envelope. Whichever method you use, keep it up until you can go through all the words, prefixes, suffixes, and roots without error. This may take you only a few hours or several days—but however long it takes, you will be well rewarded not only by a much higher SAT score but by a greatly increased vocabulary that will benefit you throughout your life.

Words Commonly Confused with Each Other

Let's start with this short list of commonly confused words. Most of them are words you probably think you know, because when you see them in context you do not misunderstand them. However, when you see such words in antonym or word-relationship questions, they may fool you—just what the makers of the SAT are expecting them to do. Also, if you are nervous and hurrying, as most of us are to some degree in a test, you can misread a word you really do know very well.

ARBITER	a supposedly unprejudiced judge	ARBITRARY	prejudiced
ASCETIC	self-denying	AESTHETIC	pertaining to beauty
AVERSE	disinclined	ADVERSE	opposed

BAN	prohibition	BANE	woe
CENSURE	to find fault	CENSOR	to purge or remove offensive passages
COMPLACENT	self-satisfied; smug	COMPLAISANT	kindly; submissive
CONTEMPTIBLE	despicable	CONTEMPTUOUS	scornful
COSMOPOLITAN	sophisticated	METROPOLITAN	pertaining to the city
CREDIBLE	believable	CREDITABLE	worthy of praise
DEMURE	modest	DEMUR	to hesitate; raise objections
DEPRECATE	to disapprove regretfully	DEPRECIATE	to undervalue
DISCREET	judicious; prudent	DISCRETE	separate
DISINTERESTED	unprejudiced	UNINTERESTED	not interested
DIVERS	several	DIVERSE	varied
ELICIT	to extract	ILLICIT	unlawful
EMEND	to correct a text or a manuscript	AMEND	to improve by making slight changes
EXULT	to rejoice	EXALT	to raise; praise highly
GOURMET	lover of good food	GOURMAND	glutton
EQUABLE	even-tempered	EQUITABLE	just; fair
INDIGENT	poor	INDIGENOUS	native
INGENIOUS	clever	INGENUOUS	frank; naive
INTERNMENT	imprisonment	INTERMENT	burial
MAIZE	corn	MAZE	confusing network
MARTIAL	warlike	MARITAL	pertaining to marriage
MENDACIOUS	untruthful	MENDICANT	begging; beggar
PERSONAL	private	PERSONABLE	attractive
PERSPICACIOUS	shrewd	PERSPICUOUS	clear; lucid
PRODIGAL	wasteful	PRODIGIOUS	extraordinarily large
REGAL	royal	to REGALE	to entertain lavishly
SANCTION	authorization; penalty	SANCTITY	holiness
SOCIAL	pertaining to human society	SOCIABLE	companionable
URBAN	pertaining to the city	URBANE	polished; suave
VENAL	corrupt; mercenary	VENIAL	pardonable

Prefixes

Prefixes would be very simple if each one always had only one meaning. Unfortunately, this is not the case. Some of the commonest prefixes have several meanings—sometimes even opposite meanings. For example, the prefix *in-* sometimes means "not," as in "inactive," but it can also mean "very"—that is, it can intensify rather than reverse the meaning of the word root it precedes—as in "insurgent." It can also mean "in" or "on," as in "infer." To complicate matters further, the prefix *in-* sometimes changes its form to *il-*, *im-*, or *ir-*, depending on the word root that it is combined with. Thus "illegitimate," "imply," and "irradiate" all have the same prefix, *in-*, but it both takes a different form and has a different meaning in each word. Finally, English takes its prefixes from several older languages, and identical prefixes can vary depending on which language they came from. Thus the *a-* prefix in "averse" is from Latin and means "away" or "from"; the *a-* prefix in "amoral" is from Greek and means "without" or "not"; and the *a-* prefix in "aboard" is from Old English and means "on" or "in."

Does this make it impossible to determine the meaning of a prefix when you see it in a particular word? No. As your verbal skills increase, you will find that you almost always sense the *correct* meaning of a prefix if you know all the things it *can* mean. For example, the word "irradiate," which was mentioned above, might confuse you if you had only a partial understanding of the prefix *in-;* you might think the word meant "not radiate." Such a meaning would not make much sense in typical contexts. But if you know that *in-* can intensify as well as reverse the meaning of the root word, you would have very little difficulty understanding "irradiate" in context; you would sense that "radiate" rather than "not radiate" was the basic meaning, even if you had never seen the word "irradiate" before.

Consequently it is well worth your while to study the prefixes that are listed here. At first you may think that you are only confusing yourself. Actually you are increasing your awareness of words and their structure. The confusion will lessen if you persevere.

The list has been divided into prefixes of Old English, Latin, and Greek origin. Usually Latin prefixes go with Latin roots and Greek prefixes go with Greek roots, but there are many exceptions.

PREFIX (AND VARIANTS)	GENERAL MEANING	EXAMPLES OF USE
Prefixes of Old English origin		
a-	in, on, of, up, to	astride, afoot
be-	around, about, away, very	behead, beset, beloved
for-	away, off, from	forsake, forbid
fore-	before, previous	foreword, forethought
mis-	badly, not, poorly	misfit, misfire
over-	over, excessively	overthrow, overcast
un-	not, opposing	unfold, unknown
Prefixes of Latin origin		
ab-, a-, abs-	from, off, away	abdicate, averse, abstract, abstain
ad-, ac-, af-, ag-, al-, an-, ap-, ar-, as-, at-	to, toward, very	advocate, accede, affiliate, aggression, allude, annul, appear, arrogate, assent, attempt
ambi-	around, both	ambition, ambidextrous
ante-, anti-	before, previous	antecedent, anticipate
bi-	two, twice	bisect, bilateral
circum-	around	circumvent, circumspect
com-, co-, col-, con-, cor-	with, together, very	commotion, complicate, coexist, collate, congenital, corrupt
contra-	against, opposing	contradict, contravene
de-	away, from, off, down	demur, demolish,
demi-	half	demigod, demitasse
dis-, di-, dif-	away, off, down, opposing	dissent, digress, diffident, diffuse
ex-, e-, ef-	away, from, out	excise, expulsion, eradicate, efface

PREFIX (AND VARIANTS)	GENERAL MEANING	EXAMPLES OF USE
Prefixes of Latin origin		
ex-	former	ex-husband, ex-convict
extra-, extro-	outside, beyond	extramural, extrovert
in-, il-, im-, ir-	in, into, within; not, opposing; very	induct, illumine, immigrate, irrigate; incapable, insoluble, inutile, illicit, implacable, irreverent; insurgent
inter-	among, between	intercede, intersperse, intermittent
intro-, intra-	inwardly, to the inside, within	introvert, introduce, intravenous
non-	not	nonessential, nonentity
ob-, oc-, of-, op-	over, against, toward, very	obtrude, obstruct, occlude, offend, opposite
per-	through, thoroughly	permeate, pernicious,
post-	after, following	postpone, postcript
pre-	before	prevent, preclude
pro-	forward, forth, favoring, in place of	protract, profuse, proslavery, pronoun
re-	back, backward, again	revoke, recede
retro-	back, backward	retroactive, retrospect
se-	away, aside	secede, seclude
semi-	half	semiannual, semiconscious
sub-, suc-, suf-, sug-, sum-, sup-, sus-	under, beneath	subsist, subjugate, suggestive, summon, sustain, succinct, suffuse
super-	over, above, extra	supervise, superscript
trans-	across, beyond	transfusion, transcend transient
ultra-	beyond, excessive	ultramodern, ultraviolet
Prefixes of Greek origin		
a-, an-	lacking, without, not	amorphous, anarchy
amphi-	around, both	amphibian, amphitheater
ana-	back, throughout, against	anagram, anachronism
anti-	against, opposing	antipathy, antithesis
apo-	from, away	apology, apostate
arch-, archi-	chief, first	architect, archbishop
cata-	down, away	catalyst, catastrophe
dia-	through, across, apart	diameter, diagnose
en-, em-	in, within, among	endemic, empirical
epi-	on, over, outside	epigram, epidermis
eu-	good, well	eulogy, euphemistic
hemi-	half	hemisphere, hemiphrase
hyper-	excessive, over	hypercritical, hyperbole
hypo-	under, beneath	hypodermic, hypothesis
meta-	change of, over	metamorphosis, metaphor
para-	beside, beyond	parallel, paraphrase
peri-	around, near	periscope, perimeter
pro-	before	prognosis, program
syn-, sym-, syl-, sys-	together	synchronize, sympathy, syllogism, systematic

PREFIXES DENOTING NUMBER

A special class of prefixes are formed from the Latin and Greek words for numbers. Usually Latin prefixes are used with Latin roots and Greek prefixes with Greek roots. Latin prefixes tend to be used with roots that are neither Greek nor Latin.

NUMBER DENOTED	LATIN PREFIX	EXAMPLE	GREEK PREFIX	EXAMPLE
half	semi-	semicircle	hemi-	hemisphere
one	uni-	uniform	mono-	monograph
two	bi-	bilateral	di-	dimeter
three	tri-	trireme	tri-	tricycle
four	quadr-	quadruped	tetra-	tetrahedron
five	quinque-, quint-	quintuple	penta-	pentagon
six	sex-	sextet	hexa-	hexameter
seven	sept-	September	hepta-	heptameter
eight	oct-	October	octa-, octo-	octopus
nine	nona-, novem-	November	ennea-	ennead
ten	dec-	decimate	deca-	decalogue
twelve	duodec-	duodecimo	dodeca-	Dodecanese
hundred	cent-	centennial	hecto-	hectograph
thousand	milli-	millennium		

Roots

In a loose sense, a root word is simply an ordinary word to which prefixes and suffixes can be added. For example, "antidisestablishmentarianism" is merely the word "establish" with an unusual number of prefixes and suffixes. But in a stricter sense, "establish" itself is constructed on the root *-stab-*. The more of these basic roots you know, the better able you will be to deal with unfamiliar words.

ROOT	MEANING	EXAMPLES
Greek roots		
-agog-	lead, leader	demagogue, synagogue
-anthrop-	man, mankind	anthropology, misanthrope
-arch-	ancient, chief	archaeology, monarch
-astr-, -aster-	star	astral, astrology, asterisk
-auto-	self	autonomy, autocratic
-bibli-	book	Bible, bibliography
-bio-	life	amphibious, biology
-chrom-	color	chromosome, chromatic
-chron-	time	synchronize, anachronistic
-cosm-	order, world	cosmic, microcosm
-crac-, -crat-	power, rule	democracy, aristocrat

ROOT	MEANING	EXAMPLES
Greek roots		
-crypt-	secret, hidden	cryptic, cryptogram
-cycl-	wheel, circle	tricycle, cyclic
-dem-	people	democracy, demagogue
-derm-	skin	epidermis, dermatology
-dox-	belief, teaching	orthodox
-dyn-	force, power	dynamic, dynasty
-erg-	work, power	energy, energetic
-gam-	mate, marry	bigamy, monogamous
-gen-	kind, race	genealogy, eugenics
-geo-	earth	geometry, geology
-gon-	corner, angle	diagonal, trigonometry
-gram-	write, writing	telegram, epigram
-graph-	write, writing	graphite, geography
-heli-	sun	heliotropism, helium
-hem-	blood	hemorrhage, hemophilia
-hetero-	other, different	heterogeneous, heterodox
-homo-	same	homogeneous, homograph
-hydr-	water	dehydrate, hydrant
-iatr-	heal, cure	iatric, geriatrics
-iso-	same, equal	isotope, isosceles
-lith-	rock	monolithic, lithography
-log-	speech, word, study	theology, epilogue
-mega-	large, enlargement	megalomania
-metr-, -meter-	measure	diameter, trigonometry
-micr-	small	microbe, microscope
-mon-	one, single	monotonous, monologue
-morph-	form	metamorphosis, amorphous
-necr-	die, dead	necrology, necromancy
-neo-	new	neolithic, neophyte
-nom-	law, rule	autonomy, economy
-onym-	name	anonymous, pseudonym
-orth-	straight, correct	orthodontist, orthodox
-pan-	all, entire	panorama, panacea
-path-	feeling, suffering	apathy, sympathy
-ped-	child	orthopedic, pediatrician
-phil-	like, love	philanthropic, bibliophile
-phon-	sound	phonetics, phonoglogy
-phor-	bear, bearing	euphoria, phosphorous
-phot-	light	photostat, photograph
-pod-	foot	tripod, podiatrist
-poly-	many	polygon, polytheistic
-proto-	first	protocol, prototype
-psych-	mind	psychology, psychosomatic
-pyr-	fire	pyromania, pyrotechnic
-scop-	seeing, watch	telescope, microscope
-soph-	wise, wisdom	sophomore, philosopher
-tax-, -tac-	arrange, arrangement	taxidermy, syntax
-techn-	art, skill	polytechnic, technology
-tele-	far, distant	telegraph, telepathy
-the-	god	theist, atheist, theology
-therm-	heat	thermal, thermometer
-tom-	cut	atom, epitome
-urg-	work, power	metallurgist
-zo-	animal	protozoa, zoology, zodiac

ROOT	MEANING	EXAMPLES
Latin roots		
-ag-, -act-	do, drive, impel	agent, active, transact
-ac-, -acr-	sharp	acute, acrid
-agr-	field	agrarian, agriculture
-ali-	other	alien, alibi, alias
-alter-, altr-	other, change	alternate, alter ego, altruism
-am-, -amic-	love, friend	amity, amicable
-anim-	mind, soul, spirit	animosity, animate
-annu-, -enni-	year	anniversary, perennial
-aqu-	water	aquarium, aqueduct
-apt-, -ept-	adjust, fit	adapt, inept, aptitude
-arm-	arm, weapon	army, disarmament
-art-	skill, craft, art	artisan, artificial
-aud-, -audit-	hear	auditory, audience
-aur-	gold	aureole, aureate
-bel-, -bell-	war	rebel, bellicose
-ben-, -bene-	well, good	benefit, benediction
-brev-	short, brief	brevity, abbreviate
-cand-	white, glowing	incandescent, candor
-cap-, -capt-, -cept-, -cip-	take, seize	capture, captivate, intercept
-capit-	head	per capita, decapitate
-carn-	flesh	incarnate, carnivorous
-ced-, -cede-, -cess-	go, yield	secede, intercede, precede
-cent-	hundred	century, centipede
-cern-, -cert-	perceive, decide, make certain	discern, certify
-cid-, -cis-	cut, kill	homicide, incision
-cit-	summon, impel	excite, citation, incite
-civ-	citizen	civilian, civilize
-clam- (-claim-)	shout, cry out	clamor, proclaim
-clar-	clear	clarify, clarity
-clin-	bend	decline, recline
-cogn-	know, be acquainted	recognize, cognizant
-clud-, -clus-	close, shut	preclude, seclusion
-corp-	body	corpuscle, corpulent
-cred-	belief, trust	credible, credulous
-cruc-	cross	cruciform, crucify
-culp-	fault, blame	culpable, culprit
-cur-, -curr-, -curs-	run, course	concur, cursory, curriculum
-cur-	care, care for	accurate, curator
-dec-	ten	decimal, decimate
-dent-	tooth	dentist, indent
-dic-, -dict-	say, speak, word	indicate, diction, predict
-dign-	worth, worthy	dignify, condign
-doc-, -doct-	teach, prove	doctrine, docile
-du-	two	duet, duplicate
-duc-, -duct-	lead	reduce, conduct, deduct
-dur-	hard, lasting	durable, obdurate
-ego-	I, self	egotist, egoist
-equ-	equal	equity, equilibrium
-err-	wander	erratic, aberration
-ev-	time, age	medieval, longevity
-fac-, -fact-, -fect-, -fic-	do, make	facile, facsimile
-ferv-	boil, bubble	fervor, effervescent
-fer-	bear, yield	fertile, transfer
-fid-	belief, faith	fidelity, infidel

ROOT	MEANING	EXAMPLES
Latin roots		
-fin-	end, limit	final, infinite
-firm-	strong	confirm, affirm
-flect-, -flex-	bend, twist	reflection, flexible
-flor-	flower	floral, florid
-flu-, -fluct-	flow	fluent, fluctuate
-form-	form, shape	formative, transform
-fort-	strong	fortify, effort
-frag-, -fract-	break	fragile, fragment, refract
-fus-	pour	transfuse, effusive
-gen-	birth, origin, kind	generate, progeny
-grad-, -gress	step, go	egress, gradual, digress
-grat-	please, favor	gratitude, gratify
-grav-	weight, heavy	grave, gravitation
-greg-	flock	congregate, aggregate
-her-, -hes-	cling, stick	adhere, coherence
-it-	go, travel	exit, transit, circuit
-jac-, -ject-	throw, hurl, cast	eject, projectile
-junct-	join	juncture, adjunct
-jud-	judge	judicious, prejudice
-jur-	swear	perjury, conjure
-labor-	work	laboratory, elaborate
-leg-	law	legislate, legitimate
-leg-, -lig-, -lect-	choose, read	legible, negligible, election
-lev-	light, rise	levitation, elevate
-liter-	letter	literal, illiterate
-loc-	place	locality, locus
-loqu-, -locut-	talk, speech	elocution, colloquial
-luc-	light	lucid, translucent
-magn-	large	magnitude, magnify
-mal-	bad	malady, malevolent
-man-, -manu-	hand	manual, manacle
-mar-	sea	marine, maritime
-mater-, -matr-	mother	maternity, matriarchy
-medi-	middle	mediocre, mediate
-merg-, -mers-	dip, plunge	submerge, immersion
-min-	less, little	minus, minimize
-mit-, -miss-	send	admittance, transmission
-mon-, -monit-	warn	admonish, monitor
-mob-, -mot-	move	mobile, remote
-mor-, -mort-	die, death	moribund, mortuary
-mor-	custom	morality, mores
-multi-	many	multiple, multifarious
-mut-	change	immutable, commute, transmute
-nav-	ship, sail	naval, navigate
-neg-	deny	negate, negative
-nomen-, -nomin-	name	nominate, nomenclature
-nov-	new	novelty, renovate
-ocul-	eye	oculist, binocular
-omni-	all	omnipotent, omnivorous
-ora-	speak, pray	orate, oracular
-orn-	decorate	ornate, ornament
-par-	equal	par, disparity, disparate
-pater-, -patr-	father	paternal, patriotic
-ped-	foot	pedal, quadruped

ROOT	MEANING	EXAMPLES
Latin roots		
-pel-, -puls-	drive	repel, compulsion
-pend-, -pens-	hang, weigh	suspend, suspense, propensity
-pet-	seek	petition, competitor
-pon-, -pos-	place, put	postpone, interpose
-port-	carry	transport, importation
-pot-	power	impotent, omnipotent
-press-	press	compress, express
-prim-	first	primary, primeval, primitive
-quir-, -quis-	ask, seek	inquire, inquisition
-reg-, -rig-, -rect-	rule, straight, right	regent, directive, incorrigible
-rid-, -ris-	laugh	ridiculous, derision
-rog-	ask	abrogate, interrogative
-rupt-	break	rupture, abrupt, interrupt
-sanct-	holy	sanction, sanctuary
-scrib-, -script-	write	transcript, manuscript
-sed-, -sid-, -sess-	sit, seat	sedentary, residual, session
-seg-, -sect-	cut	segment, dissect
-sent-, -sens-	feel	sensory, sentient
-sequ-, -secut-	follow	sequel, subsequent, consecutive
-sol-	alone	solitude, solitaire
-solv-, -solu-, -solut-	loosen, free	solvent, solution, dissolute
-son-	sound	resonant, unison
-spec-, -spic-, -spect-	look, see	specimen, perspicacity
-spir-	breathe	expire, inspiration
-stab-	firm, standing	stable, establish
-string-, -strict-	bind tight	stringent, constrict
-struct-	build	structure, construct
-suad-, -suas-	advise	persuade, dissuade
-sum-, -sumpt-	take	resume, presumption
-tang-, -ting-, -tact-, -tig-	touch	tangible, contingent, tactile
-tempor-	time	contemporary, temporary
-tenu-	thin	tenuous, extenuate
-ten-, -tin-, -tent	hold, contain	tenure, detention, retentive
-tend-, -tens-, -tent-	stretch	distend, tendency, pretense
-terr-, -ter-	land	terrestrial, subterranean
-test-	witness	testify, testimony
-tort-, -tors-	twist	distort, contortion
-tract-	draw, pull	extraction, distract
-trib-	assign	attribute, contribution
-trud-, -trus-	thrust	obtrude, abstruse
-turb-	agitate	turbid, turbulence, disturbance
-umbr-	shade	umbrella, adumbrate
-und-	wave	undulant, innundate
-uni-	one	unity, uniform, unison
-urb-	city	urban, suburban
-vac-	empty	vacant, evacuate
-vad-, -vas-	go	pervade, evasion
-ven-, -vent-	come	intervene, event
-verb-	word	verbose, verbal
-ver-	true	veracity, verisimilitude
-vert-, -vers-	turn	convert, aversion
-vest-	dress	vestment, divest
-vid-, -vis-	see	visualize, provident
-vinc-, -vict-	conquer	victor, convince
-vit-	life	vitality, vitamin

ROOT	MEANING	EXAMPLES
Latin roots		
-viv-	life, lively	vivacious, survivor
-voc-, -vok-	call	vocal, provoke
-vol-	wish	volition, volunteer
-volv-, -volu-, -volut-	roll, turn	evolve, revolve
-vulg-	common	vulgar, divulge

Suffixes

Suffixes are somewhat simpler than prefixes, because there are fewer of them and their forms do not change as much. Usually a suffix determines what part of speech a word is. For example, *-ment* added to the verb "establish" makes the noun "establishment." Consequently, a suffix not only affects the meaning of a word but determines the role the word plays in the sentence of which it is a part. This makes suffixes particularly valuable as clues to meanings of sentences and entire passages.

ADJECTIVE AND NOUN SUFFIXES

SUFFIX	SPEECH PART	MEANING	EXAMPLES
Suffixes of Old English origin			
-dom	n.	state, rank, condition	wisdom, serfdom
-en	adj.	made of, like	wooden, golden
-er	n.	doer, actor, maker	writer, swimmer
-ful	adj.	full of, marked by	grateful, careful
-hood	n.	state, condition	manhood, statehood
-ish	adj.	rather, suggesting, like	warmish, girlish
-less	adj.	without, lacking	hopeless, senseless
-ly	adj.; adv.	like, in the manner of	friendly, meanly
-ness	n.	quality, state	greatness, smallness
-ship	n.	condition, office	friendship, clerkship
-some	adj.	showing, tending to	lonesome, bothersome
-t	n.	act, state, quality	flight, weight
-th	n.	act, state, quality	breadth, growth
-ward (s)	adj.; adv.	in the direction of	forward, backward
-y	adj.	showing, suggesting	hilly, wavy
Suffixes of foreign origin			
-able	adj.	able, fit, likely	capable, tolerable
-age	n.	state, place, process	passage, bondage
-al	n.; adj.	pertaining to doing; act	animal, capital
-an	adj.; n.	one belonging to	human, Asian
-ance, -ancy	n.	act, state, condition	acceptance, allowance
-ant	adj.; n.	doing, showing; agent	servant, pendant
-ar	adj.; n.	pertaining to; marked by	regular, capitular
-ard, -art	n.	one doing	wizard, braggart

SUFFIX	SPEECH PART	MEANING	EXAMPLES
Suffixes of foreign origin			
-ary	adj.; n.	belonging to; showing	adversary, primary
-ate	n.	rank, office	delegate, sublimate
-ation	n.	action, state, result	occupation, manifestation
-cy	n.	state	accuracy, democracy
-ee	n.	one showing	employee, refugee
-eer	n.	worker at, maker, doer	engineer, auctioneer
-ence, -ency	n.	act, state, condition	emergency, evidence
-ent	adj.; n.	doing, showing, agent	confident, solvent
-er	n.	doer, for; action, result	buyer, fencer
-ery	n.	action, skill, state	robbery, surgery
-escent	adj.	beginning, becoming	obsolescent, quiescent
-ese	adj.; n.	of a place, style, language	Chinese, Japanese
-esque	adj.	in the style of, like	statuesque, grotesque
-ess	n.	feminine	governess, lioness
-et, -ette	n.	diminutive; feminine	midget; suffragette
-fic	adj.	making, causing	terrific, beatific
-ian	adj.; n.	pertaining to; marked by	Asian, reptilian
-ible	adj.	able, likely, fit	edible, incorrigible
-ic	adj.; n.	dealing with, like, caused by; person or thing showing	cosmetic, alcoholic
-ice	n.	act, state, quality	justice, service
-id	n.; adj.	marked by, showing	solid, rancid
-ile, -il	adj.; n.	marked by, showing	juvenile, fossil
-ine	adj.; n.	marked by, dealing with	canine, marine
-ion	n.	action, state, result	fusion, opinion
-ism	n.	act, manner, state, doctrine	barbarism, socialism
-ist	n.; adj.	practicer, believer, doer	anarchist, atheist
-ite	adj.; n.	formed, showing, marked by	favorite, composite
-ition	n.	action, state, result	malnutrition, expedition
-ity	n.	state, quality, condition	acidity, placidity
-ive	adj.; n.	belonging to, tending to	defective, detective
-ment	n.	result, means, action	refreshment, adornment
-mony	n.	resulting state, condition	matrimony, ceremony
-or	n.	doer, office, action, state	elevator, honor
-ory	adj.; n.	doing, pertaining to	accessory, olfactory
-ose	adj.	marked by, given to	bellicose, morose
-ous	adj.	marked by, given to, full of	religious, riotous
-ry	n.	condition, practice	dentistry, rivalry
-tion	n.	action, state, condition	relation, creation
-tude	n.	quality, state, result	fortitude, multitude
-ty	n.	quality, state	activity, enmity
-ure	n.	act, result, state, means	culture, signature
-y	n.	result, action, quality	jealousy, inquiry

VERB SUFFIXES

SUFFIX	MEANING	EXAMPLES
-ate	become, form, treat	separate, sublimate
-en	become, cause to be	deepen, darken
-esce	grow, continue, become	convalesce, acquiesce
-fy	make, cause, cause to have	glorify, fortify
-ish	do, make, perform	punish, embellish
-ize	make, cause to be, treat with	sterilize, cauterize

Word List

The words that follow are particularly apt to appear on the SAT. The definitions are necessarily brief; if you do not understand a definition, look the word up in a regular dictionary.

Abase	to humiliate
Abate	to lessen
Abdicate	to give up
Abet	to aid
Abeyance	inactivity
Abhor	to hate
Abominate	to abhor
Aborigine	original inhabitant
Abort	to miscarry
Abrade	to scrape out
Abrograte	to abolish
Abscond	to run off
Absolve	to pardon
Abstract	summary
Abstruse	hard to understand
Abut	to touch, to border on
Accolade	honor, award
Accrue	to accumulate
Acerbity	bitterness
Acme	peak
Acrimony	acerbity
Adamant	unyielding
Adjudicate	to decide a case
Adjunct	assistant
Admonish	to warn
Adroit	skillful
Adulation	praise
Advocate	to recommend
Aesthetic	pertaining to beauty
Affable	friendly; courteous
Affinity	attraction
Affluence	wealth
Affront	insult
Aggrandize	to enlarge
Agility	quickness, nimbleness
Agnostic	one who believes God is unknowable
Agrarian	pertaining to farming
Alacrity	liveliness
Alimentary	supplying food
Allay	to calm; to reduce intensity
Allegory	narrative using figurative language
Allocate	to set aside; to apportion
Allude	to refer to indirectly
Altercation	angry dispute
Altruism	unselfish devotion
Ameliorate	to improve
Amiable	pleasant: kind
Amnesty	pardon
Amulet	object worn to ward off evil
Anachronism	something out of its proper time
Anarchy	absence of government
Anathema	ban, curse
Animate	to move to action
Animosity	hatred; dislike
Annals	historical records
Anneal	to toughen
Anomaly	irregularity; exception from the norm
Anthology	collection of literary pieces
Antipathy	dislike
Antipodes	opposite side of the earth
Antithesis	direct opposite
Apathy	indifference; lack of interest
Aphorism	brief statement; proverb
Aplomb	self-confidence
Apoplexy	sudden loss of consciousness
Apostate	one who foresakes one's principles
Apothecary	druggist
Appall	to terrify; to shock
Apposite	appropriate
Arabesque	ornate design; ballet position
Arable	able to be plowed
Archaeology	study of remains of past cultures
Archaic	no longer in use
Archipelago	group of islands
Archives	place where records are kept
Arduous	difficult
Arrears	in debt
Articulate	to speak clearly; to write
Artifice	trickery
Ascendant	rising
Aspersion	slanderous remark
Assay	to test; to analyze
Assiduity	care; diligence
Assuage	to make less severe: to calm
Astral	pertaining to the stars
Astute	shrewd
Atheist	one who denies the existence of God
Athwart	across
Atoll	island which encloses a lagoon

Atrophy	to waste away
Attest	to confirm
Audacious	bold
Audible	able to be heard
Augment	to increase
Augur	to predict
August	majestic; imposing
Auspices	protection
Auspicious	favorable
Austerity	severity
Authoritative	commanding
Autocratic	arrogant
Autonomy	self-government
Auxiliary	giving aid
Avarice	greed
Avoirdupois	weight
Avow	to declare openly
Awry	off the right course
Badger	to harass; to nag
Balk	to hinder; to stop short
Balm	something that soothes
Banal	meaningless, commonplace
Bandy	to exchange (as words)
Bane	a curse; denunciation
Baneful	creating destruction; ruinous
Beatitude	state of bliss
Beguile	deceive
Bellicose	warlike
Benediction	blessing
Beneficence	kindness
Benign	harmless, kindly
Benignant	gentle; kindly
Benison	blessing
Beset	to attack
Bestial	savage
Bestow	to present (as a gift)
Bestride	to mount
Bewitch	to charm
Bicker	to quarrel
Biennial	occurring every two years
Bilk	to cheat
Bland	mild
Blasphemy	a profane remark about God
Blatant	objectionably loud
Blazon	to adorn; to proclaim
Blithe	gay
Bluster	to speak boastfully
Bondage	slavery
Boorish	rude
Botch	to ruin through clumsiness
Bounty	reward
Bourgeois	pertaining to the middle class
Bovine	pertaining to the ox, cow, etc.
Brandish	to wave menacingly
Breech	lower part of the body, buttocks
Brigand	robber
Broach	to introduce (a subject)
Browbeat	to intimidate
Bruit	to spread news
Brusque	abrupt in manner
Bucolic	pertaining to the country
Buffoon	clown
Bullion	gold or silver (as in bars)
Bumptious	arrogant
Bureaucracy	government through bureaus
Burlesque	to represent in a ridiculous way
Burnish	to polish
Butte	hill
Cacophony	harsh sound
Cajole	to coax
Caliber	degree of worth
Caliph	head of a Moslem state
Calumny	slander
Cant	tilt; whining speech
Canter	to trot
Canvass	to make a survey
Capitulate	to surrender
Caprice	an impulsive change of mind
Captious	finding fault
Captivate	to fascinate
Careen	to swerve
Caret	sign ∧ meaning "it is missing"
Carnage	slaughter
Carnivorous	flesh-eating
Carp	to complain constantly
Carrion	decaying flesh
Castigate	to punish
Cataclysm	a violent change
Cathartic	cleansing
Cavil	to quibble
Censure	to criticize sharply
Centrifugal	moving away from the center
Chaff	worthless matter
Chagrin	embarrassment
Chamberlain	chief steward
Charlatan	faker
Chaste	morally pure
Chauvinism	fanatical patriotism
Chattel	slave
Chicanery	deception
Chide	to scold
Chilblain	inflammation
Chimerical	imaginary
Chronicle	record of historical events
Circumspect	watchful
Citadel	fortress
Civility	politeness
Clandestine	secret
Claret	dark red
Clime	climate

Coerce	to force
Coffer	strongbox
Cogent	convincing
Cogitation	meditation
Cognate	related
Cognizant	aware
Collate	to put together in proper order
Collocation	arrangement
Colloquy	conversation
Collusion	secret agreement to defraud another
Commensurate	proportionate
Commissary	a store selling food and equipment
Commodious	spacious
Compassion	pity for distress of another
Complacent	contented
Compunction	uneasiness; remorse
Concatenation	act of linking together
Conclave	secret meeting
Concomitant	accompanying
Condolence	expression of sympathy
Configuration	shape; arrangement
Congeal	to freeze solid
Congenital	existing from birth
Congruent	agreeing
Coniferous	cone-bearing
Conjecture	to guess; to suppose
Connote	to suggest; to imply
Connubial	pertaining to marriage
Conscript	to force into service
Consort	husband or wife companion; to be in agreement
Constellation	group of stars
Consternation	sudden confusion; panic
Constrict	to shrink
Contemn	to despise
Contiguous	adjacent; touching
Contingent	possible
Contumely	rudeness
Conundrum	riddle
Convivial	sociable; jovial
Convoke	call together
Convolution	coiled state
Copious	abundant
Corollary	inference; result
Corona	crown; luninous circle
Corporeal	pertaining to the body
Corpulent	fat
Corroborate	to confirm
Covenant	contract
Covert	concealed; secret
Cower	to cringe in fear
Cozen	to trick
Crabbed	ill-tempered
Crag	steep, projecting rock
Credible	believable
Credulity	gullibility; readiness to believe
Crimp	to bend into shape
Crony	close friend
Crux	vital point
Cudgel	club
Culpable	deserving blame
Cult	group sharing a common interest
Curry	to seek favor by flattery
Cursory	superficial
Dank	chilly and wet
Daunt	to discourage
Dearth	scarcity
Debase	to lower in rank
Debauch	to corrupt
Debility	weakness
Decadence	decay
Decant	to pour
Declivity	descending a slope
Decorous	proper
Decrepit	weakened by age
Defection	desertion
Deference	respect
Definitive	explicit; conclusive
Deleterious	harmful
Delineate	to describe
Delta	mouth of a river; Greek letter
Demeanor	behavior
Demure	shy
Deplore	to regret
Deposition	removal (as from office)
Depraved	sinful
Deprecate	to disapprove
Depreciate	to lessen in value
Derision	ridicule
Descant	discussion; melody
Descry	to discover
Desecrate	to profane
Desiccate	to make dry
Desist	to stop doing something
Desultory	disconnected; rambling
Dichotomy	division into two parts
Dictum	a positive statement
Didactic	instructive
Diffident	shy
Digress	to stray away from the subject
Dilate	to make wider or larger
Dilemma	difficult situation
Diligent	industrious
Diocese	church district
Disconcert	to disturb
Disconsolate	without hope
Discountenance	to disapprove of; to dismay
Discursive	rambling

Disdain	to scorn
Disparage	to belittle
Disparity	difference
Disseminate	to spread widely
Dissident	disagreeing
Dissipate	to scatter wastefully
Dissonant	out of harmony
Diurnal	daily
Diverge	to branch off
Divers	several
Divest	to deprive
Docile	easy to handle
Doggerel	poorly written verse
Dogmatic	arrogant; unyielding
Dolorous	mournful
Dormant	asleep; inactive
Dour	gloomy
Dregs	leftovers
Dross	waste matter
Ductile	easily molded
Dulcet	pleasing to the ear
Dudgeon	sullen resentment or anger
Duplicity	deception
Duress	force; compulsion
Dynamic	energetic
Ebullition	bubbling over
Eclat	brilliance
Eclogue	poem about the country
Edify	to enlighten
Effete	worn out
Efficacious	potent to produce an effect
Effigy	a dummy
Effrontery	boldness
Egregious	outstanding
Ejaculate	to exclaim
Elicit	to draw forth
Emaciated	haggard
Emanate	to issue forth
Embezzle	steal
Emollient	softener
Empirical	experimental
Emulate	to excel or imitate
Encomium	praise
Engender	to cause
Enhance	to increase
Enigma	puzzle
Enjoin	to prohibit
Ephemeral	short-lived
Epicure	lover of good living
Epitaph	writing on a tombstone
Epitome	brief statement
Equable	calm; uniform
Equitable	just; fair
Equivocal	uncertain; obscure
Erotic	tending to arouse sexual love or desire
Erratic	wandering
Erudite	learned
Esoteric	relating to knowledge
Ethical	accepted standards of conduct
Ethnic	relating to races of people
Eulogy	high praise
Euphony	sweet sound
Exacerbate	to make more violent or bitter
Excise	to remove by cutting out
Exculpate	to clear from blame
Execration	to denounce; curse
Exigency	urgency
Exonerate	to clear from blame
Exotic	strikingly different
Expatiate	wander
Expectorate	to spit
Expedient	advisable under the circumstances
Expiate	to atone for
Expound	to state in detail
Expunge	to erase
Extant	in existence
Extol	to praise highly
Extradite	to surrender a prisoner to another authority
Extraneous	not belonging
Extricate	to free
Extrinsic	not an essential part
Fabricate	to construct; to devise a deception
Facade	front part of a building
Facetious	humorous
Facile	easy
Factitious	artificial
Fastidious	hard to please
Fatuous	foolish
Feasible	possible
Fecund	fertile
Felicity	great happiness
Ferret	to hunt out
Fervent	emotional; ardent
Fetish	an object believed to have magical powers
Fetid	stinking
Fettle	good condition
Fiasco	complete failure
Fief	estate held in feudal times
Fiord	long, narrow inlet from the sea
Flaccid	soft and flabby
Flagitious	vicious; wicked
Flagrant	shocking
Flail	to thrash
Flamboyant	showy
Flaunt	to show off
Fledge	to grow feathers
Florid	rosy-colored; excessively ornate
Foible	a minor weakness
Foment	whip up

Font	basin
Forensic	pertaining to debate
Formidable	huge; dreadful
Forte	strong point
Fortitude	courage
Fortnight	two weeks
Fortuitous	by chance
Fray	(v.) to unravel; (adj.) noisy quarrel
Frizzle	to curl
Frugal	thrifty
Fulmination	violent explosion; severe censure
Fulsome	disgusting
Furtive	stealthy; foxy
Galaxy	large system of stars like the Milky Way
Gambol	to romp; to frolic
Gamut	entire range
Garble	to distort
Garrulous	talkative
Gauntlet	medieval glove
Generic	pertaining to a class or group
Germane	related; fitting
Ghoul	demon; grave robber
Gibe	to scoff; to jeer
Gird	to encircle
Glib	fluent; smooth
Goad	to spur
Gradient	slope
Gratuitous	without charge; without justification
Gregarious	sociable
Grist	grain to be ground
Grueling	very tiring; severe
Guild	organization of persons with common interests
Halcyon	calm; peaceful
Hale	healthy
Hallow	to make holy
Hallucination	illusion
Hapless	unfortunate
Harangue	long speech
Harass	to annoy by repeated attacks
Harbinger	omen
Hearth	floor or a fireplace; home
Heinous	hateful; abominable
Herbivorous	feeding on plants
Heresy	anti-religious opinion
Heretic	person who maintains opinions contrary to church doctrine
Heterogeneous	different in kind
Heyday	period of success
Hiatus	gap
Hibernal	wintry
Hirsute	hairy
Histrionic	theatrical
Hoax	joke
Holocaust	destruction by fire
Homily	sermon
Homogeneous	essentially the same
Horticulture	study of how to grow plants
Hurtle	to speed
Husbandry	study of raising crops and animals
Hybrid	mongrel; mixed breed
Hydrophobia	rabies; fear of water
Hyperbole	exaggeration
Hypochondriac	one who worries about his health
Iconoclast	destroyer of idols or images
Idyl	short poem describing simple living
Ignoble	dishonorable
Ignominious	shameful
Illicit	unlawful
Imbibe	to absorb
Imbroglio	confused state of affairs
Immutable	unchangeable
Impalpable	vague; not understandable
Impasse	dead end; difficult situation
Impeach	to accuse
Impecunious	poor; penniless
Impertinent	irrelevant; rude
Implicit	absolute; suggested
Importune	to trouble with requests
Impugn	to attack by words; to challenge as false
Impute	to attribute something bad to another
Incendiary	inflammatory
Inchoate	just begun; rudimentary
Incipient	just beginning to appear
Incongruous	not harmonious; inappropriate
Incredulous	skeptical
Increment	increase
Incubus	nightmare
Indigent	poor
Indolent	lazy
Indulgence	act of pampering
Ineffable	beyond words; unspeakable
Ineluctable	inevitable
Ineptitude	awkwardness
Inert	without power to move or react
Inexorable	unyielding
Infernal	hellish; outrageous
Infidel	unbeliever
Ingenuous	innocent
Inherent	inborn
Iniquitous	unjust
Injunction	judicial order
Innate	inherent
Innocuous	harmless
Insidious	treacherous
Insipid	dull

Insolvent	bankrupt
Insular	pertaining to an island
Intangible	not able to be touched; impalpable
Interdict	to prohibit
Interminable	endless
Interpolate	to insert something additional
Interregnum	interruption in continuity
Intransigent	refusing to compromise
Intrepid	fearless
Intrinsic	essential
Intuition	insight
Invective	denunciation
Inveigle	to entice
Investiture	act of bestowing rank or office
Inveterate	firmly established
Invidious	causing ill will; offensive
Irascible	easily angered
Ironic	contrary to what was expected
Isthmus	a narrow strip of land having water on either side
Iterate	to say or do something repeatedly
Itinerant	traveling from place to place
Jargon	meaningless talk; gibberish
Jaundice	yellowness of the skin
Jetty	wall built out into the water
Jocose	joking; humorous
Juxtapose	to place side by side
Kaleidoscopic	constantly changing as in form
Ken	range of vision or knowledge
Labyrinth	maze
Lachrymal	pertaining to tears
Laconic	using few words; concise
Laity	all the people as distinguished from the clergy
Lampoon	strong satire in written form
Languid	lacking in spirit or interest
Lascivious	full of lust; oversexed
Latent	hidden; dormant
Lethargic	sluggish
Levity	lack of seriousness
Levy	to impose a tax, fine, etc.
Lexicon	dictionary
Libretto	words of an opera
Licentious	lewd; immoral
Liege	feudal term for lord or subject
Limpid	clear, as air or water
Litany	prayer with responses
Lithe	bending easily; flexible
Litigation	lawsuit
Liturgy	form of public worship
Livid	discolored due to bruise; dull blue
Loquacious	talkative
Lucid	shining; easily understood
Lucre	money (in a bad sense)
Ludicrous	ridiculous
Lugubrious	mournful
Lymph	clear, yellowish fluid
Machiavellian	crafty; cunning
Machination	evil design
Magnanimous	generous; noble
Malevolent	wishing evil; malicious
Malign	to slander
Malinger	to pretend sickness
Martinet	very strict disciplinarian
Maudlin	tearfully sentimental
Maw	mouth
Maxim	short statement of a truth
Mellifluous	smoothly flowing
Mendacious	untruthful
Mendicant	begging; beggar
Menial	servile; low
Meretricious	showily attractive; tawdry
Mesa	flat-topped land with steep sides
Metamorphosis	change of form
Metaphysics	philosophy that is subtle and difficult to comprehend
Mete	to allot
Mettle	spirit; courage
Microcosm	world in miniature
Mien	manner; way of carrying oneself
Misanthrope	hater of mankind
Misgiving	doubt
Missal	book containing prayers
Mnemonic	assisting in the memory
Mollify	to appease
Monetary	pertaining to money
Moot	doubtful; debatable
Moribund	dying
Mosque	Mohammedan temple
Mulct	to punish by fine
Mundane	pertaining to the world; commonplace
Muting	to muffle the sound
Myriad	very great number
Nadir	lowest point
Narcissism	self-love
Nativity	birth
Nebulous	vague; indistinct
Nefarious	extremely wicked
Neophyte	beginner; convert
Niggardly	stingy
Nocturnal	pertaining to night
Noisome	offensive; harmful
Nuance	delicate variation in meaning, tone, shade
Nugatory	worthless
Nurture	to feed; to bring up

Obdurate	stubborn
Obeisance	show of respect
Oblation	offering of a sacrifice
Obscure	not clear
Obsequious	overly submissive
Obsolescence	passing out of use
Obstreperous	boisterous
Obtrude	to intrude
Obtuse	stupid
Obviate	to prevent
Odious	hateful; disgusting
Officious	meddlesome
Oligarchy	government by a few
Ominous	threatening
Omnivorous	eating all kinds of food
Onerous	burdensome
Opprobrious	shameful
Opulence	wealth
Oracular	predicting
Orbit	path
Organic	fundamental; essential
Orifice	opening; mouth
Ornate	overadorned; showy
Orthography	spelling
Ossify	to change into bone
Ostentatious	pretentious
Ostracize	to banish
Overt	not concealed
Palatable	tasty
Pall	to become wearisome
Palpable	obvious
Palsy	paralysis
Panacea	cure-all
Pander	to act as a go-between in intrigue
Parable	short story to convey a moral
Paradox	seemingly absurd statement that is nevertheless true
Parsimony	stinginess
Pastoral	pertaining to shepherds
Patent	evident
Pathos	quality of arousing pity
Patriarch	father-ruler of a family or tribe
Patrician	aristocratic
Patrimony	property inherited from one's father or ancestors
Paucity	scarcity
Pavilion	building used for exhibits
Peccadillo	minor sin; slight fault
Pectoral	pertaining to the chest
Pecuniary	pertaining to money
Pedant	one who possesses mere book learning; narrow-minded teacher
Paradox	seemingly contradictory statement
Perdition	hell
Peremptory	dictatorial; unconditional
Perennial	enduring; lasting all year or year after year
Perfidious	treacherous
Perforce	of necessity
Perfunctory	mechanical; indifferent
Periphery	outside boundary
Pernicious	destructive; fatal
Perpetuate	to cause to continue
Perquisite	something additional to regular pay
Perspicacious	having insight
Pert	forward; impertinent
Perturbation	agitation
Pervade	to spread throughout
Philanthropist	lover of mankind
Philistine	narrow-minded person
Phlegmatic	calm; not easily disturbed; dull
Pinnacle	peak
Piquant	pleasantly tasting; stimulating
Pithy	concise
Placate	to pacify; to conciliate
Placid	peaceful; calm
Platitude	trite remark
Plebian	common
Plethora	overabundance
Poignant	keen; moving
Portend	to foretell
Portentous	ominous; serious
Portly	stately; stout
Pragmatic	practical
Prate	to speak foolishly
Precarious	risky
Preclude	to shut out; to eliminate
Precursor	forerunner
Prevaricate	to lie
Probity	incorruptibility
Proclivity	inclination; natural tendency
Prodigal	wasteful
Prodigious	huge; enormous
Profligate	dissipated; wasteful
Prolific	abundantly fruitful
Prolix	verbose; drawn out
Propensity	natural inclination
Propitiate	appease
Prosaic	commonplace
Protract	prolong
Proximity	nearness
Prurient	lascivious in thought or desire
Pseudonym	pen name
Puerile	childish
Pugnacious	combative
Puissant	powerful
Pulchritude	beauty

Punctilious	laying stress on form; precise
Pungent	stinging; caustic
Punitive	punishing
Purge	to clean by removing impurities
Purport	intention; meaning
Pusillanimous	cowardly
Quaff	to drink with relish
Qualms	misgivings
Quay	dock; landing place
Quell	to put down; quiet
Querulous	whining
Quintessence	purest and highest state
Quixotic	idealistic
Quizzical	bantering
Rabid	like a fanatic; furious
Rampant	unrestrained
Rancid	offensive
Rancor	bitterness; hatred
Rant	to rave
Rapacious	excessively grasping
Raucous	harsh and shrill
Ravenous	extremely hungry
Raze	to destroy completely
Recalcitrant	stubborn
Recant	to repudiate
Reciprocal	mutual
Recluse	hermit
Recondite	abstruse
Recreant	coward
Rectitude	uprightness
Redolent	fragrant
Redress	remedy
Refractory	stubborn
Refulgent	radiant
Relegate	to banish; to consign to an inferior position
Relevance	pertinence
Relinquish	to abandon
Remonstrate	to protest; to admonish
Renegade	deserter
Replenish	to fill up again
Reprehensible	deserving blame
Reprobation	severe disapproval
Repugnance	loathing
Requisite	necessary requirement
Rescind	to cancel
Respite	delay in punishment
Reticence	reserve; unwillingness to speak
Ribald	wanton; profane
Rococo	ornate; highly decorative
Roseate	rosy; optimistic
Rote	repetition
Rotundity	roundness
Ruminate	to ponder
Ruse	trick; stratagem
Sacrilegious	profane; desecrating
Sagacious	having insight; wise
Salubrious	healthful
Sanguinary	bloody
Sanguine	cheerful; optimistic
Sardonic	disdainful
Sate	to satisfy to the full
Saturnine	gloomy
Schism	division
Scintillate	sparkle
Scourge	whip; severe punishment
Scrupulous	conscientious; honest
Scurrilous	obscene; indecent
Secular	worldly; not pertaining to the church
Sedate	composed; grave
Sedentary	requiring sitting; inactive
Sedulous	diligent
Semblance	outward appearance
Sententious	full of meaning; pompous
Serrated	having a sawtoothed edge
Servile	slavish; cringing
Severance	division
Shibboleth	slogan
Simile	comparison of one thing with another using "like" or "as"
Simulate	feign; pretend
Sinecure	well-paid position with little responsibility
Sinuous	winding
Sloth	laziness
Slovenly	untidy
Somnolent	almost asleep
Sonorous	resonant
Specious	seemingly reasonable but incorrect
Spectral	ghostly
Splenetic	spiteful, irritable
Sporadic	occurring irregularly
Spurious	false; counterfeit
Squalid	dirty; neglected
Stentorian	extremely loud
Stoic	person who is indifferent to feeling
Stolid	dull; impassive
Strident	loud and harsh
Stringent	binding; rigid
Stupor	state of apathy
Suavity	urbanity; polish
Subservient	behaving like a slave; servile
Sublimate	to refine; to purify
Subsistence	existence; means of support
Subterfuge	pretense; evasion
Succinct	brief; terse
Succor	aid; assistance

Suffuse	to spread over
Sully	to tarnish; to soil
Sultry	sweltering
Supercilious	contemptuous; haughty
Supine	lying on back
Supplicate	to entreat; to beseech
Surfeit	to overfeed; to cloy
Surly	rude
Surreptitious	secret
Surveillance	watching; guarding
Sycophantic	servilely flattering
Sylvan	rustic; pertaining to the woods
Tacit	understood; not put into words
Taciturn	silent
Tactile	pertaining to the sense of touch
Talisman	a charm
Tantalize	to tease
Tautological	needlessly repetitious
Temerity	boldness; rashness
Tenacious	holding fast
Tenet	doctrine; dogma
Tentative	provisional; experimental
Tenuous	thin; rare; slim
Tepid	lukewarm
Terse	concise; abrupt
Testy	irritable
Thrall	slave; bondage
Thwart	to block; to oppose
Timidity	lack of self-confidence
Tirade	scolding; denunciation
Toady	to flatter for favors
Torpid	dormant; dull; lethargic
Tortuous	winding; full of curves
Toxic	poisonous
Tractable	docile
Transcend	to exceed; to surpass
Transcribe	to copy
Transient	fleeting
Translucent	partly transparent
Transmute	to change
Travail	painful labor
Tremulous	trembling; wavering
Trenchant	cutting; keen
Trepidation	fear
Trite	commonplace; hackneyed
Truculent	aggressive; savage
Tumid	swollen; pompous
Turbid	muddy; having the sediment disturbed
Turbulence	state of violent agitation
Turgid	swollen; distended
Tyro	beginner; novice
Ubiquitous	being everywhere; omnipresent
Umbrage	resentment; anger
Unbridled	violent
Uncouth	outlandish; clumsy
Unctuous	oil; bland
Undulate	to move with a wavelike motion
Unequivocal	plain; obvious
Unkempt	disheveled
Unmitigated	harsh; severe
Untenable	unsupportable
Upbraid	to scold; to reproach
Urbane	suave; refined
Usury	lending money at illegal rates of interest
Uxorious	excessively devoted to one's wife
Vacillation	fluctuation; wavering
Vacuous	empty; inane
Vagary	caprice; whim
Vainglorious	boastful
Validate	confirm; ratify
Vanguard	forerunners
Vantage	position given an advantage
Vapid	insipid; inane
Variegated	many-colored
Vaunted	boasted; bragged
Veer	to change direction
Vehement	impetuous
Venal	capable of being bribed
Veneer	thin layer; cover
Venerate	revere
Venial	forgivable; trivial
Veracious	truthful
Verbose	wordy
Verdant	green; fresh
Verity	truth; reality
Vernal	pertaining to the spring
Vertigo	dizziness
Vestige	trace
Vicarious	acting as a substitute
Vicissitude	change of fortune
Vie	to contend; to compete
Vigilance	watchfulness
Vilify	to slander
Vindicate	to clear of charges
Virago	shrew
Virile	manly
Virtuoso	highly skilled artist
Virulent	extremely poisonous
Visage	face; appearance
Viscous	sticky; gluey
Vitiate	to spoil the effect of
Vitriolic	corrosive; sarcastic
Vituperative	abusive; scolding
Vivacious	animated; gay
Vociferous	noisy; talkative
Volatile	explosive; changeable
Volition	act of making a conscious choice

Voluble	fluent; glib
Voluptuous	gratifying to the senses
Voracious	ravenous
Votary	follower of a cult
Vulnerable	susceptible to wounds
Waggish	mischievous
Wan	having a pale or sickly color
Wane	to grow gradually smaller
Wanton	unruly; excessive
Wary	very cautious
Whet	to sharpen; to stimulate
Wily	cunning; artful
Winsome	agreeable; gracious
Wizened	shriveled
Wont	custom; habitual procedure
Wraith	ghost; phantom of a living person
Wreak	inflict
Wrest	to pull away; to take by violence
Zealot	fanatic
Zenith	point directly overhead in sky; summit
Zephyr	soft gentle breeze; west wind

Vocabulary Exercises

These exercises are exactly like the antonym questions on the SAT. Use the techniques you have learned, especially the technique of eliminating impossible answers first. Also, use what you have learned about prefixes, roots, and suffixes.

Each practice exercise below consists of a word in capital letters, followed by five lettered words or phrases. Choose the word or phrase that is most nearly *opposite* in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, consider all the choices before deciding which is best.

PRACTICE EXERCISE 1

1. IMMUTABLE: (A) silent (B) inconstant (C) worthless (D) resolute (E) perfect

2. REFULGENT: (A) impolite (B) dull (C) unfair (D) changeable (E) presumptuous

3. CONCLAVE: (A) an open meeting (B) a public speech (C) a secret convention (D) a political rally (E) a course of conduct

4. ENHANCE: (A) diminish (B) enlarge (C) solidify (D) beautify (E) sweeten

5. LUDICROUS: (A) expensive (B) laughable (C) congruous (D) absurd (E) strange

6. CONGENITAL: (A) kind (B) complementary (C) acquired (D) irascible (E) solid

7. IMPALPABLE: (A) hesitant (B) insensitive (C) touchable (D) immaterial (E) discourteous

8. ABORTIVE: (A) productive (B) restrictive (C) pretentious (D) interrupted (E) honest

9. CONJECTURE: (A) preventive measure (B) speculation (C) certainty (D) discarded opinion (E) speculation

10. PERFUNCTORY: (A) comprehensive (B) hasty (C) commonplace (D) rude (E) caustic

PRACTICE EXERCISE 2

1. RECREANT: (A) cowardly (B) faithless (C) craven (D) bold (E) docile

2. ELICIT: (A) unlawful (B) put into (C) draw out of (D) exact (E) emerge

3. CONSTERNATION: (A) amazement (B) skepticism (C) composure (D) dismay (E) strictness

4. LUCID: (A) cloudy (B) limpid (C) transparent (D) intelligible (E) insane

5. IMPEACH: (A) upbraid (B) vindicate (C) accuse (D) remove from office (E) mutate

6. PERFORCE: (A) by necessity (B) forcefully (C) by chance (D) unavoidably (E) predetermined

7. MISGIVING: (A) apprehension (B) confidence (C) anxious (D) presentiment (E) qualm

8. COPIOUS: (A) meager (B) imitative (C) offensive (D) hypercritical (E) dull

9. RECALCITRANT: (A) reverent (B) flouting (C) defiance (D) harden (E) stubborn

10. EGREGIOUS: (A) shocking (B) undistinguished (C) extraordinary (D) outgoing (E) effervescent

PRACTICE EXERCISE 3

1. INGENUOUS: (A) naive (B) real (C) open (D) wise (E) urbane

2. COERCE: (A) force (B) constrain (C) annoy (D) implore (E) impel

3. EFFETE: (A) productive (B) sterile (C) torpid (D) worn out (E) phlegmatic

4. LOQUACIOUS: (A) garrulous (B) voluble (C) taciturn (D) verbose (E) prolix

5. TEMERITY: (A) modesty (B) impertinence (C) boldness (D) efficiency (E) hostility

6. RAPACIOUS: (A) greedy (B) eager (C) excessively grasping (D) miserly (E) magnanimous

7. COLLUSION: (A) head-on crash (B) cooperation in secret (C) fraud (D) fair trade (E) alliance

8. PARSIMONIOUS: (A) economical (B) prodigal (C) frugal (D) tight-fisted (E) modest

9. RANCOR: (A) amity (B) malevolence (C) ill will (D) status (E) spite

10. COMPLACENT: (A) discontented (B) kind (C) smug (D) courteous (E) well-adjusted

PRACTICE EXERCISE 4

1. EFFRONTERY: (A) boldness (B) impertinence (C) shamelessness (D) severity (E) diffidence

2. EBULLITION: (A) passiveness (B) power (C) dynamic (D) glow (E) limpidity

3. COMPUNCTION: (A) remorse (B) repentance (C) obdurateness (D) sorrow for wrongdoing (E) solicitude

4. IGNOMINY: (A) humiliation (B) honor (C) crudeness (D) offensiveness (E) disgrace

5. CHARLATAN: (A) quack (B) physician (C) pretender (D) impostor (E) fraud

6. SPLENETIC: (A) morose (B) curt (C) jocund (D) extravagant (E) dejected

7. DOCILE: (A) intractable (B) learned (C) obedient (D) subservient (E) gentle

8. CHICANERY: (A) deception (B) duplicity (C) candor (D) evasion (E) subtle reasoning

9. DOGMATIC: (A) opinionated (B) assertive (C) insistent (D) impervious (E) uncertain

10. HYPERBOLE: (A) literal statement (B) idiosyncracy (C) active sportswoman (D) overstatement (E) exaggeration

PRACTICE EXERCISE 5

1. CIRCUMSPECT: (A) rash (B) wary (C) circular (D) discreet (E) cautious

2. CLANDESTINE: (A) secret (B) covert (C) undisguised (D) underhanded (E) concealed

3. CALUMNY: (A) moisture (B) libel (C) adulation (D) slander (E) deception

4. CAPRICE: (A) whimsy (B) constancy (C) capability (D) error (E) fitfulness

5. LATENT: (A) punctual (B) dormant (C) hidden (D) quiescent (E) manifest

6. QUERULOUS: (A) complaisant (B) captious (C) unstable (D) inquisitive (E) quarrelsome

7. CAPTIOUS: (A) charming (B) fault-finding (C) credulous (D) fraudulent (E) bigoted

8. HIATUS: (A) gap (B) interruption (C) continuum (D) brief period (E) vacancy

9. DISPARAGE: (A) denigrate (B) extol (C) upbraid (D) accuse (E) argue

10. LANGUOR: (A) reluctance (B) dilatory (C) unconcern (D) briskness (E) controversial

PRACTICE EXERCISE 6

1. CASTIGATE: (A) criticize (B) reject (C) claim (D) eulogize (E) reprove

2. BRUSQUE: (A) affable (B) rude (C) whiskery (D) imaginative (E) discourteous

3. GARRULOUS: (A) prolix (B) verbose (C) laconic (D) loquacious (E) glib

4. WHET: (A) blunt (B) hone (C) lubricate (D) squash (E) protrude

5. BUCOLIC: (A) pastoral (B) urban (C) diseased (D) rustic (E) poetic

6. BUMPTIOUS: (A) haughty (B) arrogant (C) obsequious (D) forward (E) demanding

7. SERVILE: (A) presumptuous (B) peasantlike (C) patriotic (D) operative (E) restrained

8. LACONIC: (A) sententious (B) terse (C) circumlocutory (D) humble (E) diffident

9. DESULTORY: (A) superficial (B) banal (C) blasphemous (D) thorough (E) hasty

10. CAJOLE: (A) coax (B) wheedle (C) command (D) jeer (E) deceive

PRACTICE EXERCISE 7

1. BENIGN: (A) royal (B) humble (C) malevolent (D) kindly (E) favorable

2. DELETERIOUS: (A) beneficial (B) tasty (C) pernicious (D) immoral (E) injurious

3. OVERT: (A) avowed (B) clandestine (C) undisguised (D) merciful (E) apparent

4. SANGUINE: (A) mournful (B) chaotic (C) hopeful (D) pessimistic (E) confident

5. MENDACIOUS: (A) truthful (B) begging (C) medicinal (D) philanthropic (E) imaginary

6. INTREPID: (A) without fear (B) bothersome (C) quarrelsome (D) craven (E) gallant

7. PROPENSITY: (A) tendency (B) disposition (C) wit (D) disinclination (E) effortlessness

8. AVARICE: (A) magnanimity (B) cupidity (C) jealousy (D) hatred (E) excessive desire

9. FLAMBOYANT: (A) ostentatious (B) reserved (C) showy (D) irritable (E) antagonistic

10. BELLICOSE: (A) belligerent (B) sonorous (C) overweight (D) assertive (E) pacific

PRACTICE EXERCISE 8

1. AUSPICIOUS: (A) inopportune (B) favorable (C) suspicious (D) trusting (E) stringent

2. OPPROBRIUM: (A) disgrace (B) protest (C) oppression (D) encomium (E) discredit

3. BANEFUL: (A) destructive (B) commonplace (C) salutary (D) antiseptic (E) trite

4. EXACERBATE: (A) excavate (B) worsen (C) mitigate (D) burden (E) incite

5. EXONERATE: (A) impugn (B) vindicate (C) exculpate (D) excuse (E) frighten

6. INDIGENT: (A) native (B) indignant (C) opulent (D) impecunious (E) destitute

7. OBDURATE: (A) callous (B) hard-hearted (C) dogged (D) inflexible (E) yielding

8. EUPHONY: (A) melody (B) concord (C) cacophony (D) pleasure (E) music

9. PREVARICATE: (A) equivocate (B) tell the truth (C) misrepresent (D) quibble (E) evade

10. CULPABLE: (A) blameworthy (B) faulty (C) delinquent (D) blameless (E) acceptable

PRACTICE EXERCISE 9

1. ADAMANT: (A) firm (B) resolute (C) flexible (D) demanding (E) intractable

2. CREDIBLE: (A) skeptical (B) faithful (C) solvent (D) crude (E) unbelievable

3. NOISOME: (A) raucous (B) quiet (C) putrid (D) salubrious (E) offensive

4. CHIDE: (A) rebuke (B) admonish (C) extol (D) mock (E) reproach

5. PHLEGMATIC: (A) torpid (B) passive (C) flexible (D) barren (E) dynamic

6. AFFLUENCE: (A) opulence (B) impoverishment (C) abandonment (D) diligence (E) lack

7. EQUITABLE: (A) fair (B) calm (C) arbitrary (D) just (E) even-tempered

8. NEOPHYTE: (A) beginner (B) novice (C) expert (D) tyro (E) apprentice

9. NEFARIOUS: (A) wicked (B) distant (C) diversified (D) virtuous (E) rowdy

10. ANOMALY: (A) abnormality (B) congruence (C) mystery (D) deviation (E) paradox

PRACTICE EXERCISE 10

1. ABET: (A) assist (B) encourage (C) restrain (D) foment (E) goad

2. REMONSTRATE: (A) decry (B) laud (C) reject (D) explain (E) plead

3. NEBULOUS: (A) definite (B) vague (C) indistinct (D) obscure (E) hazy

4. ABSTRUSE: (A) complicated (B) abstract (C) esoteric (D) profound (E) obvious

5. NADIR: (A) bottom (B) empty (C) acme (D) foundation (E) beginning

6. IMPUGN: (A) indict (B) punish (C) pardon (D) execute (E) imprison

7. PERSPICACIOUS: (A) discerning (B) obtuse (C) shrewd (D) insightful (E) precise

8. EQUABLE: (A) fair (B) even-tempered (C) distracted (D) just (E) calm

9. IMPECUNIOUS: (A) indigent (B) opulent (C) selfish (D) antagonistic (E) destitute

10. MACHINATION: (A) open agreement (B) collusion (C) handiwork (D) intrigue (E) fraud

ANSWERS, PRACTICE EXERCISES 1–10

Practice Exercise 1

1. (B)	2. (B)	3. (A)	4. (A)	5. (C)
6. (C)	7. (C)	8. (A)	9. (C)	10. (A)

Practice Exercise 2

1. (D)	2. (B)	3. (C)	4. (A)	5. (B)
6. (C)	7. (B)	8. (A)	9. (A)	10. (B)

Practice Exercise 3

1. (E)	2. (D)	3. (A)	4. (C)	5. (A)
6. (E)	7. (D)	8. (B)	9. (A)	10. (A)

Practice Exercise 4

1. (E)	2. (A)	3. (C)	4. (B)	5. (B)
6. (C)	7. (A)	8. (C)	9. (E)	10. (A)

Practice Exercise 5

1. (A)	2. (C)	3. (C)	4. (B)	5. (E)
6. (A)	7. (C)	8. (C)	9. (B)	10. (D)

Practice Exercise 6

1. (D)	2. (A)	3. (C)	4. (A)	5. (B)
6. (C)	7. (A)	8. (C)	9. (D)	10. (C)

Practice Exercise 7

1. (C)	2. (A)	3. (B)	4. (D)	5. (A)
6. (D)	7. (D)	8. (A)	9. (B)	10. (E)

Practice Exercise 8

1. (A)	2. (D)	3. (C)	4. (C)	5. (A)
6. (C)	7. (E)	8. (C)	9. (B)	10. (D)

Practice Exercise 9

1. (C)	2. (E)	3. (D)	4. (C)	5. (E)
6. (B)	7. (C)	8. (C)	9. (D)	10. (B)

Practice Exercise 10

1. (C)	2. (B)	3. (A)	4. (E)	5. (C)
6. (C)	7. (B)	8. (C)	9. (B)	10. (A)

II.
THE TEST OF STANDARD WRITTEN ENGLISH

Introduction

The verbal part of the Scholastic Aptitude Test also includes a 30-minute test of standard written English to measure your familiarity with clearly written prose. Three kinds of questions are included in this test: (1) usage questions, in which you are asked to recognize the error, if any, in a sentence; (2) sentence-correction questions, in which you are asked to select the best rephrasing of part of the sentence from among the choices presented; and (3) sentence-choice questions, in which you are asked to select the best sentence.

Although it is rare, sometimes the SAT will include more than one section of standard written English. Yet, what is curious about this part of the test is that your score on it is *not included* in your final SAT score. Only your verbal and math scores are included. However, the standard written English score *is* used by colleges for the purpose of placement.

It would be unrealistic to suggest that you now begin to study English grammar. Students with a poor foundation in this area should try to *memorize* the correct answers so that they may recognize them when they see them. This is not to suggest that this is the best way to improve and broaden your knowledge in this area, but in the limited time that you have to prepare for this section this procedure will indeed help. Also, if you think about each of your wrong answers and try to analyze why it is wrong, you may discover that you *do* know the grammatical rule that the question is based on—you have just forgotten it.

Standard Written English Test 1

Time—30 Minutes
50 Questions

Directions: The following sentences contain problems in grammar, usage, diction (choice of words), and idiom.

Some sentences are correct.
No sentence contains more than one error.

You will find that the error, if there is one, is underlined and lettered. Assume that all other elements of the sentence are correct and cannot be changed. In choosing answers, follow the requirements of standard written English.
If there is an error, select the one underlined part that must be changed in order to make the sentence correct, and blacken the corresponding space on the answer sheet.
If there is no error, mark answer space E.

EXAMPLES:

I. He practices (A) every day (B) so that he (C) would become proficient (D) in his field. No error (E)

Sample Answers

	A	B	C	D	E
I.	■	□	□	□	□

II. He was rude (A) and disrespectful (B) to we (C) guests (D). No error (E)

II. A B C D E
☐☐■☐☐

1. The man, as well as (A) his three (B) brothers, were attending (C) his father's (D) funeral. No error (E)

2. Neither Harry nor (A) his friends was (B) able to attend (C) the first (D) meeting of the club. No error (E)

3. All of the doctors (A) who were (B) at the meeting was (C) concerned about the issue of malpractice (D). No error (E)

4. I know (A) that others besides Fred has (B) been irritated (C) by the click (D) of the door. No error (E)

5. Hoping (A) to achieve (B) excellence (C), the football team practice (D) every day. No error (E)

6. Us (A) gardeners are (B) never given (C) as much as we (D) deserve. No error (E)

7. Alex and me (A) want (B) to borrow (C) my mother's (D) car. No error (E)

8. He was told (A) by his (B) father to give (C) the horse it's (D) oats. No error (E)

9. The band of (A) robbers objected (B) to him (C) taking most (D) of the gold. No error (E)

10. The person who (A) would have (B) to take out the garbage turned out to be (C) I (D). No error (E)

11. If Jack was (A) at the party, we (B) would be having (C) a much better (D) time. No error (E)

12. Walking (A) down the quiet (B) street, the traffic (C) light turned (D) red. No error (E)

13. He (A) told me (B) that Edwin is (C) the kinder (D) of the three. No error (E)

14. The boy sitting (A) at that (B) desk is smarter (C) than any (D) student in the class. No error (E)

15. In (A) the summer, I always (B) enjoy (C) swimming and to jog (D). No error (E)

16. When (A) Gus finally (B) arrived at the store, the owner sells (C) him (D) a watch. No error (E)

17. Mrs. Pinkerton (A) went (B) to her present doctor (C) for the last three (D) years. No error (E)

18. The weatherman (A) says (B) that he (C) agrees with (D) the proposal the station made him. No error (E)

19. Their (A) teacher says that he is as impertinent (B) or more impertinent (C) than his (D) brother. No error (E)

20. After skiing (A) down the hill at a steep angle (B), the snow became (C) very (D) wet. No error (E)

21. If I (A) was (B) you, I'd (C) go to (D) the game. No error (E)

22. Although both roads lead (A) to New York City, take (B) the longest (C) because it is smoother (D). No error (E)

23. Walking (A) down the road, my (B) father saw (C) both Barbara and she (D). No error (E)

24. While reading (A) the directions with his (B) friend's (C) lighter, the car rolled off (D) the cliff. No error (E)

25. The adults did not wish (A) to go (B) with you (C) and I (D). No error (E)

26. The other (A) team had five (B) more hits than (C) us (D). No error (E)

27. The appraiser (A) told me that fifty dollars were (B) too (C) much to pay (D) for the ring. No error (E)

28. Every one of the dozen eggs that (A) he had (B) purchased were (C) broken on the bumpy ride (D) home. No error (E)

29. It is (A) I who (B) is (C) to be appointed acting (D) chairman. No error (E)

30. There is (A) several practical (B) ways to go (C) from our house to theirs (D). No error (E)

Directions: In each of the following sentences, some part of the sentence or the entire sentence is underlined. Beneath each sentence you will find five ways of phrasing the underlined part. The first of these repeats the original; the other four are different. If you think the original is better than any of the alternatives, choose answer A; otherwise choose one of the others. Select the best version and blacken the corresponding space on your answer sheet.

This is a test of correctness and effectiveness of expression. In choosing answers follow the requirements of standard written English; that is, pay attention to grammar, choice of words, sentence construction, and punctuation. Choose the answer that produces the most effective sentence—clear and exact, without awkwardness or ambiguity. Do not make a choice that changes the meaning of the original sentence.

31. The old man cried that he had got nothing for his years of hard labor.

(A) he had got nothing
(B) "he had got nothing
(C) "He had gotten nothing
(D) he had gotten nothing
(E) he had got no thing

32. He gave the toy to one of the boys who have been playing on the nursery floor.

(A) who have
(B) who has
(C) whom has
(D) whom have
(E) which have

33. Because the track was muddy, neither of the horses ran it's best race.

(A) it's best race
(B) their best race
(C) its best race
(D) their best races
(E) its better race

34. Each of the candidates shakes the hand of whomever he meets.

(A) shakes the hand of whomever he meets
(B) shake the hand of whomever he meets
(C) shakes the hand of whoever he meets
(D) shakes the hand of whomever they meet
(E) shaked the hand of whomever he meets

35. Between you and I, Harvey's an idiot.

(A) Between you and I, Harvey's
(B) Between you and me, Harvey's
(C) Between you and I, Harveys
(D) Among you and I, Harvey's
(E) Among you and me, Harvey's

36. To who is the envelope addressed?

(A) To who is the envelope
(B) To who is the envelop
(C) To whom is the envelope
(D) To whom is the envelop
(E) To whom are the envelope

37. He told me that his father never had liked Martin and she.

(A) his father never had liked Martin and she.
(B) "his father never had liked Martin and she."
(C) his father had never liked Martin and her.
(D) "His father had never liked Martin and she.
(E) "His father had never liked Martin and her."

38. If my brothers friend was smarter, he would be easier to be with.

(A) brothers friend was smarter, he
(B) brother's friend was smarter, he
(C) brothers friend were smarter, he
(D) brother's friend were smarter, he
(E) If my brothers friend were smarter, he

39. Realizing the pressure of the situation, a home run was hit by the batter.

(A) a home run was hit by the batter
(B) a home run had to be hit by the batter
(C) a home run should be hit by the batter
(D) the batter hit a home run
(E) a home run was to be hit by the batter

40. To use this tool properly, this cap must be screwed on correctly.

(A) this cap must be screwed on correctly
(B) this cap must be screwed on correct
(C) this cap is to be screwed on correctly
(D) these cap must be screwed on correctly
(E) you must screw on this cap correct

41. He might have been smarter than any other student in the country.

(A) might have been smarter than any other
(B) might of been smarter than any other
(C) might of been smarter than any
(D) might of been more smarter than any other
(E) might have been smarter then any other

42. He told us that he neither enjoys boating or fishing.

(A) neither enjoys boating or fishing
(B) enjoys neither boating or fishing
(C) neither enjoys boating nor fishing
(D) enjoys neither boating nor fishing
(E) neither enjoys to boat or fishing

43. When I arrived at the meeting, you are already there.

(A) you are already there.
(B) you are all ready there.
(C) you were already there.
(D) you were all ready there.
(E) you was already there.

44. Being as you are my brother, I will help you.

(A) Being as you are my brother, I
(B) Being as you are mine brother, I
(C) Being as you are my brother I
(D) Since you are my brother, I
(E) Since you is my brother, I

45. Will every one please sign his name.

(A) every one please sign his
(B) every one please sign their
(C) everyone please sign his
(D) every one please signs his
(E) everyone please sign their

46. Herman and myself were asked to clean up the room and to remove the garbage.

(A) myself were asked to clean up the room and to remove the garbage
(B) myself was asked to clean up the room and to remove the garbage
(C) I am asked to clean up the room and to remove the garbage
(D) I were asked to clean up the room and to remove the garbage
(E) myself were asked to clean up the room and that we should remove the garbage

47. That was the sort of a look that strikes fear into men's hearts

(A) sort of a look that strikes fear into men's
(B) sort of look that strikes fear into men's

(C) sort of a look that strikes fear into mens
(D) sort of look who strikes fear into men's
(E) sort of a look who strikes fear into men's

48. His teacher told him, "You has come a long way, but you still have a long ways to go."

(A) has come a long way, but you still have a long ways to go."
(B) have come a long way, but you still have a long ways to go."
(C) has come a long way, but you still have a long way to go."
(D) have come a long way, but you still have a long way to go."
(E) have come a long way but you still have a long ways to go."

49. Tom doesn't want to go; therefore, lets stay at home.

(A) go; therefore, lets stay at home
(B) go, therefore, lets stay at home
(C) go: therefore, lets stay at home
(D) go, therefore, let's stay at home
(E) go; therefore, let's stay at home

50. Jogging down the street, he ran right between the two ladies.

(A) right between the two
(B) write between the two
(C) right among the two
(D) write among the two
(E) right between the too

Answer Sheet—Standard Written English Test 1

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
25	☐	☐	☐	☐	☐
26	☐	☐	☐	☐	☐
27	☐	☐	☐	☐	☐
28	☐	☐	☐	☐	☐
29	☐	☐	☐	☐	☐
30	☐	☐	☐	☐	☐
31	☐	☐	☐	☐	☐
32	☐	☐	☐	☐	☐
33	☐	☐	☐	☐	☐
34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐
36	☐	☐	☐	☐	☐
37	☐	☐	☐	☐	☐
38	☐	☐	☐	☐	☐
39	☐	☐	☐	☐	☐
40	☐	☐	☐	☐	☐
41	☐	☐	☐	☐	☐
42	☐	☐	☐	☐	☐
43	☐	☐	☐	☐	☐
44	☐	☐	☐	☐	☐
45	☐	☐	☐	☐	☐
46	☐	☐	☐	☐	☐
47	☐	☐	☐	☐	☐
48	☐	☐	☐	☐	☐
49	☐	☐	☐	☐	☐
50	☐	☐	☐	☐	☐

Standard Written English Test 2

Time—30 Minutes
50 Questions

Directions: The following sentences contain problems in grammar, usage, diction (choice of words), and idiom.

Some sentences are correct.
No sentence contains more than one error.

You will find that the error, if there is one, is underlined and lettered. Assume that all other elements of the sentence are correct and cannot be changed. In choosing answers, follow the requirements of standard written English.
If there is an error, select the one underlined part that must be changed in order to make the sentence correct, and blacken the corresponding space on the answer sheet.
If there is no error, mark answer space E.

1. A bag of potato chips were (A) always present when (B) the family dined (C) on hot dogs or (D) hamburgers. No error (E)

2. The boss would (A) like to see (B) anyone who (C) are (D) here. No error (E)

3. He gave the shovel to one of the men who (A) was (B) struggling to uproot (C) the big oak (D) tree. No error (E)

4. Henry or you has (A) been appointed to serve (B) as the disinterested (C) judge in this (D) case. No error (E)

5. One must tell yourself (A) not to lose (B) sight (C) of the line in (D) the center of the road. No error (E)

6. Twenty-five (A) feet are (B) the entire (C) length of the Smiths' (D) lawn. No error (E)

7. Alex and me (A) want (B) to borrow (C) my mother's (D) car. No error (E)

8. Carry (A) this package to whoever (B) you happen to see (C) there. (D) No error (E)

9. Marty always (A) wanted to buy (B) a house of his own. (C) (D) No error (E)

10. Because (A) of the heat, Mrs. Micawber (B) decided against him (C) doing all (D) of the work. No error (E)

11. Between (A) you and I, (B) Hector should never have (C) been allowed (D) to go alone. No error (E)

12. Looking (A) at each of us, (B) Sam asked for who (C) this joke was intended. (D) No error (E)

13. If he was (A) my (B) friend, I would (C) be very happy (D). No error (E)

14. Realizing (A) that it was too (B) late, the car was (C) turned around (D). No error (E)

15. To make sure (A) that no robbers could enter (B) the house, the doors were (C) locked and bolted (D). No error (E)

16. This (A) painting is, in my (B) opinion, more lovelier (C) than (D) the other one. No error (E)

17. The drummer played (A) more than (B) any (C) other (D) person in the band. No error (E)

18. This man was (A) the most unique (B) person that (C) I have (D) ever met. No error (E)

19. Franklin decided (A) that he either wants (B) to go now or to wait (C) until (D) next week. No error (E)

20. The leader of (A) the vast army performed (B) this (C) task eagerly and with speed (D). No error (E)

21. As soon as (A) Jerry drove (B) to the hospital, his (C) wife has (D) a baby. No error (E)

22. Chris' (A) family stayed (B) in their (C) present house in the country every summer for six (D) years. No error (E)

23. Even if (A) we had (B) asked, Sally would not (C) have come here anyways (D). No error (E)

24. Miles hadn't (A) hardly enough (B) time to finish (C) his (D) work. No error (E)

25. She could (A) best (B) describe Rick's (C) dwelling as a sort of a (D) shack. No error (E)

Directions: In each of the following sentences, some part of the sentence or the entire sentence is underlined. Beneath each sentence you will find five ways of phrasing the underlined part. The first of these repeats the original; the other four are different. If you think the original is better than any of the alternatives, choose answer A; otherwise choose one of the others. Select the best version and blacken the corresponding space on your answer sheet.

This is a test of correctness and effectiveness of expression. In choosing answers follow the requirements of standard written English; that is, pay attention to grammar, choice of words, sentence construction, and punctuation. Choose the answer that produces the most effective sentence—clear and exact, without awkwardness or ambiguity. Do not make a choice that changes the meaning of the original sentence.

26. I like him more than any other friend.
    (A) him more than any other
    (B) him more than any
    (C) he more than any
    (D) him more then any other
    (E) him more then any

27. The robber, as well as his three cohorts, are going to be put on trial.

(A) his three cohorts, are
(B) his 3 cohorts, are
(C) his 3 cohorts, is
(D) its three cohorts, is
(E) his three cohorts, is

28. There is several deserts on the menu.

(A) There is several deserts
(B) There is several desserts
(C) There are several deserts
(D) There are several desserts
(E) There was several deserts

29. Us doctors must be honest; furthermore, we must set an example.

(A) Us doctors must be honest; furthermore, we
(B) We doctors must be honest; furthermore, we
(C) Us doctors must be honest, furthermore, we
(D) Us doctors must be honest: furthermore, we
(E) We doctors must be honest, furthermore, we

30. If he were here, you're mother would feel much better.

(A) were here, you're
(B) was here, you're
(C) would be here, you're
(D) were here, your
(E) was here, you're

31. I do not ride horses as much as him.

(A) horses as much as him
(B) horses as much as he
(C) hoarses as much as he
(D) hoarses as much as him
(E) horses as much as his

32. If I would have been at home, the robbers could not have entered.

(A) would have been at home, the robbers
(B) would be at home, the robbers
(C) have been at home, the robbers
(D) had been at home, the robbers'
(E) had been at home, the robbers

33. To be able to graduate, you must first complete your senior year.

(A) you must first complete your senior year
(B) You must first complete you're senior year
(C) your senior year must first be completed
(D) you're senior year must first be completed
(E) you must first complete you're senior year

34. Our British guest thought that our house was much more lovelier than his.

(A) British guest thought that our house was much more lovelier than
(B) British guest thought that our house was much lovelier than
(C) British guest thought that our house was much more lovely then
(D) british guest thought that our house was much more lovelier than
(E) british guest thought that our house was much lovelier than

35. Horace Greeley uttered the words with eloquence and clearly.

(A) uttered the words with eloquence and clearly
(B) utter the words with eloquence and clearly
(C) uttered the words with eloquence and with clarity
(D) uttered the words eloquently and with clarity
(E) uttered the words with eloquence and clearness

Directions: In each of the following examples, there are five variations of the same sentence. Select the best version and blacken the corresponding space on your answer sheet.

36. (A) My mother pointed to the gift on the table which she had bought.
(B) My mother is pointing to the gift on the table which she had bought.
(C) My mother pointed to the gift she had bought, which was on the table.
(D) My mother pointed to the gift in the table which she had bought.
(E) My mother had pointed to the gift on the table which she had bought.

37. (A) "I know," she cried, "that Abe is braver than any man"!
(B) "I know," she cried, "that Abe is more braver than any man!"

(C) "I know," she cried," that Abe is more braver than any other man!"
(D) "I know," she cried, "that Abe is braver than any man!"
(E) "I know," she cried, "that Abe is braver than any other man!"

38. (A) Alton B. Parker, who was a great man, addressed the crowd with candor and wittily.
(B) Alton B. Parker, whom was a great man, addressed the crowd with candor and wittily.
(C) Alton B. Parker, who were a great man, addressed the crowd with candor and with wit.
(D) Alton B. Parker, who was a great man, addressed the crowd with candor and with wit.
(E) Alton B. Parker, whom was a great man, addressed the crowd with candor and with wit.

39. (A) The man said that he has went to his present doctor for two years.
(B) The man said that he has went to his present doctor for too years.
(C) The man said that he went to his present doctor for two years.
(D) The man said that he has gone to his present doctor for two years.
(E) The man said that he has gone to his present doctor for too years.

40. (A) They decided to buy a car, and one was soon chosen.
(B) They decided to buy a car and one was soon chosen.
(C) They decided to buy a car; and one was soon chosen.
(D) They decided to buy a car and soon chose one.
(E) They decided and buy a car, and one was soon chosen.

41. (A) Walking to work, uncle Dick found a box of money.
(B) Walking to work, uncle Dick founded a box of money.
(C) Walking to work, a box of money was found by uncle Dick.
(D) Walking to work, a box of money was found by uncle Dick
(E) Walking to work, Uncle Dick found a box of money.

42. (A) If he was here, he would settle this dispute among the two of us.
(B) If he were here, he could settle this dispute between the two of us.
(C) If he were here, he could settle this dispute among the two of us.
(D) If he was here, he could settle this dispute between the two of us.
(E) If he was here, he could settle this dispute between the too of us.

43. (A) Elaine said that she had told everyone except you and I.
(B) Elaine said that she has told everyone except you and I.
(C) Elaine said that she had told every one except you and I.
(D) Elaine said that she had told everyone except you and me.
(E) Elaine said that she had told everyone except you and myself.

44. (A) Bill admitted that he has less money than I.
(B) Bill admitted that he has fewer money than I.
(C) Bill admitted that he has less money than me.
(D) Bill admitted that he has less money then I.
(E) Bill admitted that he has fewer money than me.

45. (A) How come you used to could do your own cooking?
(B) How come you used to be able to do your own cooking?
(C) How come you used to be able to do you're own cooking?
(D) How come you used to could do you're own cooking?
(E) How come you used to be able to do you're own cooking?

46. (A) They cut down the tree because of it's swaying.
(B) They cut down the tree because of it swaying.
(C) They cut down the tree because of its swaying.
(D) They cut down the tree due to it's swaying.
(E) They cut down the tree due to it swaying.

47. (A) Each of the players are going to accept their awards.
(B) Each of the players is going to accept their awards.

(C) Each of the players are going to accept his awards.
(D) Each of the players are going to accept they're awards.
(E) Each of the players is going to accept his award.

48. (A) Neither of the boys was going to find their money.
(B) Neither of the boys was going to find his money.
(C) Neither of the boys were going to find their money.
(D) Neither of the boys was going to found their money.
(E) Neither of the boys were going to find his money.

49. (A) The Russians, they are coming.
(B) The Russians they are coming.
(C) The russians, they are coming.
(D) The Russians are coming.
(E) The Russians is coming.

50. (A) Clarence or you has to find the man who escaped.
(B) Clarence or you have to find the man who escaped.
(C) Clarence or you has to find the man whom escaped.
(D) Clarence or you has to find the man that escaped.
(E) Clarence or you have to find the man whom escaped.

Answer Sheet—Standard Written English Test 2

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
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23	☐	☐	☐	☐	☐
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41	☐	☐	☐	☐	☐
42	☐	☐	☐	☐	☐
43	☐	☐	☐	☐	☐
44	☐	☐	☐	☐	☐
45	☐	☐	☐	☐	☐
46	☐	☐	☐	☐	☐
47	☐	☐	☐	☐	☐
48	☐	☐	☐	☐	☐
49	☐	☐	☐	☐	☐
50	☐	☐	☐	☐	☐

Standard Written English Test 3

Time—30 Minutes
50 Questions

Directions: The following sentences contain problems in grammar, usage, diction (choice of words), and idiom.

Some sentences are correct.
No sentence contains more than one error.

You will find that the error, if there is one, is underlined and lettered. Assume that all other elements of the sentence are correct and cannot be changed. In choosing answers, follow the requirements of standard written English.
If there is an error, select the one underlined part that must be changed in order to make the sentence correct, and blacken the corresponding space on the answer sheet.
If there is no error, mark answer space E.

1. A group of prisoners have (A) just been driven (B) to the jail, which (C) is (D) in the center of town. No error (E)

2. Either Felix or I (A) are (B) going to drive (C) to the airport to meet my mother's (D) friend. No error (E)

3. The clothes (A) that (B) he wore was (C) always (D) too small. No error (E)

4. Will (A) you make (B) this effort for (C) Mary and she (D)? No error (E)

5. They thought (A) that the man who (B) stole the car was (C) him (D). No error (E)

6. The host gave (A) the glass of (B) wine to Mr. Jones, whom (C) quickly (D) drank it. No error (E)

7. The witness claimed (A) that some of the bullets had pierced (B) the front (C) door of his (D) sporting goods store. No error (E)

8. We could see (A) that the group of (B) soldiers was (C) upset when the strangers took they're (D) seats. No error (E)

9. No one but (A) David or we (B) could (C) drive the company's (D) truck. No error (E)

10. Seeing (A) the enemy of our (B) family, my feelings turned (C) from happy to sad (D). No error (E)

11. The young campers are (A) more (B) calmer now that (C) the storm has passed (D). No error (E)

12. Alice is as abrasive (A) or more (B) abrasive than (C) her (D) friend. No error (E)

13. Dick told his (A) mother that he neither enjoys (B) running nor playing (C) tennis. (D) No error (E)

14. My (A) uncle's (B) friend likes (C) to play golf and sailing. (D) No error (E)

15. When Mr. Smith finally approached (A) the house, the young woman (B) walks (C) right by him. (D) No error (E)

16. His (A) son, who (B) was here (C) last Christmas, attended (D) his present college for two years. No error (E)

17. The leader assured (A) his (B) troops that (C) everything would be alright. (D) No error (E)

18. Because (A) his employees quit (B), Mr. Jones will have to do (C) all of the work hisself. (D) No error (E)

19. Be (A) sure and (B) turn off (C) the lights before (D) you leave the house. No error (E)

20. The foreman of (A) the construction workers lives (B) in an (C) suburb of New York. (D) No error (E)

21. Were (A) very happy to see (B) that your (C) family could (D) come. No error (E)

22. Barry said that (A) he would like (B) to come, (C) but he must spend the day in New Jersey. (D) No error (E)

23. My teacher asked, (A) "What (B) is the answer to (C) this problem"? (D) No error (E)

24. The teacher told John's (A) mother that he (B) had always (C) been good in english. (D) No error (E)

25. Hank told me (A) that he hadn't (B) heard from cousin (C) Moe for three (D) years. No error (E)

Directions: In each of the following sentences, some part of the sentence or the entire sentence is underlined. Beneath each sentence you will find five ways of phrasing the underlined part. The first of these repeats the original; the other four are different. If you think the original is better than any of the alternatives, choose answer (A); otherwise choose one of the others. Select the best version and blacken the corresponding space on your answer sheet.

This is a test of correctness and effectiveness of expression. In choosing answers, follow the requirements of standard written English; that is, pay attention to grammar, choice of words, sentence construction, and punctuation. Choose the answer that produces the most effective sentence—clear and exact, without awkwardness or ambiguity. Do not make a choice that changes the meaning of the original sentence.

26. The man selling fruit told us that a bag of peaches were still a good buy.

(A) told us that a bag of peaches were
(B) told us that a bag of peaches was
(C) told we that a bag of peaches were
(D) told we that a bag of peaches was
(E) told us concerning that a bag of peaches were

27. The group of men, they is waiting in the rain.

(A) The group of men, they is waiting
(B) The group of men, they are waiting
(C) The group of men is waiting
(D) The group of men, it is waiting
(E) The group of man is waiting

28. You must always think that you are right.

(A) always think that you are right.
(B) all ways think that you are right.
(C) always think that one is right
(D) always think that you are write.
(E) all ways think that one is right.

29. They approved of the company's giving all it's employees a long vacation.

(A) company's giving all it's
(B) company's giving all its
(C) company giving all it's
(D) company giving all its
(E) company's giving all its'

30. The terrible accident did not injure me as much as he.

(A) injure me as much as he.
(B) injured me as much as he.
(C) injure I as much as he.
(D) injure I as much as him.
(E) injure me as much as him.

31. If Pete was an athlete, he would have made more money.

(A) If Pete was an athlete, he
(B) If Pete had been an athlete, he
(C) If Pete were a athlete, he
(D) If Pete were an athlete, he
(E) If Pete would be an athlete, he

32. Walking swiftly through the town, a straight coarse was kept by the man.

(A) the straight coarse was kept by the man.
(B) the straight course was kept by the man.
(C) the man kept a straight coarse.
(D) the man kept a straight course.
(E) a straight course must be kept.

33. He tried and be better than any player.

(A) tried and be better than any
(B) tried to be better than any
(C) tried to be better then any
(D) tried to be better than any other
(E) tried and be better than any other

34. His mother says that he must shower, dress, and to leave.

(A) must shower, dress, and to leave.
(B) has got to shower, dress, and to leave.
(C) must shower, to dress, and to leave.
(D) has got to shower, to dress, and leave.
(E) must shower, dress, and leave.

35. They lived in their present house for the past ten years.

(A) lived in their present house for the past
(B) lived in their present house for the passed
(C) did live in their present house for the past
(D) have lived in their present house for the passed
(E) have lived in their present house for the past

Directions: In each of the following examples, there are five variations of the same sentence. Select the best version and blacken the corresponding space on your answer sheet.

36. (A) He saw the bird in the sky that was almost extinct.
(B) He saw the bird in the sky that were almost extinct.
(C) He saw the bird in the sky that had almost been extinct.
(D) In the sky, he saw the bird that was almost extinct.
(E) In the sky, he saw the bird that was all most extinct.

37. (A) He neither enjoys speaking in public nor to teach.
(B) He neither enjoys speaking in public nor teaching.
(C) He enjoys neither speaking in public or teaching.
(D) He enjoys neither speaking in public nor to teach.
(E) He enjoys neither speaking in public nor teaching.

38. (A) The man whom you just met is rowdy, boisterous, and sometimes lets his temper go.
(B) The man who you just met is rowdy, boisterous, and sometimes lets his temper go.
(C) The man that you just met is rowdy, boisterous, and sometimes lets his temper go.
(D) The man who you just met is rowdy, boisterous, and temperamental.
(E) The man whom you just met is rowdy, boisterous, and bad-tempered.

39. (A) A team can't trade a player who played for his present team for five years.
(B) A team can't trade a player whom played for his present team for five years.
(C) A team can't trade a player who has played for his present team for five years.
(D) A team cant trade a player who has played for his present team for five years.
(E) A team can't trade a player who played for his present team for 5 years.

40. (A) When you are in Hong Kong, one must eat at the Peninsula Hotel.
(B) When you are in Hong Kong, one should eat at the Peninsula Hotel.
(C) When you are in Hong Kong, you must eat at the Peninsula Hotel.
(D) When you are in Hong Kong, one could eat at the Peninsula Hotel.
(E) When you are in Hong Kong, one ought to eat at the Peninsula Hotel.

41. (A) Observing the dog in the road, the bus stopped immediately.
(B) While observing the dog in the road, the bus stopped immediately.
(C) Observing the dog in the road, the driver immediately stopped the bus.
(D) Observing the dog in the road, the bus was stopped immediately.
(E) After observing the dog in the road, the bus has stopped immediately.

42. (A) If I had the money, I would have bought the 24-carat gold ring.
(B) If I have the money, I would have bought the 24-carat gold ring.
(C) If I had the money, I would have bought the 24-carrot gold ring.
(D) If I had had the money, I would have bought the 24-carrot gold ring.
(E) If I had had the money, I would have bought the 24-carat gold ring.

43. (A) Deliver this check to whomever is behind the clerks desk.
(B) Deliver this check to whomever is behind the clerk's desk.
(C) You deliver this check to whomever is behind the clerks desk.
(D) Deliver this check to whoever is behind the clerk's desk.
(E) Deliver this check to who ever is behind the clerks desk.

44. (A) The horse always kicked over its bucket of water.
(B) The hoarse always kicked over its bucket of water.
(C) The horse always kicked over it's bucket of water.
(D) The horse all ways kicked over it's bucket of water.
(E) The hoarse always kicked over it's bucket of water.

45. (A) He asked, "How do you know that the culprit is him?"
(B) He asked, "How do you know that the culprit is he?"
(C) He asked, "How do you know that the culprit is him"?
(D) He asked, "How do you know that the culprit was he?"
(E) He asked, "How do you know that the culprit was him?"

46. (A) "Are any of the workers still in the room?" the boss asked.
(B) "Is any of the workers still in the room?" the boss asked.
(C) "Was any of the workers still in the room?" the boss asked.
(D) "Are any of the workers still in the room"? the boss asked.
(E) "Are any of the workers' still in the room?" the boss asked.

47. (A) Harvey or I are coming to your house.
    (B) Harvey or I is coming to your house.
    (C) Harvey or I am coming to your house.
    (D) Harvey or I are coming to you're house.
    (E) Harvey or I am coming to you're house.

48. (A) There is many men who would like to be in your shoes.
    (B) There is many men whom would like to be in your shoes.
    (C) There are many men whom would like to be in your shoes.
    (D) There are many men who would like to be in your shoes.
    (E) There was many men who would like to be in your shoes.

49. (A) He told us that not only the farmer but also me were going to be put in jail.
    (B) He told us that not only the farmer but also I were going to be put in jail.
    (C) He told us that not only the farmer but also me am going to be put in jail.
    (D) He told us that not only the farmer but also I was going to be put in jail.
    (E) He told us not only that the farmer but also me were going to be put in jail.

50. (A) Carl, as well as his two brothers, was invited to the picnic.
    (B) Carl, as well as his two brothers, were invited to the picnic.
    (C) Carl, as well as his too brothers, was invited to the picnic.
    (D) Carl, as well as his two brothers, are invited to the picnic.
    (E) Carl, as well as his too brothers, were invited to the picnic.

Answer Sheet—Standard Written English Test 3

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐	31	☐	☐	☐	☐	☐	41	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐	32	☐	☐	☐	☐	☐	42	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐	33	☐	☐	☐	☐	☐	43	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐	34	☐	☐	☐	☐	☐	44	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐	35	☐	☐	☐	☐	☐	45	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	26	☐	☐	☐	☐	☐	36	☐	☐	☐	☐	☐	46	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	27	☐	☐	☐	☐	☐	37	☐	☐	☐	☐	☐	47	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	28	☐	☐	☐	☐	☐	38	☐	☐	☐	☐	☐	48	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	29	☐	☐	☐	☐	☐	39	☐	☐	☐	☐	☐	49	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	30	☐	☐	☐	☐	☐	40	☐	☐	☐	☐	☐	50	☐	☐	☐	☐	☐

Standard Written English Test 4

Time—30 Minutes
50 Questions

Directions: The following sentences contain problems in grammar, usage, diction (choice of words), and idiom.

Some sentences are correct.
No sentence contains more than one error.

You will find that the error, if there is one, is underlined and lettered. Assume that all other elements of the sentence are correct and cannot be changed. In choosing answers, follow the requirements of standard written English.
If there is an error, select the one underlined part that must be changed in order to make the sentence correct, and blacken the corresponding space on the answer sheet.
If there is no error, mark answer space E.

1. Some of the men who (A) was (B) present grumbled (C) when the game was postponed. (D) No error (E)

2. The major drawback are (A) the people who (B) wish to block (C) the plan before it (D) is put into practice. No error (E)

3. The boys knew (A) that the worker who (B) was (C) there was him. (D) No error (E)

4. Give (A) this parcel to (B) whomever (C) is (D) at the appointed place. No error (E)

5. The accident did (A) not injure the driver as (B) much as (C) they. (D) No error (E)

6. If Sally would have been (A) there, (B) the youngsters would not have been (C) so sad. (D) No error (E)

7. Upon arriving (A) at the stadium, (B) the ticket windows were (C) mobbed with (D) people. No error (E)

8. Mr. Jackson is (A) the best (B) salesman of the two (C) whom (D) we have seen. No error (E)

9. This movie is as outrageous as (A) or more (B) outrageous than (C) the one we saw (D) last night. No error (E)

10. The frantic wife cried, (A) "Do you only like (B) to sleep and reading (C) the newspaper? (D)" No error (E)

11. Robert was (A) always (B) kind, considerate, and liked to think (C) of others. (D) No error (E)

12. When (A) the runner reached (B) second base, the batter already (C) struck out. (D) No error (E)

13. Mrs. Magwitch will (A) not be able to buy (B) another ring like that (C) one any where (D). No error (E)

14. The consensus of opinion (A) among (B) the board of (C) directors was (D) to sell the company. No error (E)

15. Because (A) of your (B) incorrect directions, the driver may of (C) made the wrong (D) turn. No error (E)

16. I know that I (A) last saw his car (B) somewheres (C) in the vicinity of that (D) clothing store. No error (E)

17. We (A) would not make (B) the decision between (C) the five of us (D). No error (E)

18. Norman is (A) taller (B) then (C) any other (D) boy in the class. No error (E)

19. The lawyer sighted (A) several examples of this type of (B) case to prove (C) his (D) point. No error (E)

20. Do (A) you mean (B) to imply (C) that he was present at the scene (D) of the crime? No error (E)

21. The young (A) salesman showed (B) great talent, (C) consequently, (D) he was given a raise in salary. No error (E)

22. "That (A) boy", (B) shouted my (C) father, "will (D) never grow up!" No error (E)

23. While he (A) was in Hong Kong (B), Mr. Smith found that many chinese (C) live (D) there. No error (E)

24. The Brazilian (A) immigrant (B) was startled (C) by the size of New York City (D). No error (E)

25. They always (A) enjoyed (B) Uncle (C) Marvin's (D) company. No error (E)

26. I disremember (A) whether or (B) not I was (C) at the game (D). No error (E)

27. Bob and myself (A) went (B) to the most (C) exciting play on Broadway (D). No error (E)

28. Daniel Dravot should of (A) known not (B) to remain (C) in Kafiristan for too (D) long. No error (E)

29. My (A) father always (B) let's (C) Arty repair (D) the television set. No error (E)

30. Walking through (A) the destroyed (B) town, the civil engineer surveyed (C) the affect (D) of the storm. No error (E)

Directions: In each of the following sentences, some part of the sentence or the entire sentence is underlined. Beneath each sentence you will find five ways of phrasing the underlined part. The first of these repeats the original; the other four

are different. If you think the original is better than any of the alternatives, choose answer (A); otherwise choose one of the others. Select the best version and blacken the corresponding space on your answer sheet.

This is a test of correctness and effectiveness of expression. In choosing answers, follow the requirements of standard written English; that is, pay attention to grammar, choice of words, sentence construction, and punctuation. Choose the answer that produces the most effective sentence—clear and exact, without awkwardness or ambiguity. Do not make a choice that changes the meaning of the original sentence.

31. Both the city council and the mayor is unsure of the solution.

(A) council and the mayor is
(B) counsel and the mayor is
(C) counsel and the mayor are
(D) council and the mayor are
(E) counsul and the mayor is

32. My father says that everyone in the army must get their hair cut short.

(A) everyone in the army must get their
(B) every one in the army must get their
(C) every-one in the army must get his
(D) everyone in the army must get his
(E) everyone in the army might get their

33. Dr. Bingley and his family takes fewer vacations than us.

(A) fewer vacations than us.
(B) less vacations than us.
(C) fewer vacations than we.
(D) less vacations than we.
(E) fewer vacations than we.

34. Although that opera star had many fans, I never liked him singing.

(A) fans, I never liked him singing.
(B) fans I never liked him singing.
(C) fans; I never liked his singing.
(D) fans I never liked his singing.
(E) fans, I never liked his singing.

35. Walking away, the guests left the host with he.

(A) the guests left the host with he.
(B) the host was left by the guests with him.
(C) the guests left the host with him.
(D) the guests leaved the host with him.
(E) the guests they left the host with him.

36. If he had been taller, he would have been a great basketball player.

(A) If he had been taller, he
(B) If he would have been taller, he
(C) If he had been tallest, he
(D) If he could have been taller, he
(E) If he was just taller he

37. To make sure that they do not break, the dishes must be handled carefully.

(A) the dishes must be handled carefully.
(B) the dishes must be handled with care.
(C) the dishes should be handled carefully.
(D) you must handle the dishes carefully.
(E) the dishes must handle carefully.

38. When I approached the old man, he greets me respectfully.

(A) he greets me respectfully.
(B) he greeted me respectfully.
(C) he greets me respectively.
(D) he is greeting me respectively.
(E) he greeted me respectively.

39. Jumping over the hurdles, the runner may of twisted his ankle.

(A) the runner may of twisted his ankle.
(B) the runner may have twisted his ankle.
(C) his ankle may of been twisted.
(D) his ankle may have been twisted.
(E) the runner may of twisted its ankle.

40. The adventurer told us that he used to be able to go anywhere.

(A) us that he used to be able to go anywhere.
(B) we that he used to be able to go anywhere.
(C) us that he used to could go anywhere.
(D) ourselves that he used to be able to go anywhere.
(E) us that he used to be able to go any where.

Directions: In each of the following examples, there are five variations of the same sentence. Select the best version and blacken the corresponding space on your answer sheet.

41. (A) They found out that the mission had been a success by the next day.
(B) They found out that the mission was a success by the next day.
(C) They are finding out that the mission had been a success by the next day.
(D) They found out that the mission by the next day was a success.
(E) By the next day, they found out that the mission was a success.

42. (A) The commander of the men says that they must march or that they will die.
(B) The commander of the men says that they must march or die.
(C) The commander of the men say that they must march or that they will die.
(D) The commander of the men say that they must march or die.
(E) The commander of the men says that they must march or that they would die.

43. (A) My former instructer always said that the work was hard.
(B) My formal instructor always said that the work was hard.
(C) My former instructor all ways said that the work was hard.
(D) My former instructor allways said that the work is hard.
(E) My former instructor always said that the work is hard.

44. (A) When the police arrived at the scene of the crime, the robbers had already escaped.
(B) When the police arrived at the scene of the crime, the robbers already escaped.
(C) When the police arrived at the scene of the crime, the robbers all ready escaped.
(D) When the police arrived at the scene of the crime, the robbers have already escaped.
(E) When the police arrived at the scene of the crime the robbers had already escaped.

45. (A) Reaching into the hat, a group of rabbits was pulled out by the magician.
(B) Reaching into the hat, a group of rabbits were pulled out by the magician.
(C) Reaching into the hat, a group of rabbits are pulled out by the magician.
(D) Reaching into the hat, a group of rabbits was pulled out buy the magician.
(E) Reaching into the hat, the magician pulled out a group of rabbits.

46. (A) If I were your father, I would make you behave.
(B) If I was your father, I would make you behave.
(C) If I were you're father, I would make you behave.
(D) If I was you're father, I would make you behave.
(E) If I am your father, I would make you behave.

47. (A) This basket of peaches is to be given to the man whom is across the hall.
(B) This basket of peaches are to be given to the man whom is across the hall.
(C) This basket of peaches are to be given to the man who is across the hall.
(D) This basket of peaches is to be given to the man who are across the hall.
(E) This basket of peaches is to be given to the man who is across the hall.

48. (A) William dislikes they're continuous smoking.
(B) William dislikes their continuous smoking.
(C) William dislikes they're continual smoking
(D) William dislikes their continual smoking
(E) William dislikes them continual smoking

49. (A) Some of the girls are two concerned with her clothes.
(B) Some of the girls is two' concerned with her clothes.
(C) Some of the girls are too concerned with her clothes.
(D) Some of the girls is too concerned with her clothes.
(E) Some of the girls are too concerned with their clothes.

50. (A) Half of their house were destroyed.
(B) Half of they're house were destroyed.
(C) Half of their house are destroyed
(D) Half out of their house were destroyed.
(E) Half of their house was destroyed.

Answer Sheet—Standard Written English Test 4

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐	31	☐	☐	☐	☐	☐	41	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐	32	☐	☐	☐	☐	☐	42	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐	33	☐	☐	☐	☐	☐	43	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐	34	☐	☐	☐	☐	☐	44	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐	35	☐	☐	☐	☐	☐	45	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	26	☐	☐	☐	☐	☐	36	☐	☐	☐	☐	☐	46	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	27	☐	☐	☐	☐	☐	37	☐	☐	☐	☐	☐	47	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	28	☐	☐	☐	☐	☐	38	☐	☐	☐	☐	☐	48	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	29	☐	☐	☐	☐	☐	39	☐	☐	☐	☐	☐	49	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	30	☐	☐	☐	☐	☐	40	☐	☐	☐	☐	☐	50	☐	☐	☐	☐	☐

Standard Written English Test 5

Time—30 Minutes
50 Questions

Directions: The following sentences contain problems in grammar, usage, diction (choice of words), and idiom.

Some sentences are correct.
No sentence contains more than one error.

You will find that the error, if there is one, is underlined and lettered. Assume that all other elements of the sentence are correct and cannot be changed. In choosing answers, follow the requirements of standard written English.
If there is an error, select the one underlined part that must be changed in order to make the sentence correct, and blacken the corresponding space on the answer sheet.
If there is no error, mark answer space E.

1. When (A) the music stopped (B), every one of the guests were (C) caught in an (D) awkward position. No error (E)

2. Harvey told me (A) that ham and eggs are (B) his (C) favorite breakfast (D). No error (E)

3. I think everyone (A) should do (B) this very easy (C) task by themselves (D). No error (E)

4. Larry's (A) friend, whom (B) is (C) on vacation, often plays (D) tennis at the school. No error (E)

5. Did (A) Douglas say that he (B) knew whom (C) that man was (D)? No error (E)

6. The interviewer at the university (A) rejected both my (B) brother and (C) he (D). No error (E)

7. Sue said to me (A) that she (B) wished (C) that Greg was (D) here. No error (E)

8. Upon entering (A) my (B) mother's (C) room, the lost glove was (D) soon found on the floor. No error (E)

9. To safeguard (A) against natural disasters, the building that (B) was (C) just completed was built (D) with steel and concrete. No error (E)

10. Their (A) mother told me (B) that John is the smarter (C) of the three (D) brothers. No error (E)

11. This fish is (A) more big (B) than (C) any other fish that (D) I have seen. No error (E)

12. The soldiers who (A) just arrived were (B) told to shower (C), to put on new clothes, and

that they would report in the morning. (D)
No error (E)

13. Living (A) life with gusto, the old man told us (B) that he (C) still liked to walk, to read, and to play golf. (D) No error (E)

14. My father always (A) believed (B) that (C) man is (D) a greedy animal. No error (E)

15. When (A) the guests left (B), the host already (C) fell (D) asleep. No error (E)

16. After tasting (A) the flavors of ice cream, Mabel said (B) that she (C) thought that they were both alike. (D) No error (E)

17. If (A) you had (B) been here (C), we could of (D) left long ago. No error (E)

18. I know (A) that Uncle Ralph (B) would not have liked (C) that kind of a (D) house. No error (E)

19. My (A) teacher says (B) that we may refer back (C) to our (D) notes. No error (E)

20. My sister told me (A) that he (B) is a (C) extremely ugly (D) man. No error (E)

21. Who's (A) book is (B) that on my (C) brother's (D) desk? No error (E)

22. The coach was (A) happy because (B) the infielder made (C) less (D) errors today. No error (E)

23. After (A) I had been given the two (B) choices for the evening meal, beans or steak, I opted (C) for the later. (D) No error (E)

24. In the passed (A), the man who (B) lives in that (C) house has gone (D) on some interesting trips. No error (E)

25. The children watched (A) respectfully (B) while their (C) father poured (D) the wine. No error (E)

26. I would love (A) to attend your (B) party, (C) however, (D) I have a prior appointment. No error (E)

27. Howard asked (A), "When (B) did that (C) man come to town"? (D) No error (E)

28. The french (A) ambassador was unable (B) to attend (C) the special banquet (D). No error (E)

29. Bob said that his (A) Brother (B) is attending (C) college in Boston (D). No error (E)

30. Speaking (A) with an impeccable accent, the English (B) visitor told us (C) of the days when Britain ruled (D) the seas. No error (E)

Directions: In each of the following sentences, some part of the sentence or the entire sentence is underlined. Beneath each sentence you will find five ways of phrasing the underlined part. The first of these repeats the original; the other four are different. If you think the original is better than any of the alternatives, choose answer (A); otherwise choose one of the others. Select the best version and blacken the corresponding space on your answer sheet.

This is a test of correctness and effectiveness of expression. In choosing answers, follow the requirements of standard written English; that is, pay attention to grammar, choice of words, sentence construction, and punctuation. Choose the answer that produces the most effective sentence—clear and exact, without awkwardness or ambiguity. Do not make a choice that changes the meaning of the original sentence.

31. If he would have known that either Edward or I was going to come, things would have turned out differently.
    (A) If he would have known that either Edward or I was
    (B) If he would know that either Edward or I was
    (C) If he had known that either Edward or I were
    (D) If he had known that either Edward or I was
    (E) If he had known either that Edward or I was

32. The policeman says that you must make sure that one knows how to both swim and run.
    (A) you must make sure that one knows
    (B) you must make sure that you know
    (C) you must make sure that yous know
    (D) you must make sure that one know
    (E) you must make sure that one no

33. All of them appreciated the compliment except he.
    (A) them appreciated the compliment except he.
    (B) they appreciated the compliment except he.
    (C) them appreciated the complement except he.
    (D) them appreciated the compliment except him.
    (E) them appreciated the complement except him.

34. Mrs. Ross' daughter is the most prettiest girl in the class.
    (A) Mrs. Ross' daughter is the most prettiest
    (B) Mrs. Rosses daughter is the most prettiest
    (C) Mrs. Ross' daughter is the prettiest
    (D) Mrs. Ross' daughter is the most prettier
    (E) Mrs. Rosses' daughter is the most prettiest

35. Jim exclaimed, "I would not have helped you anyways!"
    (A) exclaimed, "I would not have helped you anyways!"
    (B) exclaimed, "I would not have helped you anyway!"
    (C) exclaimed, "I would not have helped you anyway"!
    (D) exclaimed "I wouldnot have helped you anyways!"
    (E) exclaimed "I would not have helped you anyways!"

Directions: In each of the following examples, there are five variations of the same sentence. Select the best version and blacken the corresponding space on your answer sheet.

36. (A) The mechanic promised my mother and she that he would fix our new car two months ago.
    (B) Two months ago the mechanic promised my mother and her that he would fix our new car.

(C) Two months ago the mechanic promised my mother and she that he would fix our new car.
(D) The mechanic promised my mother and she two months ago that he would fix our new car.
(E) The mechanic promised my mother and she that he would fix two months ago our new car.

37. (A) My father always liked to say, "You're a better man than I am, Gunga Din!"
(B) My father always liked to say, "Your a better man than I am, Gunga Din!"
(C) My father all ways liked to say, "You're a better man than I am, Gunga Din!"
(D) My father always liked to say, "Your're a better man then I am, Gunga Din!"
(E) My father always liked to say, "You're a better man than I am Gunga Din"!

38. (A) Driving very slowly, the storm almost blew our car off the road.
(B) Driving very slowly, my father struggled to keep the car on the road.
(C) Driving very slowly, the storm almost blew our car off the road.
(D) Driving very slowly, our car was almost blown off the road.
(E) Driving very slowly, the storm all most blew our car off of the road.

39. (A) He wanted to go with Lisa and she, but he was unable to.
(B) He wanted to go with Lisa and she but he was unable to.
(C) He wanted to go with Lisa and her, but he was unable to.
(D) He wanted to go with Lisa and herself, but he was unable to.
(E) He wanted to go with Lisa and her; but he was unable to.

40. (A) If I was you, I'd be opposed to his going alone.
(B) If I am you, I'd be opposed to his going alone
(C) If I were you, I'd be opposed to him going alone.
(D) If I was you, I'd be opposed to him going a lone.
(E) If I were you, I'd be opposed to his going alone.

41. (A) Mr. Braner knew that everybody could do it for theirselves.
(B) Mr. Braner knew that every body could do it for theirselves.
(C) Mr. Braner knew that everybody could do it for himself.
(D) Mr. Braner knew that everybody could do it for themselves.
(E) Mr. Braner knew that everybody could do it for itself.

43. (A) All that were on the floor was one piece of the machine.
(B) All that was on the floor were one piece of the machine.
(C) All that are on the floor is one piece of the machine.
(D) All that were on the floor were one piece of the machine.
(E) All that was on the floor was one piece of the machine.

44. (A) Here rest the bodies of men who gave his life for his country.
(B) Here rests the bodies of men who gave his life for his country.
(C) Here rest the bodies of men who gave their lives for their country.
(D) Here rest the bodies of men who gave their lives for his country.
(E) Here rest the bodies of men whom gave his life for his country.

45. (A) Neither Adam or I would want to go to your party.
(B) Neither Adam or me would want to go to your party.
(C) Neither Adam nor me would want to go to your party.
(D) Neither Adam or I would want to go to you're party.
(E) Neither Adam nor I would want to go to your party.

46. (A) Talking to whomever was present, the old man told all his stories.
(B) Talking to whomever was present, many stories were told by the old man.
(C) Talking to whoever was present, the old man told all his stories.
(D) Talking to whoever was present, many stories were told by the old man.
(E) While talking to whomever was present, the old man told all his stories.

47. (A) Everyone wants to live somewheres.
    (B) Everyone wants to live somewhere.
    (C) Every one wants to live somewheres.
    (D) Everyone want to live somewhere.
    (E) Every one want to live somewhere.

48. (A) A bucket of lobsters were bought for supper any way.
    (B) A bucket of lobsters are to be bought for supper anyway
    (C) A bucket of lobsters were bought for supper anyways.
    (D) A bucket of lobsters was bought for supper anyway.
    (E) A bucket of lobsters are going to be bought for supper anyway.

49. (A) A box of ice cream can't hardly feed this crowd.
    (B) A box of ice cream can't hardly feed these crowd.
    (C) A box of ice cream can hardly feed this crowd.
    (D) A box of ice cream can hardly feed these crowd.
    (E) A box of ice cream are hardly able to feed these crowd.

50. (A) Talking in the shower, his voice could be heard next door.
    (B) While talking in the shower, his voice could be heard next door.
    (C) Talking in the shower, the boy could be heard next door
    (D) Talking in the shower, his voice was heard next door.
    (E) Talking in the shower, his voice was able to be heard next door.

Answer Sheet—Standard Written English Test 5

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐	31	☐	☐	☐	☐	☐	41	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐	32	☐	☐	☐	☐	☐	42	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐	33	☐	☐	☐	☐	☐	43	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐	34	☐	☐	☐	☐	☐	44	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐	35	☐	☐	☐	☐	☐	45	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	26	☐	☐	☐	☐	☐	36	☐	☐	☐	☐	☐	46	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	27	☐	☐	☐	☐	☐	37	☐	☐	☐	☐	☐	47	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	28	☐	☐	☐	☐	☐	38	☐	☐	☐	☐	☐	48	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	29	☐	☐	☐	☐	☐	39	☐	☐	☐	☐	☐	49	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	30	☐	☐	☐	☐	☐	40	☐	☐	☐	☐	☐	50	☐	☐	☐	☐	☐

ANSWERS, STANDARD WRITTEN ENGLISH TESTS 1–5

Test 1

1. C (correct form is was attending)
2. B (correct form is were)
3. C (correct form is were)
4. B (correct form is have)
5. E (team can be considered either singular or plural, so practice is not incorrect)
6. A (correct form is we)
7. A (correct form is I)
8. D (correct form is its)
9. C (correct form is his)
10. D (correct form is me)
11. A (correct form is were)
12. A (dangling modifier)
13. D (correct form is kindest)
14. D (correct form is any other)
15. D (correct form is jogging)
16. C (correct form is sold)
17. B (correct form is has gone)
18. D (correct form is to)
19. B (correct form is as impertinent as)
20. A (dangling modifier)
21. B (correct form is were)
22. C (correct form is longer)
23. D (correct form is her)
24. A (dangling modifier)
25. D (correct form is me)
26. D (correct form is we)
27. B (correct form is was)
28. C (correct form is was)
29. C (correct form is am)
30. A (correct form is are)
31. D (had got nothing is common in British writing but considered poor in American usage)
32. A
33. C
34. A
35. B
36. C
37. C
38. D
39. D
40. A
41. A
42. D
43. C
44. D
45. C
46. D
47. B
48. D
49. E
50. A

Test 2

1. A (correct form is was)
2. D (correct form is is)
3. B (correct form is were)
4. A (correct form is have)
5. A (correct form is oneself)
6. B (correct form is is)
7. A (correct form is I)
8. B (correct form is whomever)
9. E
10. C (correct form is his)
11. B (correct form is me)
12. C (correct form is whom)
13. A (correct form is were)
14. A (dangling modifier)
15. E
16. C (correct form is lovelier)
17. E
18. B (correct form is most unusual)
19. B (correlatives must be placed before parallel terms)
20. D (lack of parallel structure; speedily is better)
21. D (correct form is had)
22. B (correct form is has stayed)
23. D (correct form is anyway)
24. A (correct form is had)
25. D (a is not necessary)
26. A
27. E
28. D
29. B
30. D
31. B
32. E
33. A
34. B
35. C (lack of parallel structure in other choices)
36. C
37. E
38. D
39. D
40. D (preferable to keep subject the same throughout the sentence)
41. E
42. B
43. D
44. A
45. B
46. C
47. E
48. B
49. D
50. B

Test 3

1. E (group can be considered either singular or plural, so have is not incorrect)
2. B (correct form is am)

3. C (correct form is were)
4. D (correct form is her)
5. D (correct form is he)
6. C (correct form is who)
7. E
8. D (correct form is their)
9. B (correct form is us)
10. A (dangling modifier)
11. B (more is not necessary)
12. A (correct form is as abrasive as)
13. B (correct form is enjoys neither)
14. D (correct form is to sail, for parallel structure)
15. C (correct form is walked)
16. D (correct form is has attended)
17. D (correct form is all right)
18. D (correct form is himself)
19. B (correct form is to)
20. C (correct form is a)
21. A (correct form is we're)
22. E
23. D (correct form is ?")
24. D (correct form is English)
25. C (correct form is Cousin)
26. B
27. C
28. A
29. B
30. E
31. B
32. D
33. D
34. E
35. E
36. D
37. E
38. E
39. C
40. C
41. C
42. E
43. D
44. A
45. B
46. A
47. C
48. D
49. D
50. A

Test 4

1. B (correct form is were)
2. A (correct form is is)
3. D (correct form is he)
4. C (correct form is whoever)
5. D (correct form is them)
6. A (correct form is had been)

7. A (dangling modifier)
8. B (correct form is better)
9. E
10. C (correct form is to read)
11. C (correct form is thoughtful)
12. D (correct form is had struck)
13. D (correct form is anywhere)
14. A (of opinion is not necessary)
15. C (correct form is may have)
16. C (correct form is somewhere)
17. C (correct form is among)
18. C (correct form is than)
19. A (correct form is cited)
20. E
21. C (correct form is ;)
22. B (correct form is ,")
23. C (correct form is Chinese)
24. E
25. E
26. A (correct form is forget)
27. A (correct form is I)
28. A (correct form is should have)
29. C (correct form is lets)
30. D (correct form is effect)
31. D
32. D
33. E
34. E (an introductory subordinate clause should be set off from the main clause by a comma)
35. C
36. A
37. D
38. B
39. B
40. A
41. E
42. B
43. E
44. A (choice (E) is incorrect because an introductory subordinate clause should be set off from the main clause by a comma)
45. E
46. A
47. E
48. D
49. E
50. E

Test 5

1. C (correct form is was)
2. B (the preferable form is is)
3. D (correct form is himself)
4. B (correct form is who)
5. C (correct form is who)
6. D (correct form is him)
7. D (correct form is were)

8. A (dangling modifier)
9. A (dangling modifier)
10. C (correct form is smartest)
11. B (correct form is bigger)
12. D (correct form is to report)
13. E
14. E
15. D (correct form is had fallen)
16. D (both is not necessary)
17. D (correct form is could have)
18. D (a is not necessary)
19. C (back is not necessary)
20. C (correct form is an)
21. A (correct form is whose)
22. D (correct form is fewer)
23. D (correct form is latter)
24. D (correct form is past)
25. E
26. C (correct form is ;)
27. D (correct form is ?")
28. A (correct form is French)
29. B (correct form is brother)
30. E
31. D
32. B
33. D
34. C
35. B
36. B
37. A
38. B
39. C (choice (E) is awkward because of the semicolon, which should usually not be used before but)
40. E
41. C
42. E
43. E
44. C
45. E
46. C
47. B
48. D
49. C
50. C

III.
THE MATHEMATICAL APTITUDE TEST

Introduction

The mathematical part of the Scholastic Aptitude Test (SAT) contains two types of questions: multiple choice and quantitative comparison. The usual number of questions is 25 for those sections which contain *only* multiple-choice answers, and 35 for those which contain the quantitative-comparison questions, since the latter require much less computation.

Since very few students can solve, on the average, *one problem per minute,* they are *not* expected to finish *any* of these sections in the time allotted. Therefore, they must work extremely quickly and be very accurate in order to achieve an above-average score.

In the booklet *About the SAT,* which comes enclosed with your application, is the following statement:

> The SAT is designed to cover a wide range of developed abilities. Questions range from easy to very hard. A question of average difficulty is answered correctly by 50 to 60 percent of the students.
>
> IF YOU ANSWER MOST OF THE QUESTIONS AND ANSWER SLIGHTLY MORE THAN HALF OF THEM CORRECTLY, YOU CAN ACHIEVE AN ABOVE-AVERAGE SCORE.

Throughout this book, we have assumed that you are an "average" student—that is, you only occasionally have that "insightful" approach to problems that some of your classmates may seem to have all the time. Obviously, the reason you bought this book is that you want to achieve the maximum score you are capable of. In order to do this, we recommend that you set three major goals for yourself:

1. Read and *digest* the *Review Guide* part of this book.

2. *Do all* the problems in the *Review Guide* and the practice Mathematical Aptitude Tests.

3. Learn some of the *techniques* of test-taking which will enable you to compensate when you are unable to attack problems in a direct manner.

In most books of this type, items 1 and 2 are emphasized almost to the exclusion of item 3. Yet, since time plays such an important role in this test, the *techniques* of test-taking can play a very important part in raising your SAT score.

Many students have told us that they *can* do the problems and they *can* get the right answers *if only they had more time.* Unfortunately, you must put your pencil down when the proctor calls out, "Stop." Every test has a time limit, so you must learn how to get the right answer even if you *don't* know how to do a problem in the traditional manner.

It is extremely important that you not only do each problem, but you *read each answer* in this book. In the *Review Guide,* we have included what we feel are the most important theorems and procedures for solving the mathematical problems that will confront you on the SAT. There are no proofs of theorems in the *Guide*—just some intuitive arguments concerning the most important theorems so that you will have an easier time remembering them. You can turn to your textbooks if you want a more complete explanation of any of these theorems.

What distinguishes this book from others on the market is a feature that we feel will do the most to raise your score: the *explanations* of the answers to the questions are *extremely detailed.* Most books present answers in such a "stripped-down" form that the average student is no better off after having read these

explanations than before he read them. So, what good purpose is served by being brief? The reason these explanations are lengthy is that they contain *many different approaches* to solving the particular problem, including:

1. Working backwards from the answers.

2. Substituting numbers instead of working with variables and algebraic manipulations.

3. Sketching figures in order to gain more information.

4. Estimating answers and eliminating impossible ones.

5. Using contradictory answers to make useful conclusions.

6. And finally, making "educated guesses" when you have a 50% option.

In practically every explanation of an answer in this book, you will see one or more of these techniques in use. Obviously, you will not have the time to use more than one of these techniques on any one of the problems. However, we feel that you should have a complete arsenal of techniques in readiness.

There is another statement in the booklet *About the SAT* that should allay some of your fears:

> The arithmetic includes the four basic operations of addition, subtraction, multiplication, and division; properties of odd and even integers; percent; and averages. The algebra includes linear equations, simple quadratic equations, factoring, and exponents, but *not* the quadratic formula, fractional or negative exponents, or logarithms. The geometry includes the properties associated with parallel lines and the informal measurement-related concepts of area, perimeter, volume, the Pythagorean Theorem, and angle measure in degrees. Knowledge of special triangles such as isosceles, equilateral, 30°–60°–90° is also assumed.

Therefore, the amount of mathematics you really need for this examination is *less* than you think. However, it is amazing what the SAT people can do in making up questions which test your knowledge of these basic operations and theorems, as you shall soon see.

Format of the Mathematical Part of This Book

On the following pages you will find:

- A sample 35-question Mathematical Aptitude Test, which you should attempt *even before* reading the *Review Guide*, just to get the feel of such a test.
- The answers to this test along with a *very detailed* explanation of each and every answer. In this first practice test we felt it important to show you not only the right answer to each question *but also* how students seem to arrive at the *wrong* answer. This is so because we want you to know that you are not alone in your frustration in dealing with these problems. Many other students throughout the years have made the same mistakes over and over again. These mistakes are prominently labeled *ERROR*. If you read the discussions of these errors you will not only decrease your chances of making them but will get a better feeling about yourself.
- A *Review Guide* in 12 sections, containing information on elementary

geometry, elementary algebra, arithmetic, inequalities, and "invented" operations.

• 120 sample SAT-type questions (10 in each of the 12 *Review* sections). You should *do* each of these problems in order to reinforce your understanding of the particular concept just explained.

• Explanations of each of these 120 questions. It is important that you not only read these explanations but understand how to apply the *techniques* of problem-solving.

• Seven sample Mathematical Aptitude Test sections, which include 195 SAT-type problems.

• Extremely detailed explanations of the answers to these 195 problems, showing *many different methods* of attacking them. Perhaps the most frequently used method for the "average" student is the method of *substitution of numbers*.

The explanations of answers are an integral part of the book. That is, even if you solved a problem and got the correct answer, it is very possible that you *still* made a mistake. How many times have you made two or more errors in a problem which seemed to cancel each other so that although the answer was correct, you really didn't understand the solution?

Another reason why you should study the explanations is that they contain information which expands upon the topic, and even at times goes on to *further related topics*.

You will notice that there is no section on verbal problems in the *Review Guide*. Instead, these verbal problems are covered in the explanations to the particular verbal problems that appear in the practice examinations. Too many students, in trying to understand the complexities of coin, work, mixture, distance, etc. problems, get bogged down and therefore frustrate themselves to such an extent that they blank out and subconsciously refuse to continue their active participation in studying for the SAT. So, we sneak these problems and explanations into the practice examination rather than overwhelm you with them in the *Review Guide*. Besides, there are students who have told us that they can do coin problems, but when it comes to stamp problems they are at a loss, even though these two types of problems are *exactly the same!* So, we do these problems as we come to them.

You *cannot* read this book as if you were reading a best seller. You *cannot sit in your easy chair and leaf through these pages and hope to gain information without effort. It just won't happen. You must* find yourself a quiet place, sharpen a few pencils, and leave yourself a few weeks for active participation in your studying. You *must do* the problems, not just read them and their explanations.

Do the practice examinations directly on the test paper itself—just as on the SAT you do all your scratchwork in the test booklet. No scrap paper is allowed on the real test, so to get used to the actual conditions of the test, do not use scrap paper when studying. Besides, this way you will not lose your answers and you will have an accurate record of your progress.

As on the real SAT, you will mark your answers on the separate answer sheets that have been provided.

Scoring the Tests

1. For those questions with five possible answers, your raw score is obtained by using the following formula:

$$\text{raw score} = \text{correct answers} - \tfrac{1}{4} \text{ of incorrect answers}$$

2. For those questions with four possible answers (the quantitative-comparison type), your raw score is obtained by using the following formula:

$$\text{raw score} = \text{correct answers} - \tfrac{1}{3} \text{ of incorrect answers}$$

3. Do not count any problem that you skipped.
4. Add the raw scores.
5. If your total raw score is equal to half the number of questions on the test, then you have obtained an *above-average* score. We can't predict exactly what score would get you what mark on the *real* SAT, as that scale is kept quite secret. But you can use the above method for at least approximating your real score.

Should you ever *guess?* This question is covered within the following pages.

A Final Word

We have taught the techniques of test-taking to many students throughout these past few years. The explanations to problems that you see in the following pages are the same explanations that we give students in our classes, almost word-for-word. *It is as if you were being seated in one of our classrooms.* Some of the explanations lead to "dead ends" before the correct answer is given. This is exactly what happens in the real classroom situation. In too many books, students get the feeling that they must be "stupid" because they couldn't get the correct answer in the most direct manner. Perhaps you will like this novel approach of seeing some of the *suffering* that goes on before a correct answer is obtained. Perhaps you won't like it—but seeing the dead ends will nevertheless help you see new approaches to a problem's solution. *Good luck!*

Mathematical Aptitude Test 1

Time—30 Minutes
35 Questions

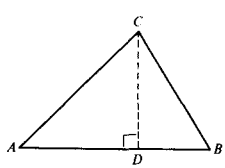
In this section solve each problem, using any available space on the page for scratchwork. Then indicate the *one* correct answer in the appropriate space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$.
The number of degrees in the arc of a circle is 360.
The measure in degrees of a straight angle is 180.

Definition of symbols:

$<$ is less than	$\leqq$ is less than or equal to
$>$ is greater than	$\geqq$ is greater than or equal to
$\perp$ is perpendicular to	$\parallel$ is parallel to



Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\measuredangle CDA$ is a right angle, then

(1) area of triangle $ABC = \dfrac{(AB)\cdot(CD)}{2}$

(2) $AC^2 = AD^2 + DC^2$

Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible *except* when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

1. If $a = b = 1$, then $a^b - b^a =$ (A) $a + b$ (B) $a - b$ (C) $\dfrac{a}{b}$ (D) ab (E) $2a + 2b$

2. If $A^2 = 9$ and $B^2 = 16$, then the least possible value of $B - A$ is

   (A) 1 (B) 0 (C) -1 (D) -7 (E) -12

3. If the degree measures of the angles of a triangle are in the ratio 3 : 4 : 5, what is the degree measure of the smallest angle?

   (A) 15 (B) 30 (C) 45 (D) 60 (E) 75

4. Of the following fractions, $^2/_3$, $^3/_4$, $^5/_8$, $^{11}/_{12}$, $^{13}/_{24}$, the difference between the largest and the smallest is

   (A) $^7/_{12}$ (B) $^1/_{12}$ (C) $^3/_8$ (D) $^1/_8$ (E) $^5/_6$

5. 1089419 is exactly divisible by

   (A) 2232 (B) 2235 (C) 2237 (D) 2238 (E) 2234

6. If $2x - 8 = 12$, then $3x - 8 =$

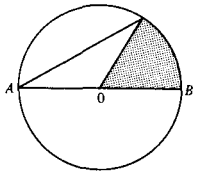
   (A) -2 (B) 10 (C) 19 (D) 22 (E) 52

7. $\dfrac{0.0303}{0.3} =$

   (A) 10.1 (B) 1.01 (C) 0.101 (D) 0.0101 (E) 0.00101

8. If a student had an average of 75 for his first three test grades and an average of 85 for his next five test grades, then his average so far is

   (A) 77.5 (B) 80 (C) 81.25 (D) 82.5 (E) 83.75



9. In the circle above, 0 is the center, $AB = 12$, and $A = 30°$. The area of the shaded region is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

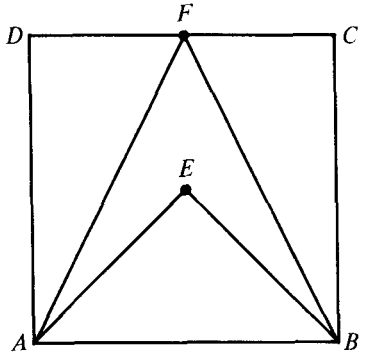
10. A certain perfume bottle is completely filled by three perfumes, A, B, and C. If there are equal amounts of A and B and four times as much C as there is A, then what percent of the perfume bottle is filled by C?

(A) 25% (B) 33⅓% (C) 40% (D) 66⅔% (E) 80%

11. Point E is the center of square $ABCD$ above. If $DF = FC = 3$, then the perimeter of $AEBF$ is

(A) $6(\sqrt{2} + \sqrt{5})$ (B) $12\sqrt{5}$ (C) $24\sqrt{2}$ (D) $12(3\sqrt{2} - 1)$ (E) $6(\sqrt{3} + 1)$

Figure for Problem 11



12. The sum of k positive consecutive integers is always an odd number if k is equal to

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

13. If $x = -3$ and $y = 1$, then $x^2y - xy^2 =$

(A) -18 (B) 0 (C) 6 (D) 12 (E) 18

14. If $2a - 3b = 0$, which of the following may be true?

I $a > b$
II $a = b$
III $a < b$

(A) I, II, and III (B) I only (C) I and II only (D) II and III only (E) I and III only

Questions 15–27 each consist of two quantities, one in Column A and one in Column B. You are to compare the two quantities and on the answer sheet blacken space

A if the quantity in Column A is greater;
B if the quantity in Column B is greater;
C if the two quantities are equal;
D if the relationship cannot be determined from the information given.

Notes: 1. In certain questions, information concerning one or both of the quantities to be compared is centered above the two columns.
2. A symbol that appears in both columns represents the same thing in Column A as it does in Column B.
3. Letters such as x, y, and n stand for real numbers.
4. Since there are only four choices, *NEVER MARK (E)*.

EXAMPLES

	Column A	*Column B*
E1.	The average of 3, 5, and 10	9
E2.	$x - y$	$y - x$
E3.	6% of 8	8% of 6

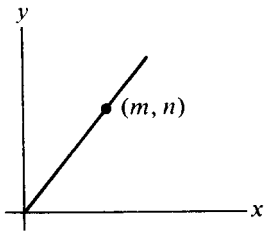
ANSWERS
E1. (B), E2. (D), E3. (C)

A if the quantity in Column A is greater;
B if the quantity in Column B is greater;
C if the two quantities are equal;
D if the relationship cannot be determined from the information given.

	Column A	*Column B*	
15.	$3(p-1)$	$3(p+1)$	A B C D

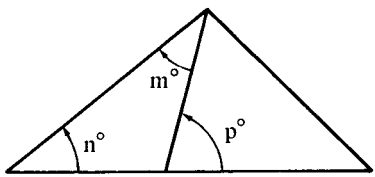
$abc = 0$
$a > b$

16.	ac	bc	A B C D
17.	$\frac{295 \times 17}{21}$	$\frac{35 \times 295}{42}$	A B C D
18.	$\frac{1}{\sqrt{5}}$	$\frac{\sqrt{5}}{5}$	A B C D



Note: Figure not drawn to scale.

19.	m	n	A B C D
20.	$\frac{11}{23}$ of 51	$\frac{51}{23}$ of 11	A B C D



21.	m	n	A B C D
22.	n	p	A B C D
23.	p	$(m+n)$	A B C D
24.	$(x-y)^2$	x^2-y^2	A B C D

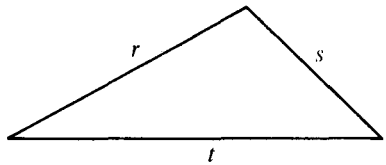
A if the quantity in Column A is greater;
B if the quantity in Column B is greater;
C if the two quantities are equal;
D if no relationship can be determined from the information given.

$x = 1$
n is an integer $\geqq 0$

	Column A	Column B	
25.	x^{n+1}	$(x + 1)^n$	A B C D

An item costs x dollars.

26.	Amount of money paid after a discount of 30%	Amount of money paid after successive discounts of 10% and 20%	A B C D



$r > s$

27.	t	$r - s$	A B C D

28. A square and a triangle have equal areas. If a side of the square, represented by t, is equal to the base of the triangle, then the altitude of the triangle in terms of t is

(A) t (B) $\frac{t}{2}$ (C) $\frac{3}{2}t$ (D) $2t$ (E) $\frac{7}{2}t$

29. If a boy can run 100 yards in 20 seconds, what is his average speed in *feet per minute?*

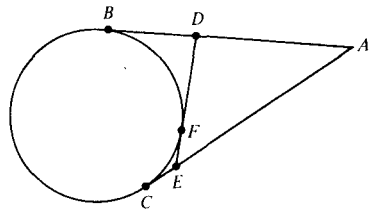
(A) 200 (B) 300 (C) 500 (D) 600 (E) 900

30. If a watch is set at 2 P.M. and loses 1.5 minutes every 2 hours, what will it read when it is actually 2 P.M. the following day?

(A) 1:30 (B) 1:36 (C) 1:40 (D) 1:42 (E) 1:51

31. The radius of a circle is increased by 20%. The area of the circle is increased by

(A) 20% (B) 33⅓% (C) 37.5% (D) 44% (E) It cannot be determined from the information given



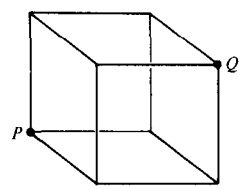
(Note: AB, AC, and DE are tangents to the circle)

32. If, in the above figure, $AB = 12$, what is the perimeter of triangle ADE?

(A) 18 (B) 24 (C) 36 (D) 44 (E) It cannot be determined from the information given

33. A truck can hold 200 cartons of apples or 240 cartons of pears. If the truck is already loaded with 150 cartons of apples, how many cartons of pears can be loaded?

(A) 40 (B) 60 (C) 70 (D) 80 (E) 90



34. The volume of the cube above is 64. What is the distance from point P to point Q?

(A) $4\sqrt{2}$ (B) $4\sqrt{3}$ (C) 12 (D) $3\sqrt[3]{12}$ (E) 16

35. If $N^* = N(N-1)(N-2)\cdots(1)$, where N is an integer greater than 1, then, $(N+1)^* = ?$

(A) $N^*[(N+1)]$ (B) $N^*[(N-1)]$ (C) $[(N-1)(N+1)]^*$ (D) $N[(N-1)^*]$
(E) $\frac{N}{2}[(N+2)^*]$

STOP

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

Answer Sheet—Mathematical Aptitude Test 1

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
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25	☐	☐	☐	☐	☐
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27	☐	☐	☐	☐	☐
28	☐	☐	☐	☐	☐
29	☐	☐	☐	☐	☐
30	☐	☐	☐	☐	☐
31	☐	☐	☐	☐	☐
32	☐	☐	☐	☐	☐
33	☐	☐	☐	☐	☐
34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐

ANSWERS AND EXPLANATIONS

1. (B)	2. (D)	3. (C)	4. (C)	5. (C)
6. (D)	7. (C)	8. (C)	9. (C)	10. (D)
11. (A)	12. (D)	13. (D)	14. (A)	15. (B)
16. (D)	17. (B)	18. (C)	19. (D)	20. (C)
21. (D)	22. (B)	23. (C)	24. (D)	25. (D)
26. (B)	27. (A)	28. (D)	29. (E)	30. (D)
31. (D)	32. (B)	33. (B)	34. (B)	35. (A)

1. (B) By substitution, $1^1 - 1^1 = 0$. Trying choice (A) $a + b = 1 + 1 = 2$, so this choice is incorrect. Choice (B) $a - b = 1 - 1 = 0$ is the correct one. Let's see what's wrong with the other choices. (C) $\frac{a}{b} = \frac{1}{1} = 1$ (D) $ab = 1 \cdot 1 = 1$ (E) $2a + 2b = 2 + 2 = 4$.

2. (D) $B = 4$ or -4, $A = 3$ or -3. Taking all the combinations for $B - A$, the least value is $(-4) - (3) = -7$.

ERRORS Forgetting the negative square roots of 16 and 9; treating $B - A$ as $(B)(-A)$.

3. (C) $3x + 4x + 5x = 180$; $12x = 180$; $x = 15$. The smallest angle is then $3x$ or $3(15) = 45$.

ERRORS Forgetting to multiply 15 by 3; getting the largest angle, 75, instead of the smallest.

4. (C) $^2/_3 = {}^{16}/_{24}$; $^3/_4 = {}^{18}/_{24}$; $^5/_8 = {}^{15}/_{24}$; $^{11}/_{12} = {}^{22}/_{24}$; $^{13}/_{24} = {}^{13}/_{24}$. When fractions have the same denominator, the one with the largest numerator is the largest fraction; the one with the smallest numerator is the smallest fraction. $^{22}/_{24} - {}^{13}/_{24} = {}^9/_{24} = {}^3/_8$.

ERRORS Assuming $^2/_3$ is the smallest fraction because it has the smallest numerator and denominator in the list; carelessness in changing fractions to their equivalent forms.

5. (C) *Do not* waste your time dividing 1089419 by each of the choices! Very few problems on the SAT require more than two minutes for their solution.

$$\begin{array}{r} \text{ABCDEF}\cdots\text{K} \\ \underline{\text{223Z}} \\ 1089419 \end{array}$$

Since the correct answer must be 223*?*, and since the last digit of 1089419 is 9, then the possibilities for digits K and Z are 1 and *9*, 3 and *3*, 7 and *7*, or 9 and *1*. So, since 2237 is the only number in the answer choices whose last digit is 9,3,7, or 1, choice (C) is the correct one.

ERROR Carelessness when dividing 1089419 by each of the answer choices.

6. (D) $2x - 8 = 12$; $2x = 20$; $x = 10$. Then, $3x - 8 = 3(10) - 8 = 30 - 8 = 22$.

ERROR Instead of *adding* 8 to both sides of the equation, some students *subtract* 8, leading to $x = 2$; then $3x - 8 = -2$.

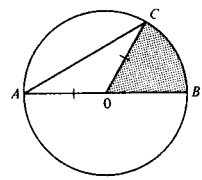
7. (C) Multiply the numerator and denominator of the fraction by 10 to make

the denominator a whole number. Then, $\frac{10}{10} \cdot \frac{0.0303}{0.3} = \frac{.303}{3} = 3\overline{)\,.303}\,.101$. Remember, in multiplying decimals by powers of 10 (10^1, 10^2, etc.), move the decimal point to the *right* 1, 2, etc. places, and add the necessary amount of zeros, if any. Example: $3.5 \times 100 = 3.5 \times 10^2 = 350$. When *dividing* decimals by powers of 10, move the decimal point to the *left*.

ERRORS Moving the decimal points the wrong way in the numerator and the denominator; making mistakes in "long" division (confusion due to all those "zeros").

8. (C) $\frac{3(75) + 5(85)}{3 + 5} = \frac{225 + 425}{8} = \frac{650}{8} = 81.25$. To get an average of 75 for three tests, the student must have had a total of 225 points. An average of 85 for five tests means a total of 425 points. Since there were eight tests altogether, you must divide the total amount of points by 8 to get the current average.

ERRORS The most common errors students make here are either averaging 75 and 85 to get 80 or simply carelessness in doing the arithmetic.

9. (C) 

$AO = OC$ (radii of the same circle are equal). In isosceles triangle AOC, then, $\measuredangle C = 30°$. $\measuredangle AOC = 180° - (30° + 30°) = 120°$. (Remember, the sum of the measure of the angles of any triangle is 180.) Therefore, $\measuredangle COB = 60°$ since it is supplementary to $\measuredangle AOC$. The shaded region, then, represents $^{60}/_{360}$ or $^1/_6$ of the area of the circle. The area of the circle is $\pi(6^2) = 36\pi$, so $^1/_6 \times 36\pi = 6\pi$.

ERRORS Obviously, unless you can find $\measuredangle COB$, you are in trouble. Perhaps you forgot the formula for the area of a circle. It is printed for you, though, at the beginning of all sections concerning mathematics on the SAT. Remember also that $AB = 12$ is the *diameter* of the circle, not the radius!

10. (D) % filled by $C = \frac{\text{amount of } C}{\text{total}} = \frac{4A}{6A} = \frac{2}{3} = 66\frac{2}{3}\%$. Total $= A + B + C$. Since $B = A$, and $C = 4A$, $A + B + C = A + A + 4A = 6A$.

ERRORS Some students see the word "four" and translate it into 40%; most of the trouble, though, is translating $\frac{2}{3}$ into a %. See the *Review Guide* for a treatment of %.

11. (A) You were probably wondering when (A) was going to be the right answer! Since $DF = FC = 3$, F must be the midpoint of a side of a square whose side is 6. Using the *Pythagorean* relationship in right triangles, ADF and FCB, $AF^2 = AD^2 + DF^2$, so, $AF^2 = 6^2 + 3^2 = 36 + 9 = 45$. Therefore, $AF = \sqrt{45} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$. Likewise, $BF^2 = BC^2 + CF^2$, or $BF = 3\sqrt{5}$. (They are congruent right triangles!) So far, the answer is $6\sqrt{5}$, which seems to eliminate choices (C), (D), and (E). Since $12\sqrt{5}$ is twice $6\sqrt{5}$, choice (B) seems to be "in the running." However, if you look at AE or BE, neither one is equal to AF or BF. Therefore, neither AE nor BE could be $3\sqrt{5}$. So . . . you *can* "back into" the answer!

If you drop perpendicular EG, then in triangle AGE, $AE^2 = AG^2 + GE^2$. $AE^2 = 3^2 + 3^2 = 9 + 9 = 18$. Now, $AE = \sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$. Since right triangle

BGE is congruent to right triangle *AGE*, $BE = 3\sqrt{2}$ also. Perimeter of *AEBF* = $AE + EB + BF + FA = 6\sqrt{5} + 6\sqrt{2} = 6(\sqrt{5} + \sqrt{2})$. Remember the distributive postulate: $A(B + C) = AB + AC$. It would be a *shame* to get as far as the right answer and *not be able to recognize it!*

ERRORS Very few students have trouble in finding *AF* and *FB*. It is *AE* and *EB* that cause consternation. Also, many students cannot "simplify radicals" or have trouble in performing the operations of arithmetic on them. See the *Review Guide* for help.

12. (D) Consecutive integers are integers such as $-3, -2, -1, 0, \ldots$ We can represent them as $x, x + 1, x + 2, \ldots$ Now, by a little trial and error (and some algebra), the sum of any *two* consecutive positive integers (positive, so the 0 doesn't "creep in") is $x + x + 1 = 2x + 1.2x$ or $4x$ or $6x$, etc. is *always* an even integer because it has a factor of 2. Thus, $2x + 1$ is always odd since it is the integer following an even one. So, the sum of any *two* positive consecutive integers is odd. However, 2 is *not* one of our choices. Trying 3, 4, 5, 6 we get: $x + x + 1 + x + 2 = 3x + 3 = 3(x + 1)$. Since x could be even *or* odd, $x + 1$ could be odd or even. 3 "times" an odd integer is odd and 3 "times" an even integer is even. (Try "plugging in" integers for x.) So, the sum of three consecutive integers could be even or odd. If you try four consecutive integers, you get $x + x + 1 + x + 2 + x + 4x + 6 = 2(2x + 3)$. Here, it doesn't matter whether or not $2x + 3$ is even or odd. Since we are going to multiply by a factor of 2 in either case, the answer will be an even integer.

If you try five consecutive integers, you get $x + x + 1 + x + 2 + x + 3 + x + 4 = 5x + 10 = 5(x + 2)$. Since $x + 2$ could be even or odd depending on x, $5(x + 2)$ can be even or odd.

Trying six consecutive integers, you get $x + x + 1 + x + 2 + x + 3 + x + 4 + x + 5 = 6x + 15 = 3(2x + 5)$. Now, assuming you are still youthful, $2x + 5$ *must* be odd because it is the representation of the sum of an even integer, $(2x)$, and an odd integer, (5). Since an odd integer "times" an odd integer is always odd, $3(2x + 5)$ is always odd, or choice (D). Easy, isn't it?

ERRORS If you decide (as 99% of my students did) to do this problem by "plugging in," you must be extremely careful. Suppose you took three consecutive integers such as 2, 3, and 4. Their sum is 9, which *is* odd. But what about 1, 2, and 3? Their sum is 6, which is *even*. So, choice (A) is out! How about five consecutive integers such as 1, 2, 3, 4, and 5? That sum is 15, which is odd. But 2, 3, 4, 5, and 6 have the sum 20, which is *even*. This sort of thing can make your life miserable. Therefore, try *many different* values of the variable when doing a problem by *plugging in*.

13. (D) If $x = -3$ and $y = 1$, then $x^2y - xy^2 = (-3)^2(1) - (-3)(1)^2 = 9 \cdot 1 - (-3 \cdot 1) = 9 - (-3) = 9 + 3 = 12$.

ERRORS Two very common errors: assuming that $9 - (-3) = 6$ (it doesn't); and assuming that $xy^2 = (xy)^2$ (which is *not* true for all values of x and y).

14. (A) $(5 \times 5) + (12 \times 12) = 25 + 144 = 169$. $169 = w \times w$, $w = 13$ *or* -13.

ERROR The only error here seems to be forgetting that a number has *two* square roots, a positive and a negative one. If $w \times w = w^2 = 169$, then $w = \sqrt{169} = 13$, or -13.

15. (B) Columns A and B contain products which are *almost* the same. The product in Column A contains one more factor of 0.4. Since four-tenths of any number is less than that number, $(.4 \times .4 \times .4) \times .4 < (.4 \times .4 \times .4)$.

16. (D) Let's substitute some values for a, b, and c, remembering that $abc = 0$ and $a > b$. If $abc = 0$, then at least one of the variables must be 0, so:

a	b	c	
3	2	0	$ac = 3 \times 0 = 0, bc = 2 \times 0 = 0; ac = bc$
0	−2	5	$ac = 0 \times 5 = 0, bc = -2 \times 5 = -10; ac \neq bc$

Since *ac could* be equal to *bc*, and *ac could* be more than *bc*, this one example shows that not enough information is given to determine whether or not any relationship exists.

17. (B) $\frac{295 \times 17}{21} = \frac{295}{1} \times \frac{17}{21}; \frac{35 \times 295}{42} = \frac{295}{1} \times \frac{35}{42}$. Since one factor of both products is the same, (295), it only remains to find out which of the fractions, $^{17}/_{21}$ or $^{35}/_{42}$, is the greater. Suspicions should be aroused by the denominators, 21 and 42. Since $^{17}/_{21} = {}^{34}/_{42}$, the product in Column B is greater.

ERROR Remember, this is a "timed" test. Speed is very important. The most common error made here is when students actually do the arithmetic involved. They multiply 295 by 17 and divide by 21. They multiply 295 by 35 and divide by 42. All this takes time and leads to mistakes.

18. (C) The value of a fraction remains unchanged when it is multiplied by a form of the number 1. $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{1\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{25}} = \frac{\sqrt{5}}{5}$. Therefore, the quantities in both columns are equivalent, or choice (C). Another way to test if two fractions are equal or not (and perhaps somewhat quicker in a problem such as this) is to compare their "cross-products." That is, if $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$, and inversely, if $\frac{a}{b} \neq \frac{c}{d}$, then $ad \neq bc$. So, in this problem, since $1 \times 5 = \sqrt{5} \times \sqrt{5} = \sqrt{25} = 5$, then the fractions are equivalent.

ERROR Not knowing the method for "rationalizing" the denominator of a fraction, i.e., converting an irrational denominator into a rational one.

19. (D) Since no units are given on the x and y axes, it is impossible to tell if $m > n$, $m = n$, or $m < n$. Remember, if you are told that the figure is *not* drawn to scale, you should not infer any information by measurement.

ERROR "It looks as if m and n are equal."

20. (C) This problem is almost like problem 17, before. When working with fractions, the word "of" is translated to mean "times." See the *Review Guide* for a rather lengthy discussion of fractions. $\frac{11}{23} \times \frac{51}{1} = \frac{11 \times 51}{23 \times 1} = \frac{51 \times 11}{23}; \frac{51}{23} \times \frac{11}{1} = \frac{51 \times 11}{23}$. Therefore the quantities in Column A and Column B are equivalent, or choice (C).

ERRORS Not knowing that "of" means "times"; not knowing how to multiply two fractions. Remember, in problems such as these, very little arithmetic is required. You are *not* supposed to do any multiplication ($51 \times 11 = 561$).

21. (D) Since we are not given any information concerning the sides of the triangle, no relationship can be determined between m and n.

ERROR "It looks like they are equal."

22. (B) There are three ways to approach this one:

(1) The angle p is an exterior angle of that triangle with m and n as non-adjacent interior angles. An exterior angle of a triangle is greater than either non-adjacent interior angle. So, $p > n$, or choice (B).

(2) The supplement of p (the angle to the left of p) is $180 - p$. That same angle is also $180 - (m + n)$, since the sum of the measures of a triangle is equal to 180. Therefore $180 - p = 180 - (m + n)$; $180 - p = 180 - m - n$; $-p = -m - n$. Multiplying both sides of the equation by -1, $p = m + n$. If a positive number (since we are talking about angles here) is equal to the *sum* of two other positive numbers, then it is greater than either one of them. So, $p > n$.

(3) Plug in some numbers. Let $n = 50$, $m = 60$, so that the left supplement of p is automatically 70. That makes $p = 110$. So, $p > n$. Let $n = 85$ (it sure doesn't look it). Then m could be equal to 1 and that left supplement of p would have to be 94 so that the angles of the triangle have a sum of 180. Therefore, p has to be 86. *Still*, $p > n$. Let $n = 45$ and $m = 45$. Then the left supplement of $p = 90$, so $p = 90$ and *still*, $p > n$.

ERROR Obviously, the more geometry you know, the easier a problem like this becomes. But even with a little work, you can prove the exterior angle theorem intuitively for yourself. But, remember, plug in a lot of numbers!

23. (C) See the discussion of problem 22.

24. (D) $(x - y)^2 = x^2 - 2xy + y^2$. We have to compare this quantity with $x^2 - y^2$. The easiest way to do this is to "plug in" a few sets of numbers.

x	y	$x^2 - 2xy + y^2$	$x^2 - y^2$
0	0	0	0
1	2	1	−3
2	1	1	3

ERRORS Some students do not know how to substitute values into algebraic expressions; some students do not "plug in" enough values to make a true determination of the relationships.

25. (D) 1 "raised" to any power is still 1. $1^0 = 1$, $1^5 = 1$, $1^{100} = 1$. In Column B, if $x = 1$, then $(x + 1) = 2$. Since n is an integer greater than *or equal to* 0, $(x + 1)^n$ could be 2^0, 2^1, 2^5, etc. Since $2^0 = 1$, the quantity in Column B is greater than *or* equal to the quantity in Column A. Therefore, no relationship can be determined.

ERROR Forgetting that n could be 0 in Column B leads to the *wrong* conclusion that the quantity in Column B is always greater than the quantity in Column A.

26. (B) What's better? A discount of 30% on the price of an item, or successive discounts (one after the other) of 10% and then 20%? Suppose the item costs \$100.00 (that's a nice number to work with when dealing with % problems). After the one-time discount of 30% the item will cost $100 - .30(100) = 100 - 30 =$ \$70.00. Start with \$100.00 again. After the first discount of 10%, the item will cost $100 - .10(100) = 100 - 10 =$ \$90.00. After the second discount of 20% the item will cost $90 - .20(90) = 90 - 18 =$ \$72.00. Therefore the quantity in Column B is greater. Since no value was given to the original price, a one-time discount of n% is better than multiple discounts that add up to n.

ERROR Not knowing how to figure a discount when given a % rate.

27. (A) In any triangle, each side is less than the sum of the two other sides and greater than the difference of the two other sides. So, in this case, $r - s < t < r + s$. Therefore $t > r - s$ or choice (A). Let's suppose for a moment that $r = 7$, $s = 4$. If $t = 11$, then no triangle can be "built" because the two sides, r and s, would "fall down" and just fit on "top" of t. So t must be less than $r + s = 11$. If t were less than $r - s = 3$, say $t = 2$, then the sides of the triangle would be 7, 4, and 2. If you "turned" the triangle around so 7 was now the base, the other two legs, 4 and 2, would "fall down" on 7 and leave a gap of 1, since $4 + 2 = 6$. Thus the intuitive theorem above.

ERROR There are many students who draw triangles and label the sides with any lengths they wish. For example, can a triangle with sides of 3, 5, and 86 really exist?

28. (D) If a side of a square is t, then its area is $t \times t$ or t^2. The area of any triangle is $\frac{1}{2} \times b \times h$ (b = base, h = height). So since the area of the square = area of the triangle, $t^2 = \frac{1}{2} \times t \times h$ (remember, the problem states that the side of the square = the base of the triangle). Multiplying both sides of the equation by 2, we get $2t^2 = th$. Since $t \neq 0$, divide both sides of the equation by t to get $2t = h$, which is choice (D). The *height* of a triangle and the *altitude* of a triangle mean the same thing.

ERROR Forgetting to take ½ of the product of the base and the altitude when figuring the area of the triangle.

29. (E) 100 yards = 300 feet. Set up a proportion:

$$\frac{\text{300 feet}}{\text{in 20 secs.}} = \frac{\text{how many feet, } f}{\text{in one minute, 60 secs.}}$$

Reduce $\frac{300}{20}$ first before "cross-multiplying" (the numbers will be easier to work with), so, $\frac{15}{1} = \frac{f}{60}; f = 15 \times 60 = 900$.

ERROR Remember, the answer must be in *feet* per minute.

30. (D) If a clock loses 1.5 minutes every two hours, how many minutes will it lose in 24 hours? (2 P.M. to 2 P.M. the following day = 24 hours). Set up a proportion: $\frac{1.5}{2} = \frac{m}{24}$; "cross-multiply," so $1.5 \times 24 = 2 \times m$; $36 = 2m$; $m = 18$ minutes lost. Therefore, at 2 P.M. the following day, the watch will read 18 minutes less than 2 P.M. or 1:42 P.M.

ERROR Perhaps you thought the time interval was 12 hours rather than 24 hours. You then would have marked choice (E).

31. (D) "Plug in" again. Let the original radius be 10. Then the new radius will be $10 + .20(10) = 10 + 2 = 12$. So, the area of the old circle is $\pi 10^2 = 100\pi$, and the area of the new circle is $\pi 12^2 = 144\pi$. The % change is figured by taking $\frac{\text{change in area}}{\text{original area}} = \frac{144\pi - 100\pi}{100\pi} = \frac{44\pi}{100\pi} = \frac{44}{100} = 44\%$. Try another set of numbers. Let the old radius be 6 and the new one $6 + .20(6) = 6 + 1.2 = 7.2$. Then the old area is 36π, and the new area is 51.84π. The change in the area is $51.84\pi - 36\pi = 15.84\pi$. $\frac{15.84\pi}{36\pi} = .44$ or 44%. Obviously the choice of 10 for the radius of the original circle made the solution of this problem much easier.

ERROR Assuming that if you are not given the radius of the original circle you cannot find out anything about the new circle. Often, with % problems, you don't need any numerical information if the answers are in terms of %.

32. (B) The theorem to use here is, "Tangents to a circle from the same external point are equal." Imagine that tangent AB is "bent" at point D and heads toward the circle to touch it at point F. Since $DB = DF$ (they are tangent also), the distance from A to D to F must remain 12. Likewise, do the same for tangent AC. Since $AC = AB = 12$, the perimeter of triangle $ADE = 12 + 12 = 24$.

ERROR This is an interesting problem because many students feel that not enough information is given to solve it. By the way, if you "see-sawed" tangent DE around so that point F moved along arc BC, you would get many, many triangles (still called ADE) and *all* their perimeters would still be 24!

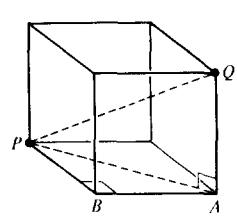
33. (B) The ratio of apples to pears is $\frac{200}{240}$ or $\frac{5}{6}$. So for every 5 cartons of apples that can be loaded, 6 cartons of pears can take their place. Since there are already 150 cartons of apples on the truck, there is room for 50 more cartons of apples (the truck can hold 200 cartons of apples). Set up a proportion:

$$\frac{\text{apples that can still be loaded}}{\text{pears that can be loaded}} = \frac{5}{6};\ \frac{50}{p} = \frac{5}{6};$$

"cross-multiply" to get $5p = 300; p = 60$ or choice (B).

There is a simple way of solving this problem that does not involve ratios. When loaded with 150 cartons of apples, the truck is obviously ¾ full $\left(\frac{150}{200} = \frac{3}{4}\right)$. In the remaining space, $\frac{1}{4} \times 240$ cartons of pears will fit; $\frac{240}{4} = 60$.

ERRORS Not understanding proportions; not understanding how to set up the numerators and the denominators; "what goes where?" See the *Review Guide* for help.



34. (B) All you need to solve this problem is the Pythagorean Theorem. PQ is the hypotenuse of right triangle PAQ. Since the volume of a cube is (side)^3, each side must be $\sqrt[3]{64}$ or 4. So, $AQ = 4$. But in order to use the Pythagorean Theorem you need the length of PA. Aha! PA is the hypotenuse of right triangle PBA, and we know that $BA = 4$ since it's a side of the cube too. In right triangle PBA, then, $PA^2 = PB^2 + BA^2; PA^2 = 4^2 + 4^2 = 16 + 16 = 32$. So $PA = \sqrt{32}$. (Don't simplify the radical. You'll see why later.)

Now that you have the length of PA, use right triangle PAQ. $PQ^2 = PA^2 + QA^2 = (\sqrt{32})^2 + 4^2 = 32 + 16 = 48$. Therefore $PQ = \sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$, or choice (B). If you got the answer $\sqrt{48}$ but did not see immediately that this is the same as $4\sqrt{3}$, you need to study Section 3 of the *Review Guide* on the arithmetic of square roots.

It is interesting to note here that to find the distance between two points in three-dimensional space, all you need is an extension of the Pythagorean Theorem: $PQ^2 = PB^2 + BA^2 + QA^2$!

ERROR Not being able to visualize a situation in which you can use the Pythagorean Theorem you learned in Geometry and which is reprinted at the beginning of the mathematical section of the SAT.

35. (A) "Plug in" numbers. $3^* = 3 \times 2 \times 1$. $4^* = 4 \times 3 \times 2 \times 1$ or $4^* = 4 \times 3^*$. In terms of N, $N^* = (N + 1)\ [N^*]$ or choice (A).

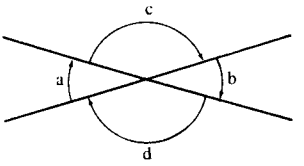
ERROR Not being able to "adjust" to *new* mathematical situations.

REVIEW GUIDE

Section 1. Angle Relationships

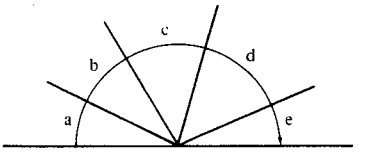
(*Note:* Unless otherwise stated, all lines are straight lines which lie in the same plane.)

When two lines intersect, pairs of vertical angles are equal.



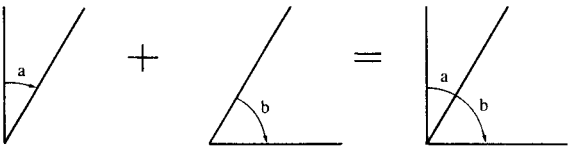
$a = b; c = d$

The sum of the angles about a point on one side of a line is 180.



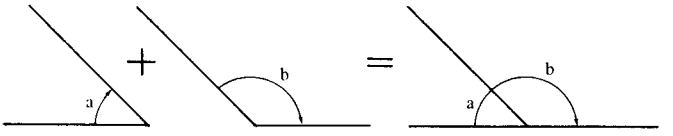
$a + b + c + d + e = 180$

Two angles are complementary if their sum is a right angle or 90.



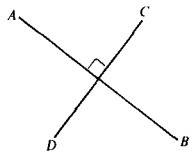
$a + b = 90$

Two angles are supplementary if their sum is a straight angle or 180.



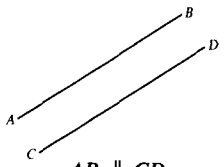
$a + b = 180$

If two lines intersect and form right angles, they are perpendicular. $AB \perp CD$ means that line AB is perpendicular to line CD.



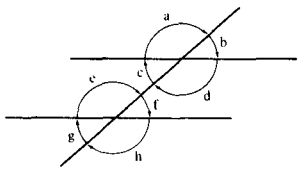
$AB \perp CD$

If two lines do not intersect, they are parallel. (Remember that a line extends infinitely in both directions.) $AB \parallel CD$ means that line AB is parallel to line CD.



$AB \parallel CD$

If two parallel lines are cut by a third line (called a transversal), then:



1. Pairs of alternate interior angles are equal.

$$c = f; d = e$$

2. Pairs of alternate exterior angles are equal.

$$a = h; b = g$$

3. Pairs of corresponding angles are equal.

$$a = e;\ c = g$$
$$b = f;\ d = h$$

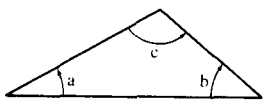
4. Pairs of interior angles on the same side of the transversal are supplementary.

$$c + e = 180$$
$$d + f = 180$$

5. Pairs of exterior angles on the same side of the transversal are supplementary.

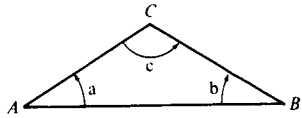
$$a + g = 180$$
$$b + h = 180$$

The sum of the measures of the angles of any triangle is 180.



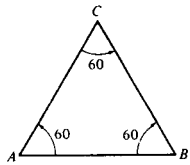
$a + b + c = 180$

If two sides of a triangle are equal, the angles opposite those sides are equal. The triangle is called an isosceles triangle.



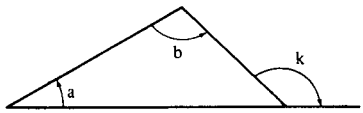
if $AC = BC$ then
$a = b$

If three sides of a triangle are equal, the three angles of the triangle are equal. The triangle is called equilateral and each angle has a measure of 60.



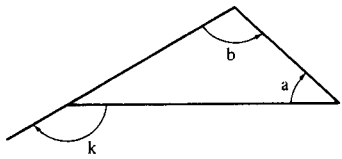
If $AB = BC = CA$ then
$a = b = c = 60$

An exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles.



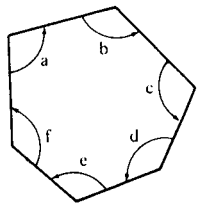
$k = a + b$

An exterior angle of a triangle is greater than either of the non-adjacent interior angles.



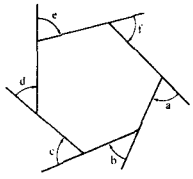
$k > a$
$k > b$

The sum of the measures of the interior angles of a polygon with n sides is $180(n - 2)$.



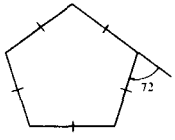
$n = 6$
$a + b + c + d + e + f = 180(6 - 2) =$
$180(4) =$
720

The sum of the measures of the exterior angles of a polygon with n sides is always 360.



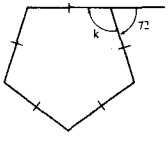
$$a + b + c + d + e + f = 360$$

Each exterior angle of a *regular* polygon of n sides is $\frac{360}{n}$.



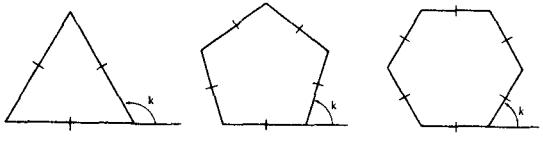
$$\frac{360}{5} = 72$$

Each interior angle of a *regular* polygon of n sides is $180 - \frac{360}{n}$.



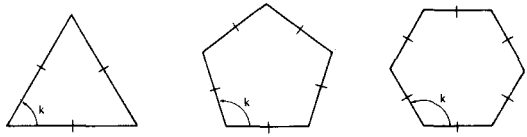
$$k = 180 - \frac{360}{5} = 180 - 72 = 108$$

As the number of sides of a *regular* polygon increases, each exterior angle of the polygon decreases.



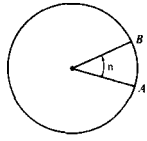
$n = 3$	$n = 5$	$n = 6$
$k = 120$	$k = 72$	$k = 60$

As the number of sides of a *regular* polygon increases, each interior angle of the polygon increases.



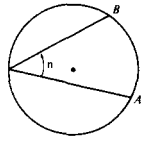
$n = 3$	$n = 5$	$n = 6$
$k = 60$	$k = 108$	$k = 120$

In a circle, the measure of a central angle is equal to the measure of its intercepted arc.



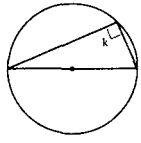
$n = 40°$
$\widehat{AB} = 40°$

In a circle, the measure of an inscribed angle is equal to ½ its intercepted arc.



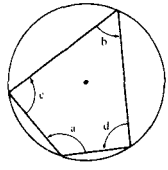
$n = 40°$
$\widehat{AB} = 80°$

An angle inscribed in a semicircle is a right angle.



$k = 90°$

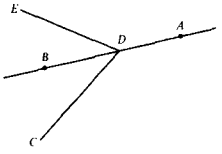
In a circle, the opposite angles of an inscribed quadrilateral are supplementary.



$a + b = 180°$
$c + d = 180°$

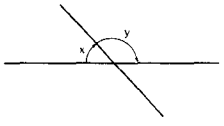
PRACTICE PROBLEMS

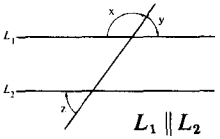
DIRECTIONS: In this practice section, and the ones that follow, indicate the correct answer by putting a circle around the letter that precedes it. If the question is of the quantitative-comparison type, circle (A) if the quantity in Column A is greater; circle (B) if the quantity in Column B is greater; circle (C) if the quantities in both columns are equal; and circle (D) if no relationship can be determined from the information given. In some of the comparison problems, information relating to the problem is centered above both columns. All variables such as x, y, m, k, etc. represent real numbers: (*Do all work on these pages. Do not use scratch paper.*)

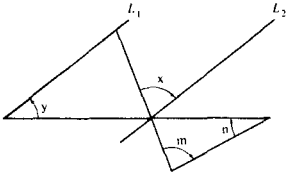


1.1 In the figure above, the measure of $\measuredangle CDE$ is 70 and line AB divides that angle into two equal angles. What is the measure of $\measuredangle ADC$?

(A) 35 (B) 55 (C) 75 (D) 110 (E) 145

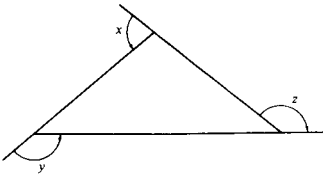
	Column A	*Column B*	
			
1.2	$180 - x$	y	A B C D

	 $L_1 \parallel L_2$		
1.3	$x + y$	$x + z$	A B C D



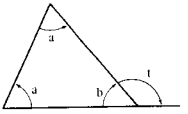
1.4 If $L_1 \parallel L_2$, then $m + n =$

(A) $x + y$ (B) $2x + 2y$ (C) $\frac{x + y}{2}$ (D) $x + y - 180$ (E) $180 - (x + y)$



1.5 In the figure above, the *sum* of the angles marked X, Y, and Z is

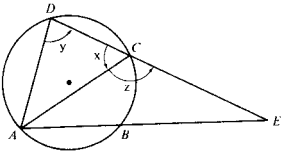
(A) 180° (B) 360° (C) 540° (D) 720° (E) It cannot be determined from the he information given



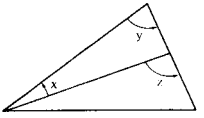
1.6 In the isosceles triangle above, t =

(A) $90 + \frac{b}{2}$ (B) $90 - \frac{b}{2}$ (C) $2a$ (D) $180 - a$ (E) $90 - b$

Column A *Column B*



1.7	z	y	A B C D
1.8	x	y	A B C D
1.9	$2y$	$\widehat{ABC}$	A B C D



1.10	$y - z$	x	A B C D

Note: In comparison problems like 1.7 to 1.10, *never mark choice* (*E*) (on the actual SAT answer sheet). In these practice problems, there *is* no choice (E) so that you will get used to the idea.

ANSWERS AND EXPLANATIONS

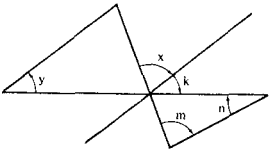
1.1 (E)	1.2 (C)	1.3 (C)	1.4 (A)	1.5 (B)
1.6 (C)	1.7 (A)	1.8 (D)	1.9 (C)	1.10 (B)

1.1 $\measuredangle CDB = 35$, so $\measuredangle ADC = 180 - 35 = 145$. Choice (E).

1.2 $x + y = 180$, so $y = 180 - x$. Choice (C).

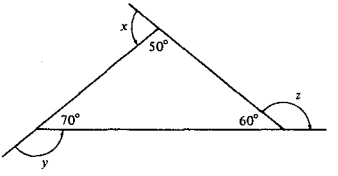
1.3 $x + y = 180$; $y = z$ (alternate exterior angles). By substitution, $x + z = 180$. Therefore $x + y = x + z$. Choice (C).

1.4



$y = k$ (corresponding angles are equal). $(x + k)$ is the exterior angle of the triangle with angles m and n. Therefore, $x + k = m + n$. By substitution, since $y = k$, $x + y = m + n$. Choice (A).

1.5



If you remember that the sum of the exterior angles of any polygon is 360°, this is an easy question. Otherwise, since the angles of a triangle add up to 180° (see the directions at the beginning of each section of the SAT), substitute numbers for the interior angles (such as 50°, 60°, and 70°). Since X, Y, and Z are supplements of these interior angles, X = 130° (180° − 50°), Y = 110° (180° − 70°), and Z = 120° (180° − 60°). So X + Y + Z = 130° + 110° + 120° = 360°. Choice (B).

1.7 z is an exterior angle of triangle ADC. Therefore $z > y$. Choice (A).

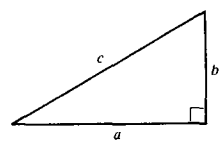
1.8 $x = \frac{1}{2}\widehat{AD}$; $y = \frac{1}{2}\widehat{ABC}$. We do not know the relationship between the two arcs. (Which one is greater? *Do not* assume that the one which *looks* greater *is* indeed greater.) Therefore, we cannot say what relationship exists between x and y. *If* arcs AD and ABC were equal, then x would indeed equal y because inscribed angles of the same circle that intercept equal arcs are equal. *However,* we must mark choice (D).

1.9 $y = \frac{1}{2}\widehat{ABC}$ (inscribed angle = ½ its intercepted arc. Therefore, $2y = \widehat{ABC}$). Choice (C).

1.10 $z > y$ (exterior angle is greater than either of the remote interior angles). Therefore $y - z < 0$. Since $x > 0$, (no negative angles on the SAT) $y - z < x$. Choice (B).

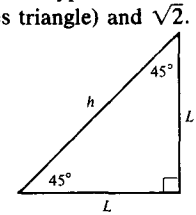
Section 2. Right Triangle Relationships

In a right triangle, $(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$. The hypotenuse is opposite the right angle (it is the longest side of the triangle) and "stands alone" in the equation above.



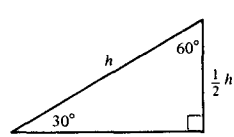
$$c^2 = a^2 + b^2$$

In a 45°-45°-90° right triangle, the hypotenuse is equal to the product of a leg (the legs are equal in this isosceles triangle) and $\sqrt{2}$.



$$h = L\sqrt{2}.$$

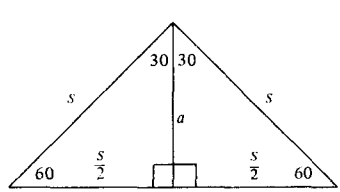
In a 30°-60°-90° right triangle, the leg opposite the 30° angle is one-half the length of the hypotenuse, and the leg opposite the 60° angle is equal to the product of one-half the hypotenuse and $\sqrt{3}$.



$$\tfrac{1}{2}h\sqrt{3}$$

$$L_1 = \tfrac{1}{2} \cdot h \qquad L_2 = \tfrac{1}{2} \cdot h\sqrt{3}$$

An equilateral triangle can be divided into two 30°-60°-90° right triangles by an altitude drawn from any angle. The altitude is then equal to the product of one-half one of the sides of the equilateral triangle and $\sqrt{3}$.



$$a = \tfrac{1}{2}s \cdot \sqrt{3}$$

Certain "triplets" of integers occur frequently on the SAT in dealing with the Pythagorean Theorem, $a^2 + b^2 = c^2$. Some of the basic ones are 3-4-5; 5-12-13; 8-15-17; 7-24-25 (and their multiples).

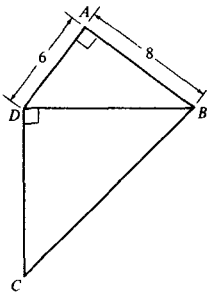
Example: $8^2 + 15^2 = 17^2$
($64 + 225 = 289$. Remember, the largest number is the hypotenuse.)

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

2.1 In a certain rectangle, the length is 2 more than twice the width. If the perimeter of the rectangle is 34, find the length of a diagonal.

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

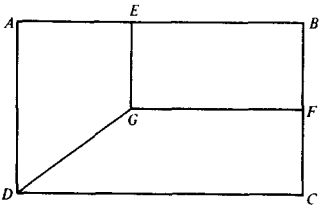


2.2 In the figure above, find the perimeter of $ABCD$, if $CD = DB$.

(A) $34 + 5\sqrt{3}$ (B) $30 + 5\sqrt{2}$ (C) $24 + 10\sqrt{3}$ (D) $34 + 10\sqrt{2}$
(E) $24 + 10\sqrt{2}$

2.3 A doorway measures 4 feet wide and 8 feet high. A man wants to carry some circular table tops through this doorway. If the diameters of these tops are 7, 8, 9, 10, and 11 feet, how many of these tops will he be able to carry through the doorway?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 0



2.4 In the above figure, $ABCD$ and $EBFG$ are rectangles. $AB = 10$, $EB = 6$, and $BF = FC = 3$. $DG =$

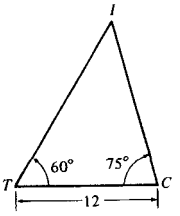
(A) 15 (B) $\sqrt{15} - 1$ (C) $4 - \sqrt{5}$ (D) 5 (E) $5 - \sqrt{5}$

2.5 In rhombus *ABCD*, diagonals *AC* and *BD* are 6 and 8, respectively. The perimeter of the rhombus is

(A) 5 (B) 10 (C) 15 (D) 20 (E) 25

2.6 If a side of an equilateral triangle is 6, its area is

(A) 12 (B) $9\sqrt{3}$ (C) 18 (D) $18\sqrt{2}$ (E) $36\sqrt{3}$



2.7 The area of triangle *TIC* is

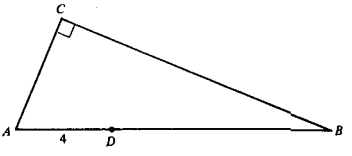
(A) $18\sqrt{3} + 54$ (B) $36\sqrt{3} + 108$ (C) $36\sqrt{3} + 18$ (D) $12\sqrt{3} + 36$
(E) $12\sqrt{3} + 54$

2.8 The area of a square is $4a^2$. The area of the square formed by connecting the midpoints of the original square in order is

(A) $a^2\sqrt{2}$ (B) $2a^2$ (C) $3a^2$ (D) $\frac{\sqrt{2}}{2}a^2$ (E) $\frac{a^2}{\sqrt{3}}$

2.9 A man walks 3 miles due east, then 4 miles due north, then 5 miles due east. How far is he from his starting point?

(A) 12 miles (B) $18\sqrt{2}$ miles (C) $4\sqrt{5}$ miles (D) 15 miles (E) $\sqrt{42.5}$ miles

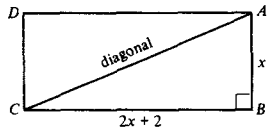


2.10 In right triangle ABC above, $\overline{AC} = 5$, $\overline{BC} = 12$ and $\overline{AD} = 4$. $\overline{DB} = ?$

(A) $\sqrt{53}$ (B) 9 (C) $\sqrt{20}$ (D) 8 (E) $6\sqrt{2}$

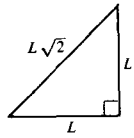
ANSWERS AND EXPLANATIONS

2.1 (A)	2.2 (E)	2.3 (B)	2.4 (D)	2.5 (D)
2.6 (B)	2.7 (A)	2.8 (B)	2.9 (C)	2.10 (B)



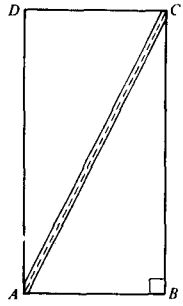
2.1 In the figure above, if the width is represented by x, then the length is $2x + 2$. The *perimeter* is the distance "around" the rectangle or $x + 2x + 2 + x + 2x + 2 = 34$; $6x + 4 = 34$; $6x = 30$; $x = 5$. The width is 5 and the length is $2(5) + 2 = 12$. Since triangle ABC is a right triangle (all the angles of a rectangle are right angles), $AC^2 = AB^2 + BC^2$; $AC^2 = 5^2 + 12^2 = 25 + 144 = 169$. Taking the square root of both sides of the equation, $AB = \sqrt{169} = 13$ (169 is a "perfect square"). *Or,* since 5-12-13 is a special triplet of Pythagorean numbers, the answer could have been obtained more quickly simply by inspection. Choice (A).

2.2 The perimeter of $ABCD = AB + BC + CD + DA$. We do not know the lengths of CD and BC. In right triangle DAB, $DB^2 = AD^2 + AB^2$; $DB^2 = 6^2 + 8^2 = 36 + 64 = 100$; $DB = \sqrt{100} = 10$. (6-8-10 is a multiple of 3-4-5.) Since $CD = DB$, $CD = 10$. Therefore, right triangle CDB is an *isosceles* right triangle.



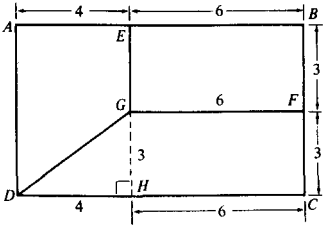
so, $CB = 10\sqrt{2}$. The perimeter of $ABCD$ is therefore $8 + 10\sqrt{2} + 10 + 6 = 24 + 10\sqrt{2}$. Choice (E).

Note: Even if you forgot the special relationship in an isosceles right triangle you could have taken a few seconds more and done the problem as follows: $BC^2 = 10^2 + 10^2 = 100 + 100 = 200$; $BC = \sqrt{200} = \sqrt{100 \cdot 2} = \sqrt{100} \cdot \sqrt{2} = 10\sqrt{2}$, etc. It is then extremely important to know how to *simplify* square roots in order to "recognize" the answer. We shall do this in the next *Review Guide* section.



2.3 The largest table top that can be carried through the doorway is the one that can be carried through on a slant, as in the diagram. The diagonal of the doorway is found by the Pythagorean Theorem: $AC^2 = AB^2 + BC^2 = 4^2 + 8^2 = 16 + 64 = 80$; $AC = \sqrt{80}$. Now, since $9 > \sqrt{80}$, only two tops can be carried through.

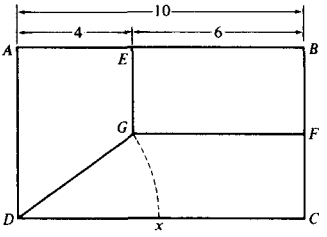
They have diameters of 7 and 8. $7^2 = 49$ and $8^2 = 64$. 9^2, 10^2, and 11^2 are all greater than 80. Choice (B).



2.4 Label the diagram. Form a right triangle by dropping a perpendicular, GH. Since $GH = FC = 3$, and since $DH = DF - HF = 10 - 6 = 4$, then in right triangle DHG, $DG^2 = DH^2 + HG^2 = 4^2 + 3^2 = 16 + 9 = 25$. So, $DG = \sqrt{25} = 5$. Choice (D). (3-4-5 right triangle!)

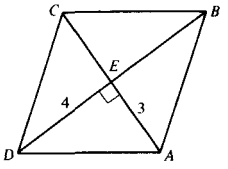
TECHNIQUE Let's start using some techniques. Perhaps you didn't realize that you could solve this problem by the Pythagorean Theorem. O.K., let's "back into" the answer by using a little *estimation* and *elimination.*

Put one finger on D and one finger on G. Using D as the "point" of your "compass," swing an arc, GX (see the figure below).



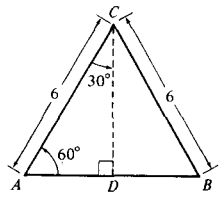
Now, $XD = GD$, and XD is certainly less than 10. That *eliminates* choice (A). Now look at choice (B). $\sqrt{15}$ is a "little bit" less than 4 (the square root of 16). So, $\sqrt{15} - 1$ is a little less than $4 - 1$ or 3. Since DX is greater than AE, DX is greater than 4, which eliminates choice (B). Now, off to choice (C). $4 -$ (any number) is certainly less than 4. Since $DG = DX$ and since DX is greater than 4, choice (C) cannot be the answer. Let's look at choice (E), $\sqrt{5}$ is certainly greater than 2. That means $5 - \sqrt{5}$ is less than 3. There goes choice (E)! So, the answer must be choice (D).

Of course, this process takes much longer than the direct attack. It also presupposes that you "knew" to use your "imaginary" compass. Well, *some* intelligence is required on this test!



2.5 *If* you knew that the diagonals of a rhombus bisect each other (it is a parallelogram) *and* are perpendicular to each other, then this problem is easy. In the

above figure, since $AC = 6$, then $AE = 3$; since $BD = 8$, $DE = 4$. To find the perimeter, we need to find just one side (since all the sides are equal in a rhombus), say DA. In right triangle DEA, $DA^2 = DE^2 + EA^2$; $DA^2 = 4^2 + 3^2 = 16 + 9 = 25$. So, $DA = \sqrt{25} = 5$. (3-4-5 again!) Therefore the perimeter is $5 + 5 + 5 + 5 = 20$. Choice (D).



2.6 The formula for the area of any triangle is given to you at the beginning of all sections containing mathematics on the SAT: Area = ½ (base) (height). Our only problem is finding the height of triangle ABC, because we know the base is 6 (all the sides of an equilateral triangle are equal). Well, triangle ADC is a 30°-60°-90° right triangle (all the angles of an equilateral triangle are 60°); therefore, the height, which is the side opposite the 60° angle, is equal to ½ (hypotenuse) $\sqrt{3}$. So the area is $\frac{1}{2}$(base) (height) $= \frac{1}{\cancel{2}}(\cancel{6})\left(\frac{1}{\cancel{2}}\cancel{6}\sqrt{3}\right) = 9\sqrt{3}$. Choice (B).

Note: Let k be a side of the equilateral triangle (instead of 6 as in the previous problem). Then the area of the equilateral triangle is ½ (base)(height) = ½ (k) (½ $k\sqrt{3}$) = ¼ $k^2\sqrt{3}$. Therefore, the area of any equilateral triangle is "side-squared over 4, times the square root of 3."

2.7 The clue to solving this problem is that 60° angle in the lower left side of the figure. Since we have to get the area of the triangle we need a base and a height. It *seems* we have the base $TC = 12$. Let's draw a height, IJ:

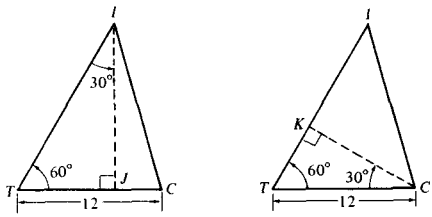


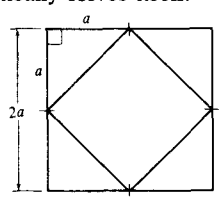
Fig. (A) Fig. (B)

Note: Figures not drawn to scale.

This *doesn't* help because although we have a 30°-60°-90° right triangle we *do not* know *any* of the sides. *However,* drawing CK *is* very helpful.
Now, angle TKC is a right angle (a height is perpendicular to a base), and angle $TCK = 30°$. Our approach will be to use IT as the base (turn the triangle around in your mind) and CK as the height. In right triangle TKC, CK is opposite a 60° angle, so it is equal to one-half the hypotenuse, TC, times the square root of 3. So CK = ½ (12) $\sqrt{3}$ or 6 $\sqrt{3}$. So, we have found the height. Now, for the base! The base of the triangle, IT, is the *sum* of two line segments, TK and KI. $TK = 6$ because it is opposite a 30° angle, so it is one-half the hypotenuse, TC. But how to find KI? In triangle CKI (also a right triangle), angle $I = 45°$. The reason for this is $\measuredangle T + \measuredangle C + \measuredangle I = 180°$. We were given $\measuredangle T = 60°$ and $\measuredangle C = 75°$. Since $60 + 70 = 135$, $\measuredangle I$ must be 45°. Had you done this at the beginning of the problem you would have realized something was fishy.

Anyway, angle $KCT = 45°$ $(75 - 30 = 45)$, so triangle KIC is a 45°-45°-90° right triangle. *Aha!* So, $KI = KC = 6\sqrt{3}$. Now, IT, a base of triangle ITC, $= TK + KI = 6 + 6\sqrt{3}$. So, the area of triangle ITC is $\frac{1}{2}(IT)(KC) = \frac{1}{\cancel{2}}(\cancel{6}^{3} + \cancel{6}^{3}\sqrt{3})(6\sqrt{3}) = (3 + 3\sqrt{3})(6\sqrt{3}) = 3 \cdot 6\sqrt{3} + 3\sqrt{3} \cdot 6\sqrt{3} = 18\sqrt{3} + 18\sqrt{9} = 18\sqrt{3} + 18 \cdot 3 = 18\sqrt{3} + 54$. Choice (A).

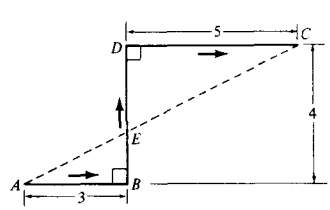
Note: Obviously, this explanation of the answer could have been shortened. In fact, other books of this type do just that, causing consternation in many students. The author of this section of this book is writing the answers to these problems exactly as he has taught them to his classes. That is, he is trying to show the student how to go about achieving the solution of a problem—including backing out of "dead ends" such as the first diagram made above. About 80–90% of students do *not* see altitude (height) CK. So don't feel badly if you didn't! And although the explanations seem quite long, in actuality, when working with pencil and paper, the problem practically solves itself.



2.8 Since the area of a square is $(\text{side})^2$, then each side of the original square can be found by applying the formula $4a^2 = \text{side}^2$, so each side $= \sqrt{4a^2} = 2a$. Therefore we have four isosceles right triangles formed, each with legs of length a and hypotenuse $a\sqrt{2}$. The hypotenuse of any of these isosceles right triangles is a side of the smaller square (which we can use to find *its* area). To find the area of the smaller square; area $= (a\sqrt{2})^2 = (a\sqrt{2})(a\sqrt{2}) = a^2\sqrt{4} = a^2 \cdot 2 = 2a^2$. Choice (B).

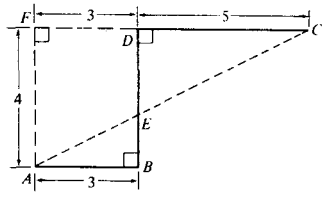
Note: You can also find the area of the smaller square by subtracting the areas of the four isosceles right triangles from the area of the larger square. The area of any right triangle $= \frac{1}{2}(\text{leg})^2$. Because a base and height have to be perpendicular, and since the two legs of a right triangle are perpendicular, either one could be called the base and either one could be called the height. So, in this case, the area of one of the isosceles right triangles is just $\frac{1}{2}a^2$. Since there are four of these triangles, the total area of the four triangles is $(4)(\frac{1}{2}a^2)$ or $2a^2$. The area of the large square-areas of the four triangles $= 4a^2 - 2a^2 = 2a^2$. Choice (B).

Note: Do *not* get the impression that if you join the midpoints of *any* geometric figure, you get ½ the area. Let's look at the equilateral triangle again. We will join the midpoints of each of its sides. Then, by a theorem you learned in Geometry, each of these smaller line segments is parallel to a side of the original triangle and is ½ its length. If the sides of the original equilateral triangle are each 10, let's say, then the sides of the smaller equilateral triangle are each 5. The area of the large triangle is $\frac{10^2}{4}\sqrt{3}$ or $\frac{100}{4}\sqrt{3} = 25\sqrt{3}$, while the area of the smaller triangle is $\frac{5^2}{4}\sqrt{3} = \frac{25}{4}\sqrt{3}$!

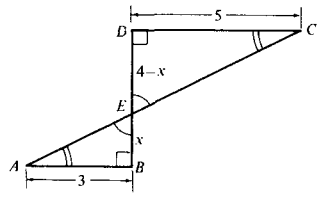


2.9 After drawing the above diagram (as most of you have done), the first inclination is to find BE and DE so you can use the Pythagorean Theorem in tri-

angles ABE and CDE. Unfortunately, all you know is $\frac{3}{AE} = \frac{5}{DE}$ by similar triangles. This *one* equation is not enough to help you find BE and DE. But what if AC were the hypotenuse of a right triangle AFC? (See below.) Then the problem is easy, isn't it? $AC^2 = AF^2 + FC^2$. $AF = 4$ (it is parallel to BD) and $FD = 3$ (it is parallel to AB), making $FC = 8$, $(3 + 5)$. So, $AC^2 = 4^2 + 8^2 = 16 + 64 = 80$, and $AC = \sqrt{80} = \sqrt{16 \cdot 5} = \sqrt{16} \cdot \sqrt{5} = 4\sqrt{5}$. Choice (C).



Perhaps you wondered why we stressed the word "one" in the preceding paragraph. Well, the problem *can* be done if you use *two* equations. See the diagram below.



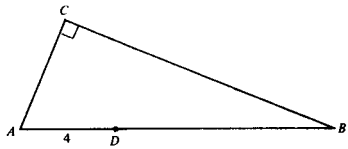
If you let $BE = x$, then $DE = 4 - x$, since $DB = 4$. Since $\measuredangle B = \measuredangle D$ and $\measuredangle BEA = \measuredangle DEC$ (vertical angles are equal), the triangles are similar by A.A. ($\measuredangle A = \measuredangle C$, also, since they are alternate interior angles. But two angles are enough. In fact, if two angles of one triangle are equal respectively to two angles of another triangle, then the third angles are equal.) Since the triangles are similar, their corresponding sides are in proportion: $\frac{AB}{BE} = \frac{DC}{DE}$; $\frac{3}{x} = \frac{5}{4 - x}$. Now, "cross-multiplying," $5x = 3(4 - x)$; $5x = 12 - 3x$; $8x = 12$; $x = 1.5$. So, $BE = 1.5$ and $DE = 4 - 1.5 = 2.5$. Now, use the Pythagorean Theorem on both triangles to get AE and EC (which you will add later to get the answer, AC). In right triangle ABE, $AE^2 = AB^2 + BE^2 = 3^2 + 1.5^2 = 9 + 2.25 = 11.25$. So, $AE = \sqrt{11.25}$. In right triangle CDE, $EC^2 = DC^2 + DE^2 = 5^2 + 2.5^2 = 25 + 6.25 = 31.25$. So, $CE = \sqrt{31.25}$. Therefore, $AC = AE + EC = \sqrt{11.25} + \sqrt{31.25}$. Since $3 < \sqrt{11.25} < 4$ and $5 < \sqrt{31.25} < 6$, then $8 < \sqrt{11.25} + \sqrt{31.25} < 10$. Thus we can eliminate choices (A), (B), (D), and (E), since the answer must be between 8 and 10.

Remember, though, very few problems should take more than a minute or two. You are not supposed to do so much work in order to get an answer. But it does work! We avoided this method because we felt you would have gone bananas had you seen this explanation first.

At this moment, you are probably thinking to yourself, "How was I supposed to know to draw AF and FD?" Perhaps the answer to that question can only be found after you have done hundreds of problems like these. The more problems you do, the better you become at doing problems! You cannot improve your score in mathematics by reading problems. You must do them, or at least try to do them. You must agonize over them and stick with them until you have convinced yourself that you have given them your best try. Unfortunately, too many students give

up after one or two minutes, turn to the answers, and persuade themselves that they are studying.

TECHNIQUE Let's do this problem by *estimation*—assuming that you couldn't do it any other way. Since the longest side of a right triangle is the hypotenuse, AE must be greater than 3 and CE must be greater than 5. That means that AC (what you are looking for) must be greater than 8. Also, since a straight line segment is the shortest distance between two points, AC must be less than $AB + BD + DC$, so AC must be less than 12. Using the inequality symbols, $8 < AC < 12$. Now, look at the answers. Choice (A) is 12. That's out! Choice (B) is $18\sqrt{2}$. That's certainly out! Choice (D) is 15. Out! And finally, choice (E) is $\sqrt{42.5}$. Well, $\sqrt{42.5}$ must be somewhere between 6 and 7, since $6 \times 6 = 36$ and $7 \times 7 = 49$. So that lets choice (E) out. Everything is out except for choice (C).



2.10 Using the Pythagorean relationship, $\overline{AC}^2 + \overline{BC}^2 = \overline{AB}^2$, so $5^2 + 12^2 = \overline{AB}^2$. $25 + 144 = \overline{AB}^2$. $169 = \overline{AB}^2$. $\sqrt{169} = \overline{AB}$. So, $\overline{AB} = 13$. Since $\overline{AD} = 4$, $\overline{DB} = \overline{AB} - \overline{AD} = 13 - 4 = 9$. Choice (B).

Should You Guess?

(*Note:* To make the explanation simpler, we shall assume that ¼ of a credit is taken off for each wrong answer and *no* credit is taken off for an answer that is omitted. On the quantitative-comparison questions, ⅓ of a credit is taken off for each wrong answer because there are only four choices instead of five.)

Let's suppose, on a test with 75 questions, you answered 40 correctly and 20 incorrectly and left out 15. Your raw score (your score before it is "scaled" into a range of 200 to 800) is then $40 - \frac{1}{4}(20) = 40 - 5 = 35$. If you had guessed at all 75 answers, then statistically you would have gotten $\frac{1}{5}$ (since there are five choices for each question) of the answers correct. Your raw score would be 15! This is not a good habit, therefore! If you just marked the 40 answers you *knew* to be correct, your raw score would be 40. So, obviously, it is a good idea to guess only when you can lower the odds against you. That is, you *should* guess, but only after you have definitely eliminated one or more of the obvious wrong answers. If you can eliminate three wrong answers, therefore, the odds of getting the right answer rise from 20% to 50%. If you do not fill in an answer blank you lose no credit, nor do

you gain. If you get the answer right, you gain one credit. If you get it wrong, you lose only ¼ of a credit. The odds are excellent in a 50% situation that you can improve your score by guessing.

An educated guess is one in which you must decide between two or three answers rather than four or five. Try to keep your guessing to a minimum unless you are in that range.

In problem 2.10, there is no room for an educated guess.

In problem 2.9, choice (E) was $\sqrt{42.5}$. Now, why do you suppose that was there? If you had done the problem by similar triangles, you would have had for your answer that $AC = \sqrt{11.25} + \sqrt{31.25}$. Now, in the rush for time, you could have easily added these two square roots to get $\sqrt{42.50}$. Choice (E). Perhaps one example will convince you that you *can't always* add two square roots like that:

ERROR $\sqrt{9} + \sqrt{16} = \sqrt{9 + 16} = \sqrt{25} = 5$

$$\begin{array}{ccccc} \downarrow & & \downarrow & & \\ 3 & + & 4 & & = 7 \end{array}$$

So obviously, $\sqrt{a} + \sqrt{b} \neq \sqrt{a + b}$. In *certain* cases, such as when $a = b = 0$, it is true. But be careful!

In the next section, you will review the rules for working with square roots.

Section 3. Arithmetic of Square Roots

The product of two square roots is the square root of their product.

$$(\sqrt{x})(\sqrt{y}) = \sqrt{xy}$$

The quotient of two square roots is the square root of their quotients.

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$$

The sum or the difference of two square roots can only be found when their radicands are equal. (The radicand is the number that appears under the square root symbol.)

Addition

$$\begin{gathered} \sqrt{32} + \sqrt{50} = \\ \sqrt{16 \cdot 2} + \sqrt{25 \cdot 2} = \\ \sqrt{16} \cdot \sqrt{2} + \sqrt{25} \cdot \sqrt{2} = \\ 4\sqrt{2} + 5\sqrt{2} = \\ 9\sqrt{2} \end{gathered}$$

Subtraction

$$\begin{gathered} \tfrac{1}{2}\sqrt{300} - 2\sqrt{48} = \\ \tfrac{1}{2}\sqrt{100 \cdot 3} - 2\sqrt{16 \cdot 3} = \\ \tfrac{1}{2}\sqrt{100} \cdot \sqrt{3} - 2\sqrt{16} \cdot \sqrt{3} = \end{gathered}$$

$$\begin{gathered}\tfrac{1}{2} \cdot 10\sqrt{3} - 2 \cdot 4\sqrt{3} = \\ 5\sqrt{3} - 8\sqrt{3} = \\ -3\sqrt{3}\end{gathered}$$

Multiplication

$$\begin{gathered}\sqrt{18} \cdot \sqrt{32} = \\ \sqrt{576} = \\ 24\end{gathered}$$

Sometimes it is easier to multiply two square roots if you *simplify* the radicand first.

$$\begin{gathered}\sqrt{18} \cdot \sqrt{32} = \\ \sqrt{9 \cdot 2} \cdot \sqrt{16 \cdot 2} = \\ 3\sqrt{2} \cdot 4\sqrt{2} = \\ 3 \cdot 4 \cdot \sqrt{2} \cdot \sqrt{2} = \\ 12\sqrt{4} = \\ 12 \cdot 2 = \\ 24\end{gathered}$$

To simplify a radicand, as you have seen in the previous examples, means to write the radicand as a product of two factors, one of which is the largest perfect square less than the radicand. Then, using the rule $\sqrt{xy} = \sqrt{x}\sqrt{y}$, the radicand is "reduced" to such an extent that it does not contain any perfect squares.

$$\begin{gathered}\sqrt{32} = \\ \sqrt{4 \cdot 8} = \\ \sqrt{4} \cdot \sqrt{8} = \\ 2\sqrt{8}\end{gathered}$$

But 8 can be simplified or reduced *further*, since $\sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$. Therefore, $\sqrt{32}$ should have been written as $\sqrt{16 \cdot 2}$. Then, $\sqrt{16}\sqrt{2} = 4\sqrt{2}$. You will get to the same point eventually—it is just quicker to use the biggest perfect square in the beginning.

Division

$$\begin{gathered}\frac{\sqrt{175}}{\sqrt{112}} = \\ \sqrt{\frac{175}{112}} = \\ \sqrt{\frac{\frac{7}{7} \cdot \frac{25}{1}}{\frac{7}{7} \cdot \frac{16}{1}}} = \\ \sqrt{\frac{25}{16}} = \\ \frac{5}{4}\end{gathered}$$

Sometimes it is easier to simplify before dividing the square roots.

$$\frac{\sqrt{175}}{\sqrt{112}} =$$

$$\frac{\sqrt{25 \cdot 7}}{\sqrt{16 \cdot 7}} =$$

$$\frac{\sqrt{25}\,\sqrt{7}}{\sqrt{16}\,\sqrt{7}} =$$

$$\frac{5\,\sqrt{7}}{4\,\sqrt{7}} =$$

$$\frac{5}{4} \cdot \frac{\sqrt{7}}{\sqrt{7}} =$$

$$\frac{5}{4} \cdot 1 =$$

$$\frac{5}{4}$$

Perfect Squares

You should memorize the following table, as it will save you time when working with square roots. The numbers under the square root symbols are the first twenty perfect squares.

$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$
$\sqrt{121} = 11$
$\sqrt{144} = 12$
$\sqrt{169} = 13$
$\sqrt{196} = 14$
$\sqrt{225} = 15$
$\sqrt{256} = 16$
$\sqrt{289} = 17$
$\sqrt{324} = 18$
$\sqrt{361} = 19$
$\sqrt{400} = 20$

Approximate Square Roots of Whole Numbers

It would also be helpful to memorize the approximate values of some square roots should you ever need them in using the technique of *estimation*. (The symbol $\approx$ means "approximately equals.")

$\sqrt{2} \approx 1.4$
$\sqrt{3} \approx 1.7$
$\sqrt{5} \approx 2.2$
$\sqrt{7} \approx 2.6$

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

3.1 $\sqrt{112} + 8\sqrt{7} =$

(A) $8\sqrt{119}$ (B) $24\sqrt{7}$ (C) $12\sqrt{7}$ (D) $\sqrt{127}$ (E) none of these

3.2 $\sqrt{16x^{16}} =$

(A) $8x^8$ (B) $4x^8$ (C) $4x^4$ (D) $8x^4$ (E) none of these

3.3 $\sqrt{200}\sqrt{500} =$

(A) $100\sqrt{10}$ (B) $10\sqrt{10}$ (C) 100 (D) $20\sqrt{7}$ (E) $100\sqrt{7}$

3.4 $\sqrt{162} - \sqrt{32} =$

(A) $\sqrt{130}$ (B) $5\sqrt{2}$ (C) $13\sqrt{2}$ (D) 5 (E) $65\sqrt{2}$

3.5 $\dfrac{\sqrt{116}}{\sqrt{29}} =$

(A) 4 (B) 2 (C) $4\sqrt{2}$ (D) $6\sqrt{3}$ (E) 18

3.6 $\dfrac{\sqrt{50} + \sqrt{32}}{\sqrt{2}} =$

(A) $\sqrt{41}$ (B) $5 + 4\sqrt{2}$ (C) $4 + 5\sqrt{2}$ (D) $9\sqrt{2}$ (E) 9

3.7 $(\sqrt{2} - 1)(\sqrt{2} + 1) =$

(A) $\sqrt{2} - 1$ (B) 1 (C) $\sqrt{2} + 1$ (D) 2 (E) 0

3.8 $\dfrac{1}{\sqrt{5}} =$

(A) $\dfrac{\sqrt{5}}{5}$ (B) $\dfrac{2\sqrt{5}}{5}$ (C) $5\sqrt{5}$ (D) 5 (E) $\sqrt{5}$

3.9 Of the following, the one which is an *irrational* number is:

(A) $\sqrt{169}$ (B) $9\sqrt{9}$ (C) $\sqrt{32} \cdot \sqrt{2}$ (D) $\sqrt{50}$ (E) $\dfrac{\sqrt{18}}{\sqrt{2}}$

3.10 If $\sqrt{x^2 + y^2} = x + y$, then which of the following must be true?

(A) $x = y$ (B) $x = 0$ (C) $x - y \neq 0$ (D) $x + y = 0$ (E) $xy = 0$

ANSWERS AND EXPLANATIONS

3.1 (C)	3.2 (B)	3.3 (A)	3.4 (B)	3.5 (B)
3.6 (E)	3.7 (B)	3.8 (A)	3.9 (D)	3.10 (E)

3.1 Remember, the only time you can add two square roots is when their radicands are equal. The *clue* here is the 7 under the square root sign in the second

summand. Obviously, 112 can be written as a product of two factors, one of which is 7. Divide 7 into 112 to get 16. $\sqrt{112} = \sqrt{16 \cdot 7} = \sqrt{16}\,\sqrt{7} = 4\sqrt{7}$. So, $\sqrt{112} + 8\sqrt{7} = 4\sqrt{7} + 8\sqrt{7} = 12\sqrt{7}$. Choice (C)

3.2 About 80 to 90% of average students get this problem wrong. It *is* true that $\sqrt{16} = 4$, *but* $\sqrt{x^{16}} = x^8$, *not* x^4. Choice (B). Remember, $4 \times 4 = 16$, but $x^4 \times x^4 = x^8$, *not* x^{16}. We shall review the operations with exponents in the next section.

3.3 It is easier to simplify each square root first, before you multiply. $\sqrt{200}\quad\sqrt{500} = \sqrt{100 \cdot 2}\quad\sqrt{100 \cdot 5} = 10\sqrt{2} \cdot 10\sqrt{5} = 10 \cdot 10\quad\sqrt{2} \cdot \sqrt{5} = 100\sqrt{2 \cdot 5} = 100\sqrt{10}$. Choice (A).

Note: If you do this problem by *estimation,* then $\sqrt{200}$ is approximately 14 and $\sqrt{500}$ is approximately 22. $14 \times 22 = 308$. In choice (B), $\sqrt{10}$ is approximately 3, so $10\sqrt{10}$ is approximately 30. Out! Choice (C) is also out of the ballpark completely. In choice (D), $\sqrt{7}$ is approximately 2.6, and $20\sqrt{7}$ is certainly not even close to 308. That choice is out. As for choice (E), $100\sqrt{7} = (100)(2 \cdot 6) = 260$. Closer than the others, but still out! So, the choice must be (A).

3.4 You can use estimation here too. But let's do it in the straightforward way. $\sqrt{162} - \sqrt{32} = ?$ First simplify $\sqrt{32}$. The factors of 32 will give you a clue as to how to simplify $\sqrt{162}$. (32 is a smaller number to work with.) $\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16}\,\sqrt{2} = 4\sqrt{2}$. So, in order to subtract, $\sqrt{162}$ must also have a factor of $\sqrt{2}$. $\sqrt{162} = \sqrt{81 \cdot 2} = \sqrt{81}\,\sqrt{2} = 9\sqrt{2}$. So, $\sqrt{162} - \sqrt{32} = 9\sqrt{2} - 4\sqrt{2} = 5\sqrt{2}$. Choice (B)

3.5 You can divide first to get $\dfrac{\sqrt{116}}{\sqrt{29}} = \sqrt{\dfrac{116}{29}} = \sqrt{4} = 2$. Choice (B)

Or, you can simplify where you can to get, $\dfrac{\sqrt{116}}{\sqrt{29}} = \dfrac{\sqrt{4 \cdot 29}}{\sqrt{29}} = \dfrac{\sqrt{4}}{1} \cdot \dfrac{\sqrt{29}}{\sqrt{29}} = \dfrac{2}{1} \cdot 1 = 2.$

3.6 Many students get this problem wrong because they are unsure of themselves when it comes to "canceling" in fractions. We shall review the operations on fractions in a later section. You can divide first to get $\dfrac{\sqrt{50} + \sqrt{32}}{\sqrt{2}} = \sqrt{25} + \sqrt{16} = 5 + 4 = 9$. Choice (E)

Or, you can simplify first to get $\dfrac{\sqrt{50} + \sqrt{32}}{\sqrt{2}} = \dfrac{\sqrt{25 \cdot 2} + \sqrt{16 \cdot 2}}{\sqrt{2}}$

$= \dfrac{5\sqrt{2} + 4\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}\,(5 + 4)}{\sqrt{2}} = \dfrac{\sqrt{2}}{\sqrt{2}}(5 + 4) = 1 \cdot 9 = 9.$

3.7 It is necessary to know how to multiply two binomials in order to solve this problem. We shall review this in Section 10. But, for now, $(a + b)(a - b) = a^2 - b^2$ (the difference of two squares). So, in this problem, $(\sqrt{2} - 1)(\sqrt{2} + 1) = \sqrt{2}^2 - 1^2 = \sqrt{2}\,\sqrt{2} - 1 = \sqrt{4} - 1 = 2 - 1 = 1$. Choice (B)

3.8 There is no easy way of dividing 1 by $\sqrt{5}$. All you can do is to change the way the fraction looks without changing its value. Multiplying the numerator and the denominator by $\sqrt{5}$, we get $\dfrac{1}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{1 \cdot \sqrt{5}}{\sqrt{25}} = \dfrac{\sqrt{5}}{5}$. Choice (A). You can test to see whether or not two fractions are equal by testing their "cross-products." We shall review this in the section on fractions later in the book. If $\dfrac{a}{b} = \dfrac{c}{d}$, then

$ad = bc$. Example: $\frac{3}{5} = \frac{9}{15}$, then $(3)(15) = (5)(9)$. Try this method with all the answers in problem 3.8. Is it true that $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{1}$, choice (E)? "Cross-multiply" to get $(1)(1) \stackrel{?}{=} (\sqrt{5})(\sqrt{5}) = \sqrt{25} = 5$. Since $1 \neq 5$, choice (E) is out. You do the other 4.

3.9 An irrational number (when working with square roots) is a square root of a non-perfect square. $\sqrt{169} = 13$; $9\sqrt{9} = 9 \times 3 = 27$; $\sqrt{32}\sqrt{2} = \sqrt{64} = 8$; $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$; so these are all *rational* numbers. $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2} = 5 \cdot (1.414213562\ldots)$. This product is an irrational number. The square root of 2 is a decimal that *never* "repeats." So, the answer is choice (D).

3.10 Here is another "bugaboo" among students. So many of them believe that $\sqrt{x^2 + y^2} = x + y$. What about this one? $\sqrt{3^2 + 4^2}$. Is it $3 + 4$? Well, $3^2 + 4^2 = 9 + 16 = 25$. So, $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$. Since $5 \neq 7$, something must be wrong. That something is the fact that $\sqrt{x^2 + y^2} \neq x + y$ for all values of x and y. However, what if either x *or* y or both x *and* y are 0? Then, if $x = 0$, $\sqrt{0^2 + y^2} = \sqrt{0 + y^2} = \sqrt{y^2} = y$. Since y can be written as $0 + y$, then it *is* true that $x^2 + y^2 = x + y$. Likewise, letting $y = 0$, you will come up with the result that $\sqrt{x^2 + y^2} = x + y$. Letting both x and y be 0, $\sqrt{0^2 + 0^2} = 0 + 0$, so it *is* true when both x and y are 0. Now, let's look at the choices.

(A) $x = y$. What if $x = 1$ and $y = 1$? Then $\sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$. $\sqrt{2} \neq 2$ (so $\sqrt{1^2 + 1^2} \neq 1 + 1$).
(B) $x = 0$. Yes, that works, *but*, read the directions carefully. The problem states, "which of the following *must* be true?" Watch out for that word "must." We have found out that x could be 0, but then again, it doesn't have to be. (y *could* be 0 *only* and $\sqrt{x^2 + y^2} = x + y$).
(C) *Must* it be true that $x - y$ can't be 0? What if both x and y were 0? Then $x - y$ does equal 0 and $\sqrt{x^2 + y^2} = x + y$.
(D) *Must* it be that $x + y = 0$? That only happens if x and y have opposite signs, such as 3 and -3. Let's try this one: $\sqrt{3^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18}$. So, does $\sqrt{18} = 3 + (-3)$? No.
(E) if $xy = 0$, then either x or y, or x and y, are 0. This *must* be the case if $\sqrt{x^2 + y^2} = x + y$. Choice (E).
Choices (B) and (E) look somewhat alike. Choice (B), however, is not inclusive enough. Also, why just $x = 0$? That is the giveaway. From the symmetry of the problem, why should only x have to be 0?

When you see "catchwords" like "must," or "never," or "always," be on your guard for that *one* case which "blows apart" the theory. For example, "All prime numbers are odd." Well, 3, 5, 7, 11, 13, 17, 19, 23, 29, and so on are all prime numbers (divisible only by themselves and 1), and in fact there are an infinite number of prime numbers, all odd. However, there is *one* even prime number, 2!

Section 4. Exponents

In general:

1. $(a^x)(a^y) = a^{x+y}$
2. $(a^x)^y = a^{xy}$
3. $a^x \div a^y = a^{x-y}$
4. $a^0 = 1$
5. if $a^x = a^y$, then $x = y$

Let's see some examples of the above rules.

1. $2^4 \times 2^2 = (2 \times \overset{2^4}{2} \times 2 \times 2)(\overset{2^2}{2 \times 2}) = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{4+2} = 2^6$

Note: Be careful! 2^4 does *not* mean $2 \times 4 = 8$. It means 2 is multiplied by itself 4 times; $2^4 = 16$. Likewise, $2^3 = 8$, *not* 6. Many students seem to make this careless error.

2. $(2^4)^2 = (2 \times \overset{2^4}{2} \times 2 \times 2)(2 \times \overset{2^4}{2} \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$.

Note: There seems to be much confusion over examples 1 and 2. In example 1, $2^4 \times 2^2$ means $16 \times 4 = 64$. In example 2, $(2^4)^2$ means 16^2 or 256.

3. $\underset{\substack{\downarrow \\ 32}}{2^5} \div \underset{\substack{\downarrow \\ 8}}{2^3} = \dfrac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} = 2 \times 2 = \underset{\substack{\downarrow \\ 4}}{2^2} = 2^{5-3}$

$2^5 \div 2^5 = \dfrac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = 1 = 2^{5-5} = 2^0$

4. (see above example) $2^0 = 1$.
5. If $2^5 = 2^x$, then obviously $x = 5$.

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

	Column A	Column B	
4.1	9^2	3^4	A B C D
4.2	2^{5-1}	$2^5 - 1$	A B C D
4.3	$x \geq 0$ $(x^{10})^2$	x^{100}	A B C D
4.4	$8^{2x} = 4^{3y}$ x	y	A B C D

4.5 If $n = 1$, then $3^n + 3^{n+1} =$

(A) 216 (B) 81 (C) 27 (D) 18 (E) 12

4.6 $7 \times 7 \times 7 \times 7 =$

(A) 4^7 (B) 7^4 (C) 14^2 (D) 4^3 (E) 49^2

4.7 $(2^0)(9^1) =$

(A) 9 (B) 18 (C) 2 (D) 1 (E) 0

4.8 If $a^x = b^{2y}$ and $b^y = a^{3z}$, then $z =$

(A) $\frac{x}{6}$ (B) $\frac{x}{2}$ (C) $\frac{2x}{3}$ (D) $\frac{5x}{6}$ (E) $\frac{x}{3}$

4.9 If $(3^{x-1}) \cdot (3^{2x+1}) = 27^y$, then $y =$
(A) 0 (B) 1 (C) x (D) $3x$ (E) $\frac{2x}{3}$

4.10 If $2^r \div 2 = 2^{r+1} \div 2^{2r}$, then $2^{r-1} =$
(A) 0 (B) 1 (C) 2 (D) 4 (E) 8

ANSWERS AND EXPLANATIONS

4.1 (C)	4.2 (B)	4.3 (D)	4.4 (C)	4.5 (E)
4.6 (B)	4.7 (A)	4.8 (A)	4.9 (C)	4.10 (B)

4.1 $9^2 = (3^2)^2 = 3^4$, so the quantities in Column A and Column B are equal. Choice (C). Since the exponents are small, you can get each value: $9^2 = 81$; $3^4 = 81$.

4.2 $2^{5-1} = 2^4 = 16$. $2^5 - 1 = 32 - 1 = 31$. So the quantity in Column B is greater. Choice (B).

4.3 $(x^{10})^2 = (x^{10})(x^{10}) = x^{20}$. So, it *seems* that x^{100} is the greater quantity. *However,* we do not know what number x is! We shall use the TECHNIQUE of *substitution.* You must be careful using this technique. You must make *several* substitutions rather than one, otherwise you will be led to an incorrect conclusion. If $x = 1$, $x^{20} = 1^{20} = 1$. Likewise, $1^{100} = 1$. So the quantities *could* be equal. If $x = 2$, $2^{20} < 2^{100}$. If $x = 1/2$, then $(1/2)^{20} > (1/2)^{100}$. (The value of a fraction that is between 0 and 1 *decreases* as it is raised to higher and higher powers. Examples: $(1/3)^2 = 1/9$; $(1/3)^3 = 1/27$. Therefore, since *no decision* can be made concerning the relationship of the quantities, we must mark choice (D).

4.4 We could do this problem by trying some numbers for x and y. Let $x = y = 1$. $8^{2 \cdot 1} \stackrel{?}{=} 4^{3 \cdot 1}$. Does $8^2 = 4^3$? $8 \times 8 \stackrel{?}{=} 4 \times 4 \times 4$. $64 = 64$. Yes! So, x *must* equal y. *But,* this was very lucky! Suppose we had a problem where it turned out that x was equal to $\frac{2y}{3}$. Then it is doubtful that you would have been so lucky when substituting your numbers. Do this problem in a straightforward manner:
Given, $8^{2x} = 4^{3y}$; $(2^3)^{2x} = (2^2)^{3y}$; $2^{6x} = 2^{6y}$; so, $x = y$. Choice (C).

4.5 $3^1 + 3^{1+1} = 3^1 + 3^2 = 3 + 9 = 12$. Choice (E). Why those other particular answers? Perhaps you thought (*ERROR*) $3^1 + 3^2 = (3 + 3)^3 = 6^3 = 216$. Or perhaps you thought (*ERROR*) $3^1 + 3^2 = 3^3 = 27$.

4.6 By definition, $A \times A \times A \times A$ means A^4. $7 \times 7 \times 7 \times 7 = 7^4$. Choice (B).

4.7 $(2^0)(9^1) = (1)(9) = 9$. Choice (A). Once again, it is very easy to be careless. Always raise to powers first, then multiply. Also remember $2^0 = 1$, not 2, or 0.

4.8. Do this in the straightforward way. If $b^y = a^{3z}$, then $b^{2y} = (b^y)^2 = (a^{3z})^2 = a^{6z}$. Since $a^x = b^{2y}$, then by substitution, $a^x = a^{6z}$, so $x = 6z$ and $z = \frac{x}{6}$. Choice (A). This problem is of medium difficulty and requires a thorough knowledge of the rules of exponents.

4.9 $(3^{x-1})(3^{2x+1}) = 3^{x-1+2x+1} = 3^{3x}$. $27^y = (3^3)^y = 3^{3y}$. Therefore, $3^{3x} = 3^{3y}$, so $x = y$. Choice (C).

4.10 $2^r \div 2$ is equivalent to $2^r \div 2^1 = 2^{r-1}$. $2^{r+1} \div 2^{2r} = 2^{r+1-2r} = 2^{1-r}$. Under the conditions of the problem, then, $2^{r+1} = 2^{1-r}$. Therefore, $r - 1 = 1 - r$. Adding r to both sides of the equation, $2r - 1 = 1$. Now adding 1 to both sides, $2r = 2$, so $r = 1$. So, $2^{r-1} = 2^{1-1} = 2^0 = 1$. Choice (B). Were you careless?

Section 5. Fractions

$$\frac{\text{N}}{\text{D}} \quad \begin{matrix}\text{(numerator)}\\ \text{(denominator)}\end{matrix}$$

Addition: Like Denominators

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Examples: $\frac{2}{3} + \frac{5}{3} = \frac{2+5}{3} = \frac{7}{3}$

$\frac{x}{y} + \frac{3x}{y} = \frac{x+3x}{y} = \frac{4x}{y}$

Subtraction: Like Denominators

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Examples: $\frac{4}{7} - \frac{1}{7} = \frac{4-1}{7} = \frac{3}{7}$

$\frac{x}{3} - \frac{1-x}{3} = \frac{x-(1-x)}{3} = \frac{x-1+x}{3} = \frac{2x-1}{3}$

Addition: Unlike Denominators

$$\frac{a}{m} + \frac{b}{n} = \frac{an+bm}{mn}$$

Examples: $\frac{2}{3} + \frac{4}{5} = \frac{(2)(5)+(3)(4)}{15} = \frac{10+12}{15} = \frac{22}{15}$

$\frac{3}{4} + \frac{7}{12} = \frac{(3)(12)+(4)(7)}{48} = \frac{36+28}{48} = \frac{64}{48} = \frac{4}{3}$

Or the more familiar way:

$$\begin{array}{rcl} \frac{3}{4} & = & \frac{9}{12} \\ + & & \\ \frac{7}{12} & = & \frac{7}{12} \\ \hline \end{array}$$

so,

$$\begin{array}{r} \frac{9}{12} \\ + \\ \frac{7}{12} \\ \hline \frac{16}{12} = \frac{4}{3} \end{array}$$

(So, even with the lowest common denominator, you can *still* "reduce.")

With only two fractions, using the formula $\frac{a}{m} + \frac{b}{n} = \frac{an + bm}{mn}$ is much quicker than seeking the "lowest common denominator" as taught in elementary school. (Actually, the formula is derived from the more familiar method: If you have to add $\frac{a}{m} + \frac{c}{n}$, multiply the left fraction by $\frac{n}{n}$ and the right fraction by $\frac{m}{m}$ to get $\frac{a}{m} \cdot \frac{n}{n} + \frac{b}{n} \cdot \frac{m}{m} = \frac{an}{mn} + \frac{bm}{mn} = \frac{an + bm}{mn}$.) A "quickie" way to add to fractions, then, is to use the following diagram;

$$\frac{3}{4} \quad \frac{5}{6} \quad \begin{matrix} 20 \\ 27 \end{matrix} \quad = \quad \frac{27 + 20}{36}$$

Subtraction: Unlike Denominators

$$\frac{a}{m} - \frac{b}{n} = \frac{an - bm}{mn}$$

Example: $\frac{2}{3} - \frac{5}{7} = \frac{(2)(7) - (5)(3)}{21} = \frac{14 - 15}{21} = \frac{-1}{21}$

Adding and Subtracting Mixed Numbers

A mixed number is an integer together with a fraction, such as 2¼, 3½, etc. To add or subtract mixed numbers, change them first to fractions, and then use the previous methods.

Examples: $3\frac{1}{2} + 4\frac{1}{4} = \frac{7}{2} + \frac{17}{4} = \frac{28 + 34}{8} = \frac{62}{8} = \frac{31}{4}$

$2\frac{3}{4} - 6\frac{2}{5} = \frac{11}{4} - \frac{32}{5} = \frac{55 - 128}{20} = -\frac{73}{20}$

Of course, by the more familiar way:

$$\begin{array}{rr} 2\frac{3}{4} & = \frac{15}{20} \\ -\ 6\frac{2}{5} & \frac{8}{20} \\ \hline -4 & \frac{7}{20} \end{array}$$

(which means $-4 + \frac{7}{20} = \frac{-80}{20} + \frac{7}{20}$

$= \frac{-73}{20}$

Remember, though, $-4\frac{7}{20}$ is WRONG. $-4\frac{7}{20}$ means $\frac{-87}{20}$ (that "pesky" negative sign makes the whole mixed number negative!)

Multiplication: Like and unlike Denominators

$$\left(\frac{a}{m}\right)\left(\frac{b}{n}\right) = \frac{ab}{mn}$$
$$\left(\frac{a}{c}\right)\left(\frac{b}{c}\right) = \frac{ab}{cc}$$

Examples: $\left(\frac{2}{3}\right)\left(\frac{5}{7}\right) = \frac{10}{21}$

$\left(\frac{3}{4}\right)\left(\frac{5}{4}\right) = \frac{15}{16}$

Division

$$\frac{a}{m} \div \frac{b}{n} = \frac{a}{m} \cdot \frac{n}{b} = \frac{an}{mb}$$

Examples: $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \cdot \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$

$2\frac{1}{2} \div 3\frac{1}{4} = \frac{5}{2} \cdot \frac{4}{13} = \frac{20}{26} = \frac{10}{13}$

Rules of Addition and Subtraction

1. To add two fractions with the same denominator, add the numerators *only*, and keep the common denominator.
2. To subtract two fractions with the same denominator, subtract the numerators *only*, and keep the common denominator.
3. To add two fractions with different denominators, change the fractions to equivalent forms in which they have the same denominator and then proceed as in Rule 1 above. (Or else use the faster symbolic method.)
4. To subtract two fractions with different denominators, change the fractions to equivalent forms in which they have the same denominator and then proceed as in Rule 2 above. (Or else use the faster symbolic method.)

Since the rules for adding and subtracting fractions seem so logical, we shall not dwell on them here. Only one silly error needs explaining. Some students add (or subtract, as the case may be) the denominators as well as the numerators. For example: $^3/_4 + {}^1/_4 = {}^4/_8$. *ERROR!*

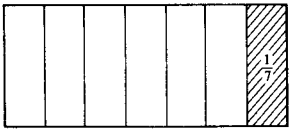
"Listen" to the problem. "Three-fourths plus one-fourth equals?" Yes, it equals four-fourths or 1, *not* $^4/_8$ or $^1/_2$.

Rules of Multiplication Explained

The rules for multiplication and division of fractions do not seem so logical to many students, so we will attempt to show their logic.

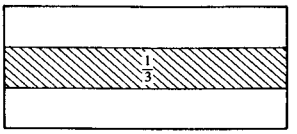
First of all, in most verbal problems, the word "of" denotes multiplication. For example, $^1/_3$ of $^1/_7$ means $^1/_3 \times {}^1/_7$. But what is $^1/_3$ of $^1/_7$? What does it "look like"? Each vertical strip below represents $^1/_7$ of the original region.

(A)

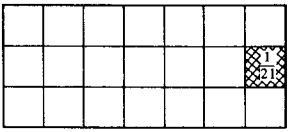


Each horizontal strip below represents $^1/_3$ of the original region.

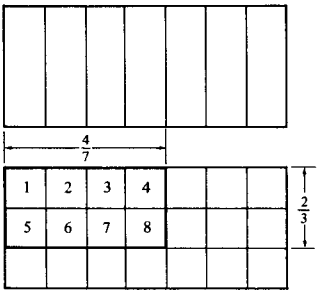
(B)



Now, if you superimpose (A) onto (B), you will get 21 equal regions.



That cross-hatched region represents $^1/_3$ of a $^1/_7$ strip, or, as you can see, $^1/_{21}$ of the entire region. It also represents $^1/_7$ of $^1/_3$ of the region. So, $^1/_3$ of $^1/_7$ is the same thing as $^1/_7$ of $^1/_3$. They both are equivalent to $^1/_{21}$. Using the same pictorial argument, $^2/_3$ of $^4/_7$ would be $^8/_{21}$.



Intuitively, then, $\frac{2}{3}$ of $\frac{4}{7}$ means $\left(\frac{2}{3}\right) \cdot \left(\frac{4}{7}\right) = \frac{(2)(4)}{(3)(7)} = \frac{8}{21}$. So, we have the multiplication postulate, $\left(\frac{a}{m}\right) \cdot \left(\frac{b}{n}\right) = \frac{ab}{mn}$.

Rules of Division Explained

It is probably the rule for division of fractions that remains most mysterious for generations of students. "When you *divide* two fractions, you *invert* the divisor and *multiply*."

1. If you multiply the numerator and the denominator of a fraction by the same number, nothing happens—that is, the value of the fraction remains unchanged. Example: $(^3/_4)(^5/_5) = {}^{15}/_{20}$, which is precisely $^3/_4$ again. Any number (and a fraction is certainly a number) multiplied by the number 1 is the same number. When you multiply the numerator and the denominator by the same number, like $^5/_5$, you are really multiplying by a form of the number 1. That is why $^3/_4 = {}^{15}/_{20}$. $^3/_4$ is also equal to $^{21}/_{28}$, if you multiply by $^7/_7$.
2. Now, what is the value of $^3/_4 \div {}^7/_8$? Using a parallel example, $6 \div 2$ can be

written as $^6/_2$. Likewise, $^3/_4 \div {}^7/_8$ can be written as $\frac{3/4}{7/8}$. In other words, we have a fraction whose numerator and denominator are both fractions.

3. We can multiply the numerator and the denominator of $\frac{3/4}{7/8}$ by any number we choose and it will still give us back the original value of $\frac{3/4}{7/8}$. So, let's multiply by $\frac{8/7}{8/7}$ (which is still 1). Why $\frac{8/7}{8/7}$? Well, let's see what happens.

$$\frac{3/4}{7/8} \cdot \frac{8/7}{8/7} = \frac{24/28}{56/56} = \frac{24/28}{1} = {}^{24}/_{28}$$

(Since any number divided by 1 is that number again.) So, the rule, invert and multiply, makes sense if you remember that what is really happening is that you are multiplying the numerator *and* the denominator by the inverted divisor!

Repeating the rule once again, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, $^3/_4 \div {}^7/_8 = (^3/_4)(^8/_7) = {}^{24}/_{28}$

"Reducing" Fractions

$^{24}/_{28} = (^2/_2)(^{12}/_{14}) = (1)(^{12}/_{14}) = {}^{12}/_{14}$. But, $^{12}/_{14} = (^2/_2)(^6/_7) = (1)(^6/_7) = {}^6/_7$. So, $^{24}/_{28}$ has been "reduced" to a form equivalent to $^{24}/_{28}$. It would have been faster to divide the numerator and the denominator of $^{24}/_{28}$ by 4 ($^{24}/_{28} = (^4/_4)(^6/_7) = (1)(^6/_7) = {}^6/_7$) rather than dividing by 2 and then by 2 again. So, the whole idea in reducing fractions is to try to find the *largest* divisor of the numerator and the denominator of the fraction that is to be reduced.

Perhaps you realized that all we are doing here is using the rule for multiplication, $\frac{ac}{bc} = \left(\frac{a}{b}\right)\left(\frac{c}{c}\right) = \left(\frac{a}{b}\right)(1) = \frac{a}{b}$.

When adding fractions or subtracting fractions with unlike denominators we had to change to equivalent forms. Perhaps we could call that process "gaining" rather than "reducing." For example, $^3/_4 = {}^{27}/_{36}$. That is, $(^3/_4)(^9/_9) = {}^{27}/_{36}$. $^3/_4$ is also equal to $^{63}/_{84}$, since $(^3/_4)(^{21}/_{21}) = {}^{63}/_{84}$. In fact, any fraction can be written in an infinite number of equivalent forms. So, when confronted with: $^3/_4 + {}^5/_6 + {}^1/_3 - {}^5/_8$ it is obvious we have to change all those denominators to some number that is divisible by 4, 6, 3, and 8. Well,

$$\begin{aligned} 4 &= 2 \times 2 \\ 6 &= 2 \times 3 \\ 3 &= \quad\;\; 3 \\ 8 &= 2 \times 2 \times 2 \end{aligned}$$

What number contains those prime factors (without duplication)? In order for 4 to divide into our number we will need at least 2×2. In order for 6 to divide into our number we will need a 2 and a 3 as factors. But we *already* have a 2 (from the factors of 4). So, our number has to have at least $2 \times 2 \times 3$ in order to be divisible by 4 and 6. Do we need another 3 in order for our third number, 3, to divide into our number? No, we have one already. Now, how about 8? 8 will not divide into $2 \times 2 \times 3$ unless we "tack on" another 2. So our final common denominator will be $2 \times 2 \times 3 \times 2$, or 24.

$$\frac{3}{4} = \frac{?}{24} \qquad \frac{5}{6} = \frac{?}{24} \qquad \frac{1}{3} = \frac{?}{24} \qquad \frac{5}{8} = \frac{?}{24}$$

We must have multiplied 4 by 6 in order to get 24 in our first fraction. So we have to multiply 3 by that same 6 to write ¾ in its equivalent form:

$(^3/_4)(^6/_6) = {}^{18}/_{24}$. Instead of wasting our time thinking, "What did we multiply 4 by in order to get 24?" we could have asked the equivalent question, "24 divided by 4 is what?" In fact, this is what most students do. They say, "4 into 24 is 6. 6 times 3 = 18." So, $^3/_4 = {}^{18}/_{24}$. That's why it is so important to be able to find a common denominator—one that is *divisible* by all the others. Therefore,

$$\frac{3}{4} + \frac{5}{6} + \frac{1}{3} - \frac{5}{8} = \frac{18}{24} + \frac{20}{24} + \frac{8}{24} - \frac{15}{24} = \frac{18 + 20 + 8 - 15}{24} = \frac{31}{24}$$

"Canceling"

Canceling and reducing are synonymous. Remember, though, you are really using the multiplication rule, $\frac{ac}{bc} = \left(\frac{a}{b}\right)\left(\frac{c}{c}\right)$. Many, many careless errors are made because students forget this fact.

Examples: $\frac{2x + 6y}{2} = \left(\frac{2}{2}\right)\left(\frac{x + 3y}{1}\right) = (1)\left(\frac{x + 3y}{1}\right) = x + 3y$

ERROR So many students are overenthusiastic cancelers. That is, they slash out things as long as they are on both top and bottom. That, in fact, is exactly how they explain what they are doing: "If you see a number on top and bottom of a fraction, just cross it out." So, in the very same problem, they do the following: $\frac{\cancel{2}x + 6y}{\cancel{2}} = x + 6y$.

The important thing to remember is that you can only cancel when you have a *product* in the numerator and a *product* in the denominator *both* of which contain the same factor(s). You *can* cancel in the situation above, but only after changing the numerator to a product:

$$\frac{2x + 6y}{2} = \frac{\cancel{2}(x + 3y)}{\cancel{2}} = x + 3y.$$

Here is another example, showing how a quadratic expression can be simplified so that you end up with factors you can cancel (see Section 10 for the formulas for simplifying quadratics):

$$\frac{x^2 - 5x + 6}{x - 3} = \frac{\cancel{(x - 3)}(x - 2)}{\cancel{(x - 3)}(1)} = \frac{x - 2}{1} = x - 2$$

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

5.1 Of the following fractions, which is the greatest?

(A) $^3/_4$ (B) $^7/_8$ (C) $^7/_{12}$ (D) $^2/_3$ (E) $^{19}/_{24}$

5.2 If $J = \dfrac{1}{\frac{1}{p} + \frac{1}{q}}$ when $p = 1$ and $q = {}^1/_3$, then $J =$

(A) $^1/_4$ (B) $^1/_2$ (C) $^3/_4$ (D) 2 (E) $^4/_3$

5.3 If $\dfrac{1}{1 + \dfrac{t}{1+t}} = 1$, then $t =$

(A) 0 (B) −1 (C) −2 (D) 1 (E) 2

5.4 If $p = 3q/8$, $q = 2n/3$, then $p =$

(A) $\frac{n}{4}$ (B) $\frac{9n}{16}$ (C) $4n$ (D) $\frac{25n}{24}$ (E) $2n$

5.5 $\dfrac{3^1/_4 + 3^1/_4 + 3^1/_4 + 3^1/_4}{4} =$

(A) $3^1/_4$ (B) $4^1/_2$ (C) $6^1/_2$ (D) $8^1/_4$ (E) 3

5.6 At North High School, 9 out of 10 seniors enter college. 4 out of 5 of these get their degrees, and 1 out of 4 of these go on to postgraduate work. What fraction of North High School's seniors go on to postgraduate work?

(A) $^3/_{10}$ (B) $^{17}/_{30}$ (C) $^{20}/_{39}$ (D) $^9/_{50}$ (E) $^1/_6$

	Column A		*Column B*	
		$\frac{1}{x} < 1$		
5.7	0		x	A B C D
		$y \neq 0$		
5.8	$1 - \frac{x}{y}$		$\frac{y - x}{y}$	A B C D

5.9 $\dfrac{x^2y^2 - 1}{xy - 1} =$

(A) $xy + 1$ (B) $xy - 1$ (C) xy (D) 1 (E) $1 - xy$

5.10 If $^2/_3$ of $^3/_5$ of x is equal to 6, then $^1/_3$ of x is equal to

(A) 2 (B) 5 (C) 9 (D) 12 (E) 15

ANSWERS AND EXPLANATIONS

5.1 (B)	5.2 (A)	5.3 (A)	5.4 (A)	5.5 (A)
5.6 (D)	5.7 (D)	5.8 (C)	5.9 (A)	5.10 (B)

5.1 Change all the fractions to equivalent forms with the same denominator. $^3/_4 = {}^{18}/_{24}$; $^7/_8 = {}^{21}/_{24}$; $^7/_{12} = {}^{14}/_{24}$; $^2/_3 = {}^{16}/_{24}$; $^{19}/_{24} = {}^{19}/_{24}$. If fractions have the same denominator, then the one with the greatest numerator is the greatest fraction. Therefore, choice (B)

5.2 Substitute $p = 1$ and $q = {}^1/_3$. J $= \dfrac{1}{\dfrac{1}{1} + \dfrac{1}{^1/_3}}$

$\frac{1}{1/3} = \left(\frac{1}{1}\right)\left(\frac{3}{1}\right) = 3$. (In fact, 1 "over" any fraction is that fraction "inverted.")
So, $J = \frac{1}{1+3} = \frac{1}{4}$. Choice (A)

5.3 The easiest way to this problem is to substitute each answer choice until a true statement occurs. Trying (A)—that is, substituting 0 for t—we get, $\frac{1}{1+\frac{0}{1+0}} = \frac{1}{1+\frac{0}{1}} = \frac{1}{1+0} = \frac{1}{1} = 1$. We were lucky in that our first substitution produced the correct answer. Choice (A).

If you wish to do the problem algebraically; $\frac{1}{1+\frac{t}{1+t}} = \frac{1}{\frac{1}{1}+\frac{t}{1+t}}$. In order to add the two fractions in the denominator, you need a common denominator. What will (1) and $(1 + t)$ divide into? $(1)(1 + t)$ or just $1 + t$. So, $\frac{1}{1} = \left(\frac{1}{1}\right)\left(\frac{1+t}{1+t}\right) = \frac{1+t}{1+t}$. So, we are down to $\frac{1}{\frac{1+t}{1+t}+\frac{t}{1+t}} = \frac{1}{\frac{1+t+t}{1+t}} = \frac{1}{\frac{1+2t}{1+t}}$. Since we are told that this fraction = 1, we can set up the equation $\frac{1}{\frac{1+2t}{1+t}} = \frac{1}{1}$, "cross-multiplying," $(1)(1) = \left(\frac{1+2t}{1+t}\right)(1)$ or $\frac{1+2t}{1+t} = 1 = \frac{1}{1}$. "Cross-multiplying" once again, $(1 + 2t)(1) = (1 + t)(1)$; therefore, $1 + 2t = 1 + t$. Subtracting t from both sides of the equation, $1 + t = 1$. Subtracting 1 from both sides of the equation, $t = 0$.

If there were no answers given to this problem, then you would have had to do this problem algebraically. (And what if the answer weren't so simple?) But nevertheless, if the answers are given, then sometimes the best way to approach the problem is by simple substitution of all the answers until one works.

5.4 If $p = {}^3/_8(q)$, then $p = ({}^3/_8)({}^2/_3)(n)$. So, $p = ({}^6/_{24})(n)$ or $({}^1/_4)(n)$. Choice (A).

Note: It is easier to multiply two fractions *after* "canceling," as the numbers are smaller to work with: $\left(\frac{3}{8}\right)\left(\frac{2}{3}\right)(n) = \frac{\cancel{3}\times\overset{1}{\cancel{2}}}{\underset{4}{\cancel{8}}\times\cancel{3}}\,n = \frac{1}{4}n$.

How about "making up" some of your own numbers for this problem? Let $q = 8$. Then $p = \left(\frac{3}{8}\right)(8)$, which is just 3. Then $8 = \frac{2}{3}(n)$. Well, $\frac{8}{1} = \frac{2(n)}{3}$; cross-multiply; $(8)(3) = (1)(2n)$, so $24 = 2n$ and $n = 12$. So, since $q = 8$, we are led to the fact that $p = 3$ and $n = 12$. The question asks, $p = ?$ Well, trying all the answers once again (remembering your "made-up" numbers), $3 = ?$ Look at choice (A), $\frac{n}{4}\cdot\frac{n}{4} = \frac{12}{4}$ using your numbers! So choice (A) is the correct choice.

5.5 You *can* add $3^1/_4 + 3^1/_4 + 3^1/_4 + 3^1/_4$ to get 12 and ${}^4/_4$ or $12 + 1$ or 13. Then you must divide 13 by 4 in order to get the correct answer, $3^1/_4$! ($3^1/_4 = {}^{13}/_4$) *However,* if you know the distributive postulate, $a(b + c + d + e) = ab + ac + ad + ae$, then you would realize that $3^1/_4 + 3^1/_4 + 3^1/_4 + 3^1/_4 = 3^1/_4(1 + 1 + 1 + 1) = 3^1/_4(4)$. Therefore, $\frac{3^1/_4 + 3^1/_4 + 3^1/_4 + 3^1/_4}{4} = \frac{\overset{1}{\cancel{4}}(3^1/_4)}{\underset{1}{\cancel{4}}} = 3^1/_4$. Choice (A).

5.6 "9 out of 10" means $\frac{9}{10}$, likewise for the other fractions. And "of" means "multiply." So, $\left(\frac{1}{4}\right)\left(\frac{4}{5}\right)\left(\frac{9}{10}\right) = \frac{1 \times 4 \times 9}{4 \times 5 \times 10} = \frac{9}{50}$. Choice (D).

5.7 If $\frac{1}{x} < 1$, then what *could* x be? Try *several* numbers. If $x = 3$, then the statement is true. So, it would seem that x could be any positive integer. $\frac{1}{3} < 1$, $\frac{1}{8} < 1$, etc. Therefore, the quantity in Column B *would* be the greater quantity. *However,* perhaps we did not try enough numbers. What if $x = -5$? Then $\frac{1}{x} = \frac{1}{-5} < 1$, also. So *now,* the quantity in Column A is the greater quantity. So, we must mark choice (D). No relationship can be determined. Be careful!

5.8 $\frac{1}{1} - \frac{x}{y} = \left(\frac{1}{1}\right)\left(\frac{y}{y}\right) - \frac{x}{y} = \frac{y}{y} - \frac{x}{y} = \frac{y - x}{y}$. Choice (C). Again, you could have substituted some numbers. Let $x = 2, y = 3$. Then the quantity in Column A is $1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} \neq \frac{1}{3}$. The quantity in Column B is $\frac{3 - 2}{3} = \frac{1}{3}$. The reason for the statement that $y = 0$ is that a fraction cannot have 0 in its denominator.

5.9 Be careful, you "gung-ho" cancelers, otherwise you will probably cross out some x's and y's as well as some 1's, and put down choice (C) as your answer. A *lot* of students do that! You *can* cancel but only when $xy - 1$ is a factor of the numerator. It is, because $x^2y^2 - 1 = (xy + 1)(xy - 1)$. See Section 10 if you have forgotten how to simplify quadratic expressions. Therefore, $\frac{(xy + 1)\overset{1}{\cancel{(xy - 1)}}}{\underset{1}{\cancel{(xy - 1)}}} =$ $xy + 1$ or choice (A). Substitution of some numbers would work well here too. Let $x = 2$, $y = 3$. Then, $\frac{2^2 \cdot 3^2 - 1}{(2)(3) - 1} = \frac{(4)(9) - 1}{6 - 1} = \frac{36 - 1}{5} = \frac{35}{5} = 7$. Which answer is 7 when you substitute 2 and 3 for x and y? Choice (A), of course!

5.10 $\left(\frac{2}{3}\right)\left(\frac{3}{5}\right)\left(\frac{x}{1}\right) = 6$. $\frac{2 \cdot 3 \cdot x}{3 \cdot 5 \cdot 1} = 6$. $\frac{2x}{5} = \frac{6}{1}$; "cross-multiply," $(2x)(1) = (5)(6)$; $2x = 30$, $x = 15$. Therefore, $\frac{1}{3}$ of $15 = 5$. Choice (B). There is another method that requires no factoring: $\frac{2}{3} \cdot \frac{3}{5} \cdot x = \frac{6x}{15} = \frac{6}{1}$. Dividing both sides by 6, $\frac{x}{15} = \frac{1}{1}$; and multiplying both sides by 15, $x = 15$.

Section 6. Decimals

Addition and Subtraction

To add numbers in decimal form, line up the decimal points and add just as if you were adding whole numbers.

Example: 5.7 + .003 + 45.06 =

```
  5.7
   .003
 45.06
 ------
 50.763
```

To subtract two numbers in decimal form, line up the decimal points and subtract just as if you were subtracting whole numbers.

Example: 27.96 − 8.588 =

```
 27.960
  8.588
 ------
 19.372
```

Note: The addition of zeros after the last digit after the decimal point does not change the value of the number: 27.96 = 27.960. Also, .3 = .30 = .300. A whole number is actually a decimal number, but the decimal point is "imaginary": 5 = 5., in other words.

Multiplication

To multiply two numbers in decimal form, multiply the numbers just as if they were whole numbers. Then count the number of decimal places (from right to left) in each of the two numbers and find that sum. That number (the sum) is the number of places to count (from right to left again) before placing the decimal point in the product, or the answer.

Example: (35.87)(5.114) =

```
    35.87   (2 places) +
    5.114   (3 places)
   ------
    14348
    3587
  3 587
 179 35
 ---------
 183.43918  (5 places)
```

Note: Sometimes, zeros must be added to account for some places.

```
   5.91   (2 places) +
   .011   (3 places)
  ------
    591
   591
  ------
 .06501   (5 places)
```

To multiply a decimal number by 10, 100, 1,000, etc., just move the decimal point 1, 2, 3, etc., places to the *right*, adding zeros as you need to.

Examples: $(3.4)(100) = 340.$ or 340
$(.007)(10) = .07$
$(65)(1000) = (65.)(1000) = 65000.$ or 65000

Perhaps a little justification of the multiplication rules are in order.

$$\frac{3}{10} \times \frac{2}{100} = \frac{3 \times 2}{10 \times 100} = \frac{6}{1000}$$
$$.3 \times .02 \qquad\qquad = .006$$
(1 place) (2 places) (3 places)

Division

Any division problem can be put into fraction form.

Examples: $56.34 \div 4 = \frac{56.34}{4}$

$28.4 \div .04 = \frac{28.4}{.04}$

$18 \div 4.5 = \frac{18}{4.5}$

$28 \div 16 = \frac{28}{16}$

All you have to remember, then, is one rule. Multiply the numerator and the denominator of these fractions by some power of 10 ($10^0 = 1$, $10^1 = 10$, $10^2 = 100$, etc.) which is determined solely by the number of places the decimal point must be moved in the denominator in order to make it a whole number.

Examples (same as above):

$56.34 \div 4 = \frac{56.34}{4}$ Since the denominator is already a whole number, just divide $4\overline{)56.340}$ (the decimal point in the quotient is located directly above the decimal point in the dividend).

$28.4 \div .04 = \frac{28.4}{.04}$ To make the denominator into a whole number, it is necessary to multiply it by 100 (move the decimal point 2 places to the right). Therefore, to *not* change the value of the fraction, you must multiply the numerator by the same power of 10, namely 100. That is, you must move the decimal point 2 places to the right in the numerator also.

So, $\frac{28.4}{.04} = \left(\frac{28.4}{.04}\right)\left(\frac{100}{100}\right) = \frac{2840}{4} = 4\overline{)2840.}$ $710.$ or just 710.

$$\begin{array}{r} 28 \\ \hline 4 \\ 4 \\ \hline 0 \end{array}$$

$18 \div 4.5 = \frac{18}{4.5}$ In this case, you have to multiply both numerator and denominator by 10. (Move the decimal point 1 place to the right.) So, $\frac{18}{4.5} = \frac{180.}{45} = 45\overline{)180}$ 4

$$\underline{180}$$

$28 \div 16 = \frac{28.}{16.}$ Since the denominator is a whole number, just divide.

```
      1.75
16) 28.00
    16
    12 0
    11 2
       80
       80
```

Therefore, the one rule takes into account four types of division problems involving decimals:

1. When the dividend (numerator) only is a decimal.
2. When the dividend and the divisor (denominator) are both decimals.
3. When the divisor only is a decimal.
4. When neither dividend nor divisor is a decimal (strictly speaking, we mean a decimal when you *must* have the decimal point).

To Change Any Fraction to a Decimal

Any fraction can be converted into its decimal equivalent by dividing the denominator into the numerator.

Example: $\frac{7}{50} =$

```
      .14
50) 7.00
    5 0
    2 00
    2 00
```

Some decimal equivalents occur so frequently that they should be memorized.

$^1/_8 = .125$
$^1/_6 = .1666666 \ldots$
$^1/_5 = .2$
$^1/_4 = .25$
$^1/_3 = .3333333 \ldots$
$^1/_2 = .5$
$^2/_3 = .6666666 \ldots$
$^2/_5 = .4$
$^3/_4 = .75$
$^4/_5 = .8$
$^7/_8 = .875$

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

6.1 $0.0707 \div 0.7 =$

(A) 0.00101 (B) 0.0101 (C) 0.101 (D) 0.111 (E) 1.010

6.2 Of the following, which is the smallest?

(A) $\frac{1}{.5}$ (B) $(.5)^2$ (C) $\frac{.5}{5}$ (D) .5 (E) .5 − .005

6.3 $4.67 \times 1000 =$

(A) .00467 (B) .0467 (C) .467 (D) 4670 (E) 46.700

6.4 If $0.02x - 0.3 = 0.015 + 0.011x$, then $x =$

(A) 2.4 (B) 35 (C) 1.76 (D) 240 (E) 0.035

6.5 $\sqrt{0.0009} =$

(A) 0.03 (B) 0.003 (C) 0.0003 (D) 0.3000 (E) 3.0000

	Column A		*Column B*	
6.6	$\frac{1}{10}$ of 0.5		$\frac{1}{20}$	A B C D
6.7	$0.2 + 0.8$		$2.1 - 0.1$	A B C D
		$x > y$		
6.8	$.3x$		$.5y$	A B C D
6.9	$0.9 + 0.09 + 0.0009$		0.999	A B C D
		$1.5x = 3$		
6.10	$10x$		200	A B C D

ANSWERS AND EXPLANATIONS

6.1 (C)	6.2 (C)	6.3 (D)	6.4 (B)	6.5 (A)
6.6 (C)	6.7 (B)	6.8 (D)	6.9 (B)	6.10 (B)

6.1 $.0707 \div .7 = .707 \div 7 = \begin{array}{r} .101 \\ 7\overline{).707} \end{array}$. Choice (C).

6.2 $\frac{1}{.5} = \frac{10}{5} = 2$; $(.5)^2 = (.5)(.5) = .25$; $\frac{.5}{5} = .1$; $.5 = .5$; $.5 - .005 = \begin{array}{r} .500 \\ -\ .005 \\ \hline .495 \end{array}$

Thus, comparing 2, .25, .1, .5, and .495, we see the smallest of these decimals is .1. Choice (C).

6.3 Move the decimal point 3 places to the right when multiplying by 1000 (3 zeros; 3 places to the right). So, $4.67 \times 1000 = 4670$. Choice (D).

6.4 If you don't like decimals, get rid of them! How? By multiplying both sides of the equation by 1000. (We use 1000, because the "worst" decimal is .011. To make that one a whole number we have to move the decimal point 3 places to the right.) So, $.02x - .3 = .015 + .011x$ is equivalent to $20x - 300 = 15 + 11x$; sub-

tracting $11x$ from both sides of the equation, $9x - 300 = 15$. Adding 300 to both sides of the equation, $9x = 315$. Dividing both sides now by 9, we get $x = 35$. Choice (B).

Of course you *could* have done this problem by substituting each of the answers into the original equation. It would have been very messy and time-consuming, though. Another method is to work *with* the decimals for a while: First subtract $.011x$ from both sides to get $.02x - .011x - .3 = .015$. Then add .3 to both sides to get $.02x - .011x = .3 + .015$. So,

$$\begin{array}{r} -\ .020x \\ .011x \\ \hline .009x \end{array} \quad \text{and} \quad \begin{array}{r} +\ .300 \\ .015 \\ \hline .315 \end{array}$$

$$\text{So, } x = \frac{.315}{.009} = \frac{35.}{9\overline{)315.}}$$

It is usually *easier* to multiply both sides of an equation by some power of 10 in order to "clear" *all* the decimals before you start "manipulating" the equation.

6.5 Which of the answers multiplied by itself will give you .0009?

$$\begin{array}{r} \times .03 \\ .03 \\ \hline .0009 \end{array}$$

Choice (A).

6.6 You can do this problem by changing everything to decimals or everything to fractions: $\frac{1}{10}$ of $0.5 = \left(\frac{1}{10}\right)\left(\frac{5}{10}\right) = \frac{1 \times 5}{10 \times 10} = \frac{5}{100}$; $\frac{1}{20} = \left(\frac{5}{5}\right)\left(\frac{1}{20}\right) = \frac{5 \times 1}{5 \times 20} = \frac{5}{100}$. So, the quantities are equal. Choice (C). Working with decimals now, $\frac{1}{10}$ of $0.5 = .1 \times .5 = .05$; $\frac{1}{20} = \frac{.05}{20\overline{)1.00}}$. Again, choice (C).

6.7 Do it simply. $.2 + .8 = \begin{array}{r} +\ .2 \\ .8 \\ \hline 1.0 \end{array}$ $\qquad 2.1 - .1 = \begin{array}{r} -\ 2.1 \\ .1 \\ \hline 2.0 \end{array}$

So, the quantity in Column B is greater. Choice (B).

6.8 Be careful here! Let $x = 10, y = 9$. Then $.3(10) = 3$, $.5(9) = 4.5$; $.3x < .5y$.
Let $x = 0, y = -10$. Then $.3(0) = 0$, $.5(-10) = -5$;
$.3x > .5y$.

It's those pesky negative numbers that seem to cause so much trouble! So, no decision can be reached as to the relationship. Choice (D).

6.9 Simple arithmetic:

$$\begin{array}{l} .9 \\ .09 \\ .0009 \\ \hline .9909 \text{ which is smaller than } .999 \end{array}$$

Choice (B).

6.10 If $1.5x = 3$, then $15x = 30$ and $x = 2$. So, $10x = 20$, which is less than 200. Choice (B).

Section 7. Percent

$x\%$ is equivalent to $\frac{x}{100}$. It is easy to remember this because the % symbol can be thought of as 0/0, 2 zeros and a 1 (yes, a "slanty" 1).

There are three basic types of percent problems, and they cause mass confusion and hysteria because they all "sound" alike.

1. 3% of what number is 9?
2. 9% of 3 is what number?
3. 9 is what % of 36?

Before we "tackle" these problems it is necessary to know how to change a decimal number into a % and vice versa.

$$.43 = \frac{43}{100} = 43\%$$

$$.8 = \frac{8}{10} = \frac{80}{100} = 80\%$$

$$2.4 = 2 + .4 = \frac{20}{10} + \frac{4}{10} = \frac{24}{10} = \frac{240}{100} = 240\%$$ (So, percents *can* be greater than 100!)

To change a decimal number to a %, move the decimal point 2 places to the right and "tack on" the % sign.

More examples:
$.05 = 5\%$
$.545 = 54.5\%$
$6 = 600\%$
$.001 = .1\%$

To change a % to a decimal, then, do the reverse! Move the decimal point 2 places to the left and remove the % symbol.

Examples:
$25\% = 25.\% = .25$
$1\% = 1.\% = .01$
$1/2\% = .5\% = .005$
$125\% = 125.\% = 1.25$
$8\,1/4\% = 8.25\% = .0825$

To change a fraction to a percent, first change it to a decimal, then move the decimal point 2 places to the right and "tack on" the % symbol.

Examples:
$1/4 = .25 = 25\%$
$7/8 = .875 = 87.5\%$
$1/3 = \frac{.3333}{100} = 33\,1/3\%$
$2/3$ is then $66\,2/3\%$

Back to our original three problems now. The key to solving these problems is to think of the word "multiply" when you see the word "of," think of the word "equals" when you see the word "is," and think of some variable, say, x, when you see the word, "what."

1. 3% of what number is 9?

Translation: $(.03)(x) = 9$.

So, $.03x = 9$. Multiply both sides of the equation by 100 to remove the decimal point. $3x = 900$; $x = 300$. So, 3% of 300 is 9.

2. 9% of 3 is what number?

Translation: $.09 \cdot 3 = x$

So, $.27 = x$. Therefore, 9% of 3 is .27.

3. 9 is what % of 36?

Translation: 9 = x% × 36 (remember, "of" with percents means "multiply").

So, dividing both sides of the equation by 36, we get $^9/_{36} = x\%$; $^1/_4 = x\%$; $.25 = x\%$; $25\% = x\%$; $x = 25$.

Therefore, 9 is 25% of 36.

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

7.1 If 60% of A is 20% of B, then B is what % of A?

(A) 3% (B) 25% (C) 30% (D) 250% (E) 300%

7.2 If $87^1/_2$% of Jack's weekly salary is $140.00, and if 25% of his weekly salary goes into his savings account, how much money does Jack have left for spending money?

(A) $40.00 (B) $80.00 (C) $100.00 (D) $120.00 (E) $160.00

7.3 Frank paid $6.40 for a shirt after receiving a discount of 20% off the list price. What was the list price before the sale?

(A) $6.60 (B) $7.00 (C) $7.40 (D) $8.00 (E) $9.20

7.4 A merchant sells all his articles so that he makes a profit of 15% of his cost. If he sells an article for $287.50, what was the cost to the merchant?

(A) $200.00 (B) $215.00 (C) $250.00 (D) $272.50 (E) $287.35

7.5 Working on an assembly line, Janet's job is to reject .05% of the parts that come before her. If she rejects 4 parts, how many parts did she examine?

(A) 8 (B) 80 (C) 800 (D) 8000 (E) 80000

7.6 What percent of $^1/_2$ is $^3/_4$?

(A) 100% (B) 120% (C) 125% (D) 135% (E) 150%

7.7 If a man's salary is increased from $400.00 to $600.00, what was the percent of increase in his salary?

(A) 20% (B) 25% (C) 30% (D) 50% (E) 60%

	Column A	*Column B*	
7.8	16% of 80	80% of 16	A B C D
7.9	$^1/_4$%	.025	A B C D

7.10 What percent of a is b?

(A) $\frac{b}{a}$ (B) $\frac{a}{b}$ (C) $\frac{100b}{a}$ (D) $\frac{100a}{b}$ (E) $\frac{a}{100b}$

ANSWERS AND EXPLANATIONS

7.1 (E)	7.2 (D)	7.3 (D)	7.4 (C)	7.5 (D)
7.6 (E)	7.7 (D)	7.8 (C)	7.9 (B)	7.10 (C)

7.1 60% of A is 20% of B. Translation: $.60A = .20B$. B is what % of A? Translation: $B = (x\%)(A)$. From the first equation, dividing both sides by .20, we get $\frac{.60A}{.20} = B$, or $3A = B$, so $B = (3)(A)$ or $(300\%)(A)$. Choice (E).

7.2 87½% of x is 140. Translation: $(.875)(x) = 140$. Multiply both sides of the equation to get rid of the decimal: $875x = 140000$. Divide both sides by 875: $875\overline{)140000}$ (160). So, Jack's weekly salary is \$160.00. Now, 25% of that weekly salary is his savings, so $(.25)(160) = x$; $x = 40$. Therefore, he has left, $160 - 40 = 120$. Choice (D).

Note: Sometimes, it is much easier to use the fractional equivalents of decimals in problems like these. We shall now do the problem again by this method.

$$87\tfrac{1}{2}\% \text{ of } x = 140;\ \frac{7x}{8} = \frac{140}{1}.\ \text{"Cross-multiply,"}\ 7x = 1120;$$

Divide, $x = 160$. Now, 25% of 160 = savings. $\frac{1}{4}(160)$ = savings; 40 = savings. $160 - 40 = 120$.

7.3 Let x = list price. If Frank received a discount of 20% off the list price, then he paid 80% of the list price. That 80% represents \$6.40. So, $.80x = 6.40$. Get rid of the decimals: $80x = 640$. Divide: $x = 8$. So, the list price was \$8.00 or choice (D). You could have worked backwards from the answers fairly simply in this problem. Take 20% off 6.60; $.20(6.60) = 1.32$. $6.60 - 1.32 = 5.28$. So, choice (A) is out, etc.

7.4 Let the cost of the article be x. Then his profit is $.15x$. Selling price = cost + profit, therefore $287.50 = x + .15x = 1.15x$. Multiply both sides of the equation by 100 (move the decimal points 2 places to the right) to get "rid" of the decimals: $28750 = 115x$. Divide both sides of the equation by 115 to get 250 or choice (C). Once again, you could have worked this problem using the answers. Multiply each answer by 1.15 until you reach the required selling price of \$287.50.

7.5 Be careful! .05% = .0005, *not* .05. The problem states that Janet rejects .05% of the parts. Therefore, letting x = parts examined: $(.05\%)(x) = 4$; $(.0005)(x) = 4$ $.0005x = 4$; $5x = 40000$; $x = 8000$. Choice (D).

7.6 What percent of ½ is ¾?

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ x & \% & \cdot\ \tfrac{1}{2} & = \tfrac{3}{4}? \end{array}$$

$$(x\%)(\tfrac{1}{2}) = \tfrac{3}{4}$$

$$\frac{x\%}{2} = \frac{3}{4}$$

"Cross-multiplying," $4x\% = 6$. Dividing by 4, $x\% = \frac{6}{4} = 1.5$. So, $x\% = 150\%$, $x = 150$. Choice (E).

7.7 Percent increase $= \dfrac{\text{amount of increase}}{\text{original amount}}$

$$\text{Percent increase} = \frac{600 - 400}{400} = \frac{200}{400} = \frac{1}{2} = 50\%. \text{ Choice (D).}$$

7.8 (.16)(80) = (.80)(16) because the digits in the products are exactly the same and the number of decimal places in the products are exactly the same, 2. Choice (C).

7.9 ¼% is equivalent to .25%, which equals .0025. Therefore, the quantity in Column B is greater. Choice (B).

7.10 What % of a is b?

$$\begin{array}{cccccc} \downarrow & \downarrow & \downarrow & & \downarrow & \\ x & \% & \cdot & a & = & b? \end{array}$$

$(x\%)(a) = b$

$x\% = \dfrac{b}{a}$. So, it "looks like" choice (A).

However, we must find x, not $x\%$. So,

$$(x\%)(a) = b; x\% = \frac{b}{a}$$

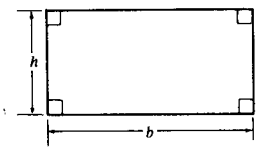
$$\frac{x}{100} = \frac{b}{a}$$

$$xa = 100b$$

$$x = \frac{100b}{a}. \text{ Choice (C).}$$

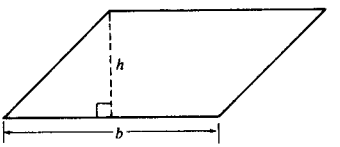
Section 8. Areas, Perimeters, and Volumes

The area of a rectangle is the product of its base and its height.



$A = bh$

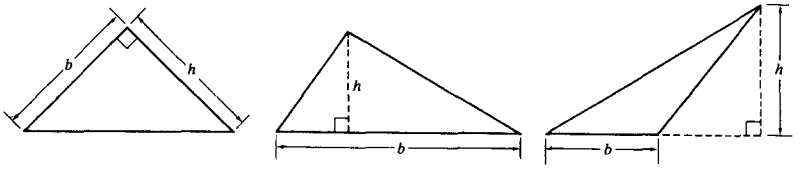
The area of a parallelogram is the product of its base and its height.



$A = bh$

Note: Be careful here! The height *must* be perpendicular to the base. The height of a parallelogram is not one of its sides, unless, of course, that parallelogram happens to be a rectangle.

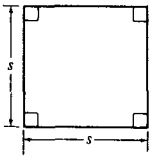
The area of any triangle is equal to $^1/_2$ the product of a base and a height (drawn to that base).



$A = {}^1/_2\, bh$ $\qquad$ $A = {}^1/_2\, bh$ $\qquad$ $A = {}^1/_2\, bh$

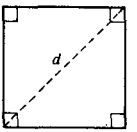
In a right triangle, either leg can be thought of as the base. Then the other leg is the height. Remember, a triangle has 3 bases and 3 heights. Use the most convenient pair.

The area of a square is (side)^2



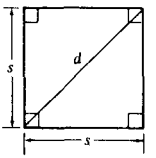
$A = s^2$

The area of a square is also equal to $^1/_2$ (diagonal)^2



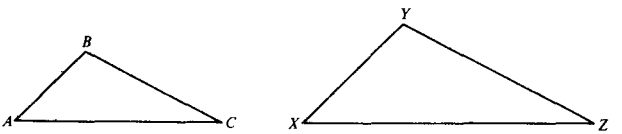
$A = {}^1/_2\, d^2$

Note: There is a very simple proof for this.



In the square, $d^2 = s^2 + s^2 = 2s^2$ Therefore, $s^2 = {}^1/_2 d^2$. Since s^2 is the area of the square, $^1/_2\, d^2$ is also the area of the square.

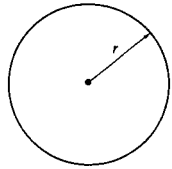
The ratio of the area of two similar triangles is equal to the ratio of the squares of corresponding sides, heights, medians, or angle bisectors.
Example:



If triangle ABC is similar to triangle XYZ, then:

$$\frac{AB^2}{XY^2} = \frac{\text{area of triangle } ABC}{\text{area of triangle } XYZ}$$

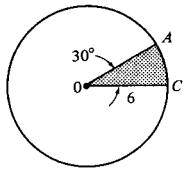
The area of a circle is equal to the product of π and the (radius)^2.



$A = \pi r^2$
(a radius is $^1/_2$ the length of a diameter)

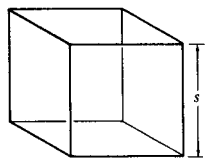
The area of a sector of a circle is equal to a fractional part of the area of the whole circle. That fractional part can be determined by taking the ratio:

$$\frac{\text{central angle}}{360°}$$



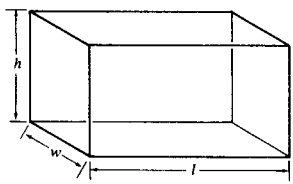
In the preceding diagram, central angle AOC of circle O is 30°. Radius $OC = 6$. The area of the whole circle is $\pi 6^2 = \pi 36$ or 36π. Since $\frac{30°}{360°} = \frac{1}{12}$, then the area of the sector is $\frac{1}{12}$ the area of the whole circle, or $\frac{1}{12} \cdot 36\pi = 3\pi$.

The area of a cube is the sum of the areas of all 6 faces (which are each squares). Therefore, if the area of one square face is s^2, then the surface area of the cube is $6s^2$.

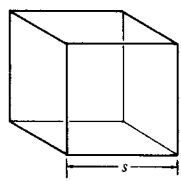


$SA = 6s^2$

The volume of a cube is s^3. The volume of any rectangular solid is length × width × height. Since each of these dimensions are the same for a cube, the volume is just s^3.



$V = l \times w \times h$



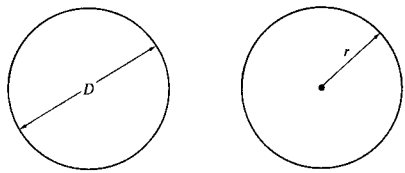
$V = s^3$

The perimeter of a plane geometric figure is simply the "distance around it."

The perimeter of a square, therefore, is $4s$, $(s + s + s + s)$.

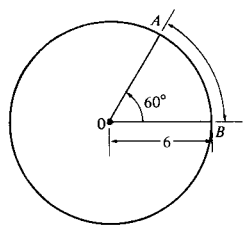
The perimeter of a triangle is $s_1 + s_2 + s_3$ (the sum of its sides).

The perimeter of a circle is called its circumference. The circumference of a circle is equal to the product of π and the diameter (remember, a diameter is twice a radius, so the formula can be written as $C = \pi 2r$ or $2\pi r$).



$C = \pi D$ $\qquad$ $C = 2\pi r$

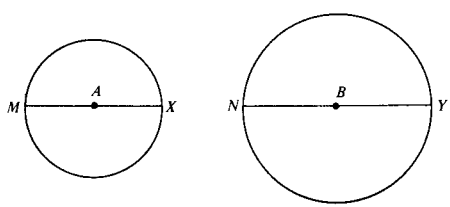
To find the arc length of a sector, find the circumference of the circle and multiply it by the fractional part of the circle that the central angle intercepts.



Example: $r = 6$,
$\measuredangle AOB = 60°$

$$C = 2\pi \cdot 6 = 12\pi. \qquad \frac{60°}{360°} = \frac{1}{6} \qquad \text{Length of arc } AB \text{ is } \frac{1}{6}\,12\pi = 2\pi.$$

The ratio of the areas of two circles is equal to the ratio of the squares of their radii, their diameters, or their circumferences.

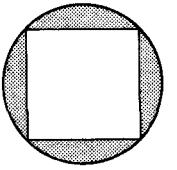


$$\frac{\text{area of circle } A}{\text{area of circle } B} = \frac{AX^2}{BY^2} = \frac{MX^2}{NY^2} = \frac{(\text{circumference of circle } A)^2}{(\text{circumference of circle } B)^2}$$

You will find numerical examples in the practice exercises which follow.

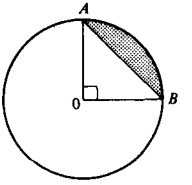
PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.



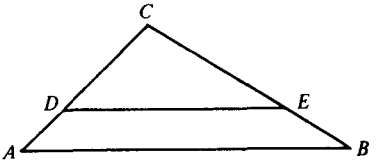
8.1 In the figure above, the perimeter of the square is 40. The area of the shaded region is

(A) $100 - 50\pi$ (B) $100 + 50\pi$ (C) $200 - 50\pi$ (D) $50\pi - 100$
(E) $100\pi - 200$



8.2 Find the area of the shaded region if $OB = 6$ in the figure above.

(A) $18 - 9\pi$ (B) $18\pi - 9$ (C) $9\pi - 18$ (D) $9 - 18\pi$ (E) $9\pi + 18$



8.3 The area of the triangle CDE above is 12. If $DE \parallel AB$, $DE = 4$, and $AB = 6$, then the area of $ADEB$ is

(A) 10 (B) 12 (C) 15 (D) 18 (E) 27

8.4 What is the area of a circle whose circumference is 4π?

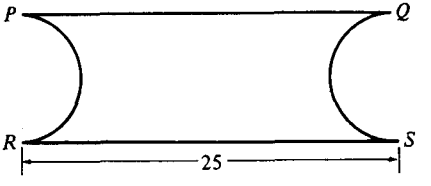
(A) 4π (B) 8π (C) 12π (D) 16π (E) 24π

8.5 The perimeter of an equilateral triangle is 18. What is the area of this triangle?

(A) 18 (B) $9\sqrt{2}$ (C) $9\sqrt{3}$ (D) $18\sqrt{2}$ (E) $18\sqrt{3}$

8.6 How many tiles $3'' \times 3''$ are needed to completely cover a floor that is $9' \times 12'$?

(A) 12 (B) 108 (C) 144 (D) 1728 (E) 2130

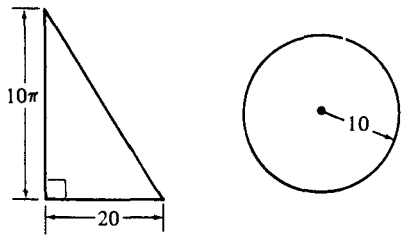


8.7 Each of the semicircles above has diameter 8. $PQ \parallel RS$. The area of the figure is

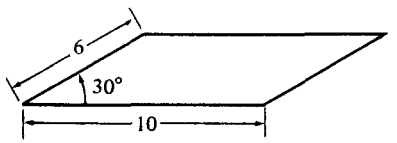
(A) $200 - 64\pi$ (B) $100 - 16\pi$ (C) $100 - 8\pi$ (D) $200 - 16\pi$ (E) $200 - 8\pi$

	Column A	*Column B*	
	d = diameter of circle A		
	r = radius of circle B		
8.8		d	A B C D

Note: Figures not drawn to scale



8.9	Area of the triangle	Area of the circle	A B C D



8.10 The area of the parallelogram above is

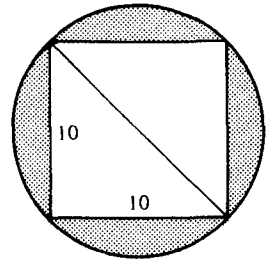
(A) 20 (B) 30 (C) 50 (D) 60 (E) 120

ANSWERS AND EXPLANATIONS

8.1 (D)	8.2 (C)	8.3 (C)	8.4 (A)	8.5 (C)
8.6 (D)	8.7 (D)	8.8 (D)	8.9 (C)	8.10 (B)

8.1 Practically all of the "shaded region" problems on the SAT are done by subtraction. The clue, in fact, is in the answers. See the minus signs? In this problem, the area of the shaded region is equal to the area of the circle minus the area of the square.

In order to get the area of the square we need a side (or a diagonal), and in order to get the area of the circle we need the radius (or the diameter). Since we are told that the perimeter of the square is 40, then each side must be 10, since $10 + 10 + 10 + 10 = 40$. So the area of the square is 100. From this fact alone, and looking at the answers, it is fairly obvious that the answer is (D). However, let's work out the solution. Since the diameter of a circle is also the diagonal of any inscribed square, we can find PQ by using the Pythagorean Theorem. $10^2 + 10^2 = PQ^2$; $100 + 100 = PQ^2$; $200 = PQ^2$; $\sqrt{200} = PQ$; $\sqrt{100 \cdot 2} = PQ$; $10\sqrt{2} = PQ$. (You can reach this point more quickly if you remember that in an



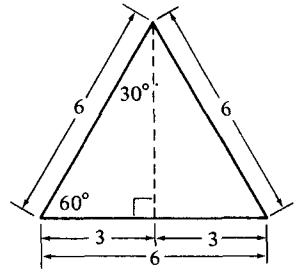
isosceles right triangle, the hypotenuse is always $\sqrt{2} \times$ one of the legs.) Since the diameter is $10\sqrt{2}$, the radius is $5\sqrt{2}$. The area of a circle is πr^2. So in this case, the area of the circle is $\pi(5\sqrt{2})^2 = \pi(5\sqrt{2})(5\sqrt{2}) = \pi 25\sqrt{4} = \pi\, 25 \cdot 2 = \pi 50$ or 50π. So the area of the shaded region is $50\pi - 100$, or choice (D).

8.2 Most problems dealing with "shaded area" involve subtraction of areas. The area of the shaded region in this one is equal to the area of the square − area of the triangle. The area of the square is 6×6 or 36. To find the area of the triangle, ECF, we need to know $\overline{CE}$ and $\overline{CF}$. Since $\overline{BC} = 6$ (all sides of a square are equal in length) and $\overline{BE} = 3$, $\overline{CE}$ is also equal to 3. Since $\overline{DF} = 2$, $\overline{FC} = 6 - 2 = 4$. So the area of $\Delta ECF = \frac{1}{2}(4)(3) = \frac{1}{2}(12) = 6$. The area of the shaded region is $36 - 6$ or 30. Choice (C).

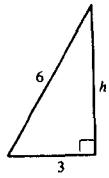
8.3 Area of $ADEB$ = area of triangle ABC − area of triangle CDE. We are told the area of triangle $CDE = 12$. All we have to do is find the area of triangle ABC. Since $DE \parallel AB$, $\measuredangle A = \measuredangle CDE$ and $\measuredangle B = \measuredangle CED$ (corresponding angles). Therefore, the triangles are similar and their corresponding sides are in proportion. The ratio of the area of two similar triangles is equal to the *squares* of the ratio of corresponding sides, so, $\frac{4^2}{6^2} = \frac{\text{area of triangle } CDE}{\text{area of triangle } ABC}$. $\frac{16}{36} = \frac{12}{x}$; $\frac{4}{9} = \frac{12}{x}$; "cross-multiplying," $4x = 108; x = 27$. So the area of $ADEB$ is $27 - 12 = 15$. Choice (C).

8.4 To get the area of a circle we need the radius or the diameter. Since we are told that the circumference is 4π, according to the formula $C = \pi d$, then the diameter is 4 so the radius is 2. The area of a circle is πr^2, so the area of this circle is $\pi 2^2 = \pi \cdot 4$ or 4π. Choice (A). It's quite *unusual* for the area and the circumference to have the same numerical value. What if the diameter of the circle was any other number but 4? Could the area equal the circumference in any other case?

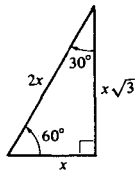
8.5 Let's draw a diagram:



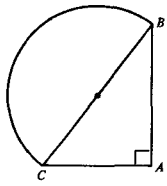
Since the perimeter of this triangle is 18, each of the equal sides is 6. If you remember the formula for the area of an equilateral triangle, $A = \frac{s^2}{4}\sqrt{3}$, then the area is $\frac{6^2}{4}\sqrt{3} = \frac{36}{4}\sqrt{3} = 9\sqrt{3}$. Choice (C). If you forget that formula, then use the formula $A = \frac{1}{2}$ (base)(height). Since the base is 6, all we have to do now is find the height. By the Pythagorean Theorem



$3^2 + h^2 = 6^2$; $9 + h^2 = 36$; $h^2 = 27$; $h = \sqrt{27} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$. So the area of the triangle is $\frac{1}{2}(6)(3\sqrt{3}) = \frac{1}{2}(18\sqrt{3}) = 9\sqrt{3}$. You could have *avoided* using the Pythagorean Theorem if you remembered the 30° – 60° – 90° right triangle relationships:



8.6 In a rectangular floor 9′ × 12′ there are 108 square *feet*. Each square foot contains 144 square inches (*not* 12 square inches). So there are 108 × 144 = 15552 square inches in the floor. Each 3″ × 3″ tile contains 9 square *inches*. So, $\frac{15552}{9} = 1728$ tiles. Choice (D).

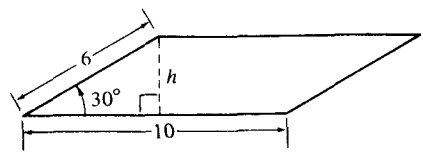


8.7 We already know $\overline{AC} = 3$ and $\overline{AB} = 4$. We need the length of semicircle BC. Since the circumference (perimeter) of a circle is $2\pi r$ we need r. But $r = \frac{1}{2}\overline{BC}$. By the Pythagorean theorem $\overline{BC}^2 = \overline{AC}^2 + \overline{AB}^2 = 3^2 + 4^2 = 9 + 16 = 25$. So $\overline{BC} = \sqrt{25} = 5$ and $r = \frac{5}{2}$. So the circumference of the *whole* circle is $2\pi r = 2\pi\left(\frac{5}{2}\right) = \frac{10\pi}{2} = 5\pi$. But *remember*, we only have $\frac{1}{2}$ a circle. So, $\widehat{AC} = \frac{5\pi}{2}$ and the answer is $3 + 4 + \frac{5}{2}\pi = 7 + \frac{5}{2}\pi$. Choice (D). Notice choice (A) $7 + 5\pi$. This is a trap for students who forget it is a semicircle, not a whole circle.

8.8 The diameter of a circle is always greater than its radius. However, what if we are dealing with two *different* circles? Assume that the diameter of circle A is 10 and the radius of circle B is 10. Or, the radius of circle B is 50 and the diameter of circle A is 20. So, no relationship can be determined from the given information. Choice (D).

8.9 The area of the right triangle is $\frac{1}{2}(10\pi)(20) = \frac{1}{2}(200\pi) = 100\pi$. The area of the circle is $\pi 10^2 = \pi 100 = 100\pi$. So the quantities in both columns are equal. Choice (C).

8.10 See the diagram below:



The area of this parallelogram is $10 \times 3 = 30$, *not* $10 \times 6 = 60$. Be careful! The height is 3 because it is the side of a 30° − 60° − 90° right triangle that is opposite the 30° angle. So, it is $\frac{1}{2}$ the hypotenuse; $\frac{1}{2}(6) = 3$. The area of a parallelogram *diminishes* as soon as it ceases to be a rectangle. Once it is "shoved over" its area decreases (its height decreases while its base remains the same).

Section 9. Signed Numbers

The absolute value of a number is that number *without* its sign.

Examples: The absolute value of +3 is 3
The absolute value of −3 is 3
The absolute value of 0 is 0

Addition

If the signs of the two numbers are the *same*, add their absolute values and use the same sign in the sum.

Examples: $-5 + -7$ Add 5 and 7 to get 12. Then use the − sign. So, $-5 + -7 = -12$.

$+5 + +7$ Add 5 and 7 to get 12. Then use the + sign. So, $+5 + +7 = +12$. *Note:* In actual use, the + sign before a number is "understood." $+5 + +7$ is actually written as just $5 + 7$.

If the signs are *different, subtract* the absolute values of the numbers and use the sign of the number which had the greater absolute value.

Examples: $-5 + 2$ "Forgetting" the signs, always subtract the smaller from the greater. $5 - 2 = 3$. The absolute value of −5 is 5. The absolute value of 2 is 2. So, "tack on" the − sign in the difference. $-5 + 2 = -3$.

$5 + -8$ $8 - 5 = 3$. The absolute value of −8 is 8. The absolute value of 5 is 5. Use the − sign. $5 + -8 = -3$

$5 + -2$ $5 - 2 = 3$. The absolute value of 5 is 5. The absolute value of −2 is 2. Use the + sign (actually, no sign) $5 + -2 = 3$.

$0 + -3$ $3 - 0 = 3$. The absolute value of −3 is 3. The absolute value of 0 is 0. Use the − sign. $0 + -3 = -3$. (Or, simply, "0 + *any* number is that number itself").

Subtraction

Change the sign of the "second" number and follow the rules for addition.

Examples: $-5 - -7$ is the same as $-5 + +7 = 2$
$-5 - +7$ is the same as $-5 + -7 = -12$
$+5 - +7$ is the same as $5 + -7 = -2$
$+5 - -7$ is the same as $5 + +7 = 12$

Perhaps a little "gimmick" would come in handy here. Whenever you see two like signs together, circle them and put a + symbol over the circle. Whenever you see two unlike signs together, circle them and put a − symbol above the circle.

Examples:

$$5 \overset{-}{\textcircled{+-}} 7 = -2$$

$$5 \overset{-}{\textcircled{-+}} 7 = -2$$

$$-5 \overset{-}{\textcircled{+-}} 7 = -12$$

$$5 \overset{+}{\textcircled{--}} 7 = 12$$

$$-5 \overset{+}{\textcircled{--}} 7 = 2$$

Multiplication

The product of two numbers with the same sign is positive. The product of two numbers with different signs is negative.

Examples: $(-3)(-5) = 15$
$(-3)(\ 5) = -15$

Assuming that you know that $3 + {}^{-}3 = 0$ (any number plus its additive inverse is zero), $-3 \cdot 0 = 0$ (any number "times" $0 = 0$), the distributive postulate, $a(b + c) = ab + ac$, and the fact that $(-3)(3) = -9$ (because $(-3)(3)$ really means $-3 + -3 + -3 = -9$), then it's really not too hard to present an *intuitive* argument why "two negatives make a positive."

$$\begin{aligned} a(b + c) &= ab + ac \\ -3(3 + {}^{-}3) &= -3(3) + (-3)(-3) \\ -3(\ \ 0\ \ ) &= -3(3) + (-3)(-3) \\ 0 &= -9 + (-3)(-3) \end{aligned}$$

but, $0 = -9 + 9$, therefore $(-3)(-3)$ *must* be 9.

Division

The quotient of two numbers with the same sign is positive. The quotient of two numbers with different signs is negative.

Examples: $\dfrac{-12}{-6} = 2$

$\dfrac{-12}{6} = -2$

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

9.1 If $(3x - 5) - (2x - 2)$ is negative, then x could be

(A) 7 (B) 5 (C) 3 (D) 1 (E) none of these

9.2 If $(-3)^4(-2)^3 = (27)(2)(x)$, then $x =$

(A) -12 (B) -3 (C) 7 (D) 11 (E) 19

9.3 The average of 40, -6, 0, 50, and -10 is

(A) 12.4 (B) 14.8 (C) 22.6 (D) 44.2 (E) 48.4

9.4 Points $P,Q,R,$ and S are located at -4, -12, 3, and 14 on a number line. $(Q - P) - (R - S) =$

(A) -27 (B) -5 (C) -3 (D) 0 (E) 3

9.5 $(-1)^{51} - (-1)^{50} =$

(A) -2 (B) -1 (C) 0 (D) 2 (E) 101

9.6 The product of -3 and -9 is how much larger than the sum of -3 and -9?

(A) 11 (B) 15 (C) 18 (D) 25 (E) 39

	Column A		*Column B*	
9.7	$3x - 1$		$3x + 1$	A B C D
9.8	ac	$abc = 0$ $a > b$	bc	A B C D
9.9	xz	$x < y$	yz	A B C D
9.10	xy	$\frac{x}{y} < 0$	$-xy$	A B C D

ANSWERS AND EXPLANATIONS

9.1 (D)	9.2 (A)	9.3 (B)	9.4 (E)	9.5 (A)
9.6 (E)	9.7 (B)	9.8 (D)	9.9 (D)	9.10 (B)

9.1 The distributive postulate, $a(b - c) = ab - ac$, is needed here. When a negative sign precedes a pair of parentheses, you can assume a negative 1. For example, $-(2x - 2) = -1(2x - 2) = (-1)(2x) - (-1)(2) = -2x + 2$. Most students when confronted with a minus sign in front of a pair of parentheses just *change all* the signs of the terms that are being added or subtracted inside the

parentheses. That is O.K., and is a much shorter process. Therefore, $(3x - 5) - (2x - 2)$ is equivalent to $3x - 5 - 2x + 2$. Combining like terms, we get $3x - 2x - 5 + 2 = x - 3$. Since the only answer that makes $x - 3$ less than 0 is $x = 1$, mark choice (D). Obviously, in this problem, no algebra at all is needed. Just "plug in" each answer until a negative number "pops out." However, you should know something about the operations on *in*equalities. In this problem, $x - 3 < 0$. If you add +3 to both sides of the inequality, you get $x < 3$. There are an *infinite* number of replacements for x that will satisfy this "inequation." But, in this problem, we must choose from the given answers. In Section 10 of the *Review Guide* you will see some of the basic rules for working with inequalities.

9.2 Do *not* multiply 27 by 2 in this problem. It is unnecessary. $(-3)^4 = 81$; $(-2)^3 = -8$. Therefore, $(81)(-8) = (27)(2)x$. Divide both sides of the equation by 27 to get $(3)(-8) = 2x$. Divide both sides of the equation by 2 to get $(3)(-4) = x$; $x = -12$. Choice (A).

9.3 The average of a set of numbers is their sum divided by n, where n = the number of numbers in the set. The average of 40, −6, 0, 50, and −10 is $\frac{(40) + (-6) + (0) + (50) + (-10)}{5} = \frac{74}{5} = 14.8$. Choice (B).

9.4 $(Q - P) - (R - S) = (-12 - -4) - (3 - 14) = (-12 + 4) - (3 - 14) = (-8) - (-11) = -8 + 11 = 3$. Choice (E).

9.5 A negative number raised to an *even power is positive*. A negative number raised to an *odd power is negative*. $(-1)^{51} = -1$. $(-1)^{50} = +1$. Therefore, $-1 - +1 = -1 - 1 = -2$. Choice (A). Remember, 1 raised to any power is still 1.

9.6 The product of −3 and −9 is 27. The sum of −3 and −9 is −12. $27 - -12 = 27 + 12 = 39$. Choice (E).

9.7 Any number minus 1 is less than that same number plus 1. Try substituting some numbers. Let $x = -3$, 0, and +3. Compare! Choice (B).

9.8 If the product of any amount of numbers is 0, then *at least one of them must be 0*. Try some possibilities for a, b, and c. Remember, though, that a is greater than b.

a	b	c	ac	bc	abc
3	1	0	0	0	0
−2	−3	0	0	0	0
5	0	−6	−30	0	0

You might as well stop here. *ac could* equal *bc*. *ac could* be less than *bc*. In fact, if $a = 5$, $b = 0$, $c = 6$, then $ac = 30$, $bc = 0$, and ac could be greater than bc. So no relationship can be determined. Choice (D).

9.9 Try some numbers (remember $x < z$).

x	y	z	xz	yz
1	3	+2	2	6
1	3	−2	−2	−6

So, at this point, you can conclude that no relationship can be determined. Remember, $2 < 6$, but $-2 > -6$. Choice (D).

9.10 If $\frac{x}{y} < 0$, then x and y must have different signs. (See the rules for dividing signed numbers at the beginning of Section 9.) Therefore, the product, xy, is negative. Now, $-xy$ is equivalent to $-1(xy)$. Therefore, $-xy$ must be positive. Since a positive number is always greater than a negative number, mark choice (B).

Section 10. Simple Equations

In the booklet *About the SAT,* there is a statement that "the algebra includes linear equations, simple quadratic equations, factoring, and exponents, *but not the quadratic formula . . .*" That is why we call this section "Simple Equations," even though it does include simple quadratic expressions as well as linear equations.

Since there are many *types* of linear equations ("linear" means that no power of the unknown is greater than 1), we feel that the best way to get into solving equations is by presenting some examples.

Examples of Linear Equations

$$\text{if } x + 3 = 5, \text{ then } x = ?$$

Since 3 is "attached" to the unknown, x, by addition, you perform the inverse operation, subtraction, to "get rid" of it. The most important thing to remember in solving equations is that whatever you "do" to one side of an equation, you must do to the other side. $x + 3 = 5; x + 3 - 3 = 5 - 3; x + 0 = 5 - 3; x = 2$.

$$\text{if } 2x - 7 = 5, \text{ then } x = ?$$

Since 7 is "attached" to the term containing the unknown by subtraction, perform the inverse operation, addition, to "get rid" of it. $2x - 7 = 5$; $2x - 7 + 7 = 5 + 7$; $2x = 12$.

Now, since 2 is "attached" to the unknown by multiplication, perform the inverse operation, division, to "get rid" of it. $2x = 12; \frac{2x}{2} = \frac{12}{2}; x = 6$.

$$\text{if } \frac{3x + 1}{5} = 8 - 2x, \text{ then } x = ?$$

Since 5 is "attached" to the whole left side of the equation by division, we can "get rid" of it by the inverse operation of multiplication. Multiply both sides of the equation by 5:

$$\left(\frac{5}{1}\right)\frac{(3x + 1)}{5} = 5(8 - 2x); \frac{5(3x + 1)}{5} = 40 - 10x; 3x + 1 = 40 - 10x$$

Now, you have a choice. First of all, you decide on which side you want the unknown and on which side you want the known (that is, in this case, the

numbers). Assume that you want the unknown on the left side. So, since there is a $-10x$ on the right side you must add $10x$ to both sides in order to "get rid" of that $-10x$. $3x + 1 + 10x = 40 - 10x + 10x$; $13x + 1 = 40$. Now subtract 1 from both sides to get $13x + 1 - 1 = 40 - 1$; $13x = 39$. Divide both sides by 13, and you'll have the answer, 3. You can check your answer by substituting it into the original problem.

If you decide to put the unknown on the right side of the equation, then you can subtract $3x$ from both sides first to get $3x + 1 - 3x = 40 - 10x - 3x$; $1 = 40 - 13x$. Then subtract 40 from both sides to get $1 - 40 = 40 - 13x - 40$; $-39 = -13x$. Divide both sides by -13 to get the answer, 3. However, we feel that a little practice will show that fewer mistakes occur when you think a little before you start solving these equations. Try not to get involved with those pesky negative signs. Often, making a foresighted decision about which side you want the unknown on helps avoid that situation.

$$\text{if } \frac{3x + 1}{5} = 8 - 2x, \text{ then } x = ?$$

Yes, this is the same problem. But let's do it a quicker way, although the mathematical principles involved are really the same. Rewrite the problem as $\frac{3x + 1}{5} = \frac{8 - 2x}{1}$. Remember, in the section on fractions, we said that if $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$. So you can "cross-multiply" here. $(3x + 1)(1) = 5(8 - 2x)$; $3x + 1 = 40 - 1x$; etc. This is probably the best way to start a problem that looks like: $\frac{A}{B} = \frac{C}{D}$

$$\text{if } \frac{x}{3} + \frac{5x}{6} = 3.5, \text{ then } x = ?$$

You can add the fractions on the left side to get $\frac{2x}{6} + \frac{5x}{6}$ or $\frac{7x}{6}$. Then $\frac{7x}{6} = \frac{3.5}{1}$. Now, cross multiply to get $7x = 21$; $x = 3$. You could have multiplied each side of the equation by 6 to start with in order to "get rid" of all the fractions: $\frac{6}{1}\left(\frac{x}{3} + \frac{5x}{6}\right) = \left(\frac{6}{1}\right) 3.5$. This will be equivalent to $\frac{6x}{3} + \frac{30x}{6} = 21$; $2x + 5x = 21$; $7x = 21$; $x = 3$.

Then again, you could have started by "getting rid" of the decimal by multiplying both sides of the equation by 10: $\frac{10x}{3} + \frac{50x}{6} = 35$. Then, you can add the fractions first and then "cross-multiply" *or* you can multiply both sides of the equation by 6 in order to "clear" the denominators.

There are so many ways of solving a given equation that only by experience will you find the methods that are easiest and quickest for you.

Equations Involving Inequalities

Actually, we should call these *inequations*. For example, "What value(s) for x will satisfy the condition $5x - 3 > 12$?"

$A > B$ means, A is greater than B.

$A < B$ means, A is less than B.

An easy way to remember this is that the arrowhead points to the *smaller* quantity.

If the same number is added or subtracted on both sides of an inequality, the sense (direction) of the inequality remains *unchanged.*

Examples: $-2 > -9$; $-2 + 5 > -9 + 5$, $(3 > -4)$
$-2 < +9$; $-2 - 5 < +9 - 5$, $(-7 < 4)$

If both sides of an inequality are multiplied or divided by the same *positive* number, the sense of the inequality remains unchanged.

Examples: $5 > 2$; $(4)(5) > (4)(2)$, $(20 > 8)$
$12 < 30$; $\frac{12}{6} < \frac{30}{6}$, $(2 < 5)$

However: If both sides of an inequality are multiplied or divided by a negative number, the sense of the inequality is *reversed.*

Examples: $5 > 2$; $(-2)(5) < (-2)(2)$, $(-10 < -4)$
$-8 < +12$; $\frac{-8}{-4} > \frac{12}{-4}$, $(2 > -3)$

$A \geq B$ means that A is greater than *or* equal to B.

Examples: $6 \geq 4$
$6 \geq 6$

$A \leq B$ means that A is less than *or* equal to B.

Examples: $2 \leq 7$
$2 \leq 2$

$A < x < B$ means that x is less than B but greater than A. In other words, x is between A and B (not including either A or B).
Examples: $-2 < 0 < 5$
$1 < 8 < 12$

$a \ngtr B$ means that A is *not* greater than B. $5 \ngtr 7$

Two Equations in Two Unknowns

Sometimes we are given two relationships that exist between two different variables. For example, $3x + 4y = 10$, and $x - 3y = -1$. From the first equation, $3x + 4y = 10$, we can infer that there are an infinite number of solutions, because when $x = 0$, $y = 2.5$; when $x = 2$, $y = 1$; when $x = -2$, $y = 4$; etc. Likewise, there are an infinite number of solutions for the equation $x - 3y = -1$. However, there is only *one* common solution.

There are two ways to approach a set of equations such as these:

$$\begin{aligned} 3x + 4y &= 10 \\ x - 3y &= -1 \end{aligned}$$

Multiply both sides of the second equation by 3. Let's see what happens:

$$\begin{aligned} 3x + 4y &= 10 \\ 3x - 9y &= -3 \end{aligned}$$

Now subtract to "get rid" of one of the variables:

$$\begin{array}{rl} -\ 3x + 4y = & 10 \\ 3x - 9y = & -3 \\ \hline 13y = & 13 \\ y = & 1 \end{array} \qquad (4 - -9 = 13;\ 10 - -3 = 13)$$

Then of course you can find x by simple substitution in either of the two original equations.

Another method is to solve for one variable in terms of another and then to substitute that variable's equivalent into the other equation.

$$\begin{array}{r} 3x + 4y = 10 \\ x - 3y = -1 \\ \hline \end{array}$$

Solve for x in the *second* equation. (To solve for x or y in the first equation leads to fractions, and you know how we all love fractions! In the second equation, to solve for y, once again you will need to involve fractions.)

So, since $x - 3y = -1$, $x = 3y - 1$. Substitute for x in the "top" equation. Since $3x + 4y = 10$, then $3(3y - 1) + 4y = 10$; $9y - 3 + 4y = 10$; $13y - 3 = 10$; $13y = 13$; $y = 1$. Then find x by substituting 1 for y in either of the two original equations.

Which method to use depends on your experience and practice. Sometimes one method is easier than another depending on the set-up. For example, if you are given that

$$\begin{array}{r} 3x + 4y = 10 \\ 5x - 7y = 3 \\ \hline \end{array}$$

then the first method we described would be preferable.

Simplifying Quadratic Expressions

Although you will not be expected to use the quadratic formula, some questions may require factoring quadratic expressions. You have already seen a question of this type in Section 3.7. Remember the following three equations:

1. $a^2 + 2ab + b^2 = (a + b)(a + b)$
2. $a^2 - b^2 = (a - b)(a + b)$
3. $a^2 - 2ab + b^2 = (a - b)(a - b)$

Occasionally you will be faced with a situation where you must simplify an expression such as $x^2 - 5x + 6$ so that you can deal with its factors separately. $x^2 - 5x + 6 = (x - 3)(x - 2)$, as you can discover by a bit of trial and error. Not all quadratic expressions can be factored, of course.

Also remember that

1. $a^2b^2 = (ab)(ab)$
2. $\dfrac{a^2}{b^2} = \left(\dfrac{a}{b}\right)\left(\dfrac{a}{b}\right)$

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

10.1 If $5x - 3 > 12$, then

(A) $x^2 < 9$ (B) $x - 4 > 0$ (C) $2x + 1 > 7$ (D) $\frac{1}{x} > \frac{1}{3}$ (E) $-x > 3$

10.2 If $1 + 0.2x = 1.010$, then $x =$

(A) 0.5 (B) 0.05 (C) 0.005 (D) 0.0005 (E) 0.00005

10.3 If $x + y = A$ and $x - y = B$, then $x =$

(A) A (B) $2A + B$ (C) $2y$ (D) $\frac{B - A}{2}$ (E) $\frac{A + B}{2}$

10.4 If $ax = dx + g$, then $x =$

(A) $\frac{g}{a - d}$ (B) $\frac{d + g}{a}$ (C) $\frac{g - d}{a}$ (D) $\frac{g}{d - a}$ (E) $\frac{g + a}{d}$

10.5 If J is an integer and $0 < J < 20$, then how many integers can be expressed in the form $\frac{3J}{2}$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

10.6 If $\frac{1}{P} + \frac{1}{Q} = \frac{1}{R}$, then $P =$

(A) $R + Q$ (B) $\frac{RQ}{Q - R}$ (C) $R - Q$ (D) $\frac{Q + R}{RQ}$ (E) $Q - R$

10.7 If $\frac{a + b + c}{3} = \frac{a + b}{3} + 1$, then $c =$

(A) 1 (B) 2 (C) 3 (D) -6 (E) 4

	Column A		*Column B*	
		$\frac{x}{y} = \frac{4}{5}$		
10.8	$\frac{2x}{y}$		$\frac{y}{x}$	A B C D
		$\frac{1}{x} < 1$		
10.9	x		1	A B C D
		$2xy = 6xy - 3$		
10.10	x^2y^2		$\frac{5}{8}$	A B C D

ANSWERS AND EXPLANATIONS

10.1 (C)	10.2 (B)	10.3 (E)	10.4 (A)	10.5 (D)
10.6 (B)	10.7 (C)	10.8 (A)	10.9 (D)	10.10 (B)

10.1 Add 3 to both sides of the inequality: $5x - 3 + 3 > 12 + 3$; $5x > 15$. Divide both sides by 5: $x > 3$. Now look at all the answers. (A) $x^2 < 9$. Since we know that $x > 3$, then certainly $x^2 > 9$. So (A) is out! (B) $x - 4 > 0$ is equivalent to $x > 4$ (adding 4 to both sides). Well, if $x > 3$ then x *could be* > 4 but *it doesn't have to be!* For example, $3.5 > 3$, but $3.5 < 4$. $3.677 > 3$ but $3.677 < 4$. There are an infinite number of real numbers that are greater than 3 without being greater than 4! (C) Since $x > 3$, $2x + 1$ represents twice a number greater than 3 that is increased by 1. Twice *any* number greater than 3 gives a result which is greater than 6. Add 1, and the sum is always greater than 7. So our answer is choice (C). However, let's continue. (D) Since $x > 3$, $\frac{1}{x}$ is *smaller* than $\frac{1}{3}$. For example, $\frac{1}{4} < \frac{1}{3}$; $\frac{1}{3.1} = \frac{1}{\frac{31}{10}} = \frac{10}{31}$, which is smaller than $\frac{1}{3}$, etc. So (D) is out! (E) The negative of a number greater than 3 is obviously less than 3. $-5 < 3$, $-10 < 3$. In fact, any negative number is less than 3!

10.2 Subtract 1 from both sides of the equation to get $0.2x = .010$. Multiply by 100 to "get rid" of the decimals: $20x = 1.0$ or just 1. Divide by 20 to get $x = 20\overline{)1.00}$ (.05). Choice (B). Of course, you could have substituted in all the answers until one "clicked." But it would be a waste of time in this case. It is easier to do it directly.

10.3
$$\begin{array}{rl} & x + y = A \\ + & x - y = B \\ \hline & 2x \quad = A + B \\ & x \quad = \dfrac{A + B}{2} \end{array}$$
Choice (E).

$x + y = A$; $x - y = B$; substitute $y = A - x$ into the second equation to get $x - (A - x) = B$; $x - A + x = B$; $2x - A = B$; $2x = A + B$; $x = \frac{A + B}{2}$. Choice (E).

If you don't remember either of these two methods, then pick some numbers for x, y, A, and B such as $x = 1$, $y = 2$. Then, $x + y = 3$. So, $A = 3$. $x - y = 1 - 2 = -1$, so $B = -1$. Now look at all the answers. (A) A Well, does $x = A$? Does $1 = 3$? (A) is out! (B) $2A + B$; $2(3) + (-1) = 6 - 1 = 5$. So $x \neq 2A + B$. (C) $2y$ Does $x = 2y$? Does $1 = 2(2) = 4$? (C) is out! (D) $\frac{B - A}{2}$; $\frac{-1 - 3}{2} = \frac{-4}{2} = -2$. But $x = 1$, not -2. So (D) is out. (E) must be the answer! But let's check it anyway. $\frac{A + B}{2} = \frac{3 + (-1)}{2} = \frac{3 - 1}{2} = \frac{2}{2} = 1$.

10.4 Get every term containing the unknown, x, on one side of the equation. Since $ax = dx + g$, subtract dx from both sides to get $ax - dx = g$. So, if you are going to guess at this point, at *least* guess (A) or (D)! Using the distributive postulate, $ax - dx = (a - d)x$. So, $(a - d)x = g$, and dividing both sides by $(a - d)$ we get $\frac{\cancel{(a - d)}x}{\cancel{(a - d)}} = \frac{g}{(a - d)}$; $x = \frac{g}{a - d}$. Choice (A). Once again, you could substitute numbers for a, d, x, and g in the original "if" statement *and* in all the answers and see which were equivalent. Be careful! Remember, you should sub-

stitute at least two sets of numbers just in case you have some freaky case where you are led astray. For example (and this has nothing to do with the numbers in this problem), suppose you substituted $A = 2$, $B = 1$ into the two expressions $2A - B$ and $3A - 3B$. In the first case you get $2(2) - 1 = 4 - 1 = 3$ and in the second case you get $3(2) - 3(1) = 6 - 3 = 3$. So, if time permits, try at least two sets of numbers and pick them far apart! Otherwise you could conclude that $2A - B = 3A - 3B$ for all numbers A and B.

10.5 This problem (although it contains inequality symbols) really concerns *divisibility,* which we shall discuss further in Section 11. If J is between 0 and 20 (that's what those symbols imply), then how many integers, J, are divisible by 2? Let's try some integers between 0 and 20; $J = 1$, $\frac{3J}{2} = \frac{3}{2}$, but that is not an integer! $J = 2$, $\frac{3J}{2} = \frac{(3)(2)}{(2)} = 3$, so we have one integer that can be expressed in the form $\frac{3J}{2}$ so far. Can you see that as long as J = an even number (less than 20), $\frac{3J}{2}$ will be an integer? If so, then when $J = 2, 4, 6, 8, 10, 12, 14, 16$, or 18, $\frac{3J}{2}$ is an integer. There are 9 of these cases, or choice (D).

10.6 So many students get a problem like this one wrong! They start out O.K.: If $\frac{1}{P} + \frac{1}{Q} = \frac{1}{R}$, then, subtracting $\frac{1}{Q}$ from both sides of the equation, they get $\frac{1}{P} = \frac{1}{R} - \frac{1}{Q}$. It is at this point that most of the wrong answers come about. These "wrong" students now invert each term of the equation to get $P = R - Q$. *ERROR.* Substitute numbers and you will see that you can't invert fractions any old time you want to. For example, $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$ (or 1, of course). Now invert each fraction to get $\frac{2}{1} + \frac{2}{1} = \frac{2}{2}$. Hmmm! Is it true that $2 + 2 = 1$? You *can* invert fractions when you have a situation like $\frac{A}{B} = \frac{C}{D}$, because if $\frac{A}{B} = \frac{C}{D}$, then $\frac{B}{A} = \frac{D}{C}$. Example: $\frac{2}{3} = \frac{4}{6}$, $\frac{3}{2} = \frac{6}{4}$. It's that + sign between the fractions that should prevent you from inverting everything in sight. Well, we were down to $\frac{1}{P} = \frac{1}{R} - \frac{1}{Q}$; subtract the fractions on the right side of the equation to get $\frac{1}{P} = \frac{Q - R}{RQ}$. *Now* you have the form $\frac{A}{B} = \frac{C}{D}$, so you *can* invert to get $P = \frac{RQ}{Q - R}$. Choice (B).

Another approach to this problem is to "clear" the fractions by multiplying by PQR. $PQR\left(\frac{1}{P} + \frac{1}{Q}\right) = PQR\left(\frac{1}{R}\right)$. So, $\frac{PQR}{P} + \frac{PQR}{Q} = \frac{PQR}{R}$; $QR + PR = PQ$. Getting all those terms containing P on one side of the equation: $QR = PQ - PR$; Using the distributive postulate now, $QR = P(Q - R)$. Dividing both sides of the equation by $(Q - R)$ we get, $\frac{QR}{(Q - R)} = \frac{P(Q - R)}{(Q - R)}$. So, $\frac{QR}{Q - R} = P$.

10.7 "Clear" the fractions by multiplying both *sides* of the equation by 3. $3\left(\frac{a + b + c}{3}\right) = 3\left(\frac{a + b}{3} + 1\right)$. This gives you

$$\frac{\cancel{3}(a + b + c)}{\cancel{3}} = \frac{\cancel{3}(a + b)}{\cancel{3}} + 3(1)$$

So, $a + b + c = a + b + 3$. Now, subtract a and then b from both sides to get $c = 3$. Choice (C). It would take a while longer to figure out what numbers could fit into the equation by trial and error.

10.8 Since we are given that $\frac{x}{y} = \frac{4}{5}$, then $\frac{2x}{y} = 2\left(\frac{4}{5}\right) = \frac{8}{5}$. Since $\frac{x}{y} = \frac{4}{5}$, then in Column B, $\frac{y}{x} = \frac{5}{4} \cdot \frac{8}{5} = \frac{32}{20}; \frac{5}{4} = \frac{25}{20}$; so the quantity in Column A is greater. Choice (A).

10.9 Since $\frac{1}{x} < 1$ can we conclude that $x > 1$? Substituting some values for x, $\frac{1}{3} < 1$ and $3 > 1$. $\frac{1}{8} < 1$ and $8 > 1$. *But* what if $x = -2$? $\frac{1}{-2} < 1$ but $-2 \not> 1$. Since we do not know if x is positive or negative, we must mark choice (D). *Try several values of the variable*.

10.10 You could find values of x and y that will "fit" into the equation but it is just as easy to do this one directly (algebraically). If $2xy = 6xy - 3$, then subtracting $2xy$ from both sides, we get $0 = 4xy - 3$. Then, adding 3 to both sides, we get $4xy = 3$. Dividing by 4 we get $xy = \frac{3}{4}$, so that

$$x^2y^2 = (xy)(xy) = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16}$$

Since $\frac{5}{8} = \frac{10}{16}$, the quantity in column B is greater. Choice (B).

There is a point in this problem where a lot of students go off course. It is a careless error and we should discuss it. When we are at the point where $4xy = 3$, it is very tempting to say, then $4x^2y^2 = 9$. *ERROR*. Did you square *both* sides? $3^2 = 9$, but $(4xy)^2 \neq 4x^2y^2$. It is equal to $16x^2y^2$. So be very careful!

Section 11. Divisibility, Odd and Even Integers

1. Integers are numbers such as $-6, -1, 0, 3, 10$, etc.
2. Whole numbers are the non-negative integers such as 0,1,2,3, etc.
3. To say that *X is divisible by Y* means that when you divide X by Y there is *no* remainder.

Examples: 96 is divisible by 8 because $^{96}/_{8} = 12$.
34 is *not* divisible by 8 because $^{34}/_{8} = 4^{1}/_{4}$.

4. Any number that is divisible by 2 is called an *even* number: -6, 8, etc.
5. Any number that is not even is *odd:* -5, 3, 7, 101, etc.
6. Any number that is divisible only by itself and 1 (except 1) is called a *prime* number: 5, 7, 11, etc.
7. Any number that is not prime is called a *composite* number.

Examples: 2, 3, 29 are prime numbers.
4, 12, 100 are composite numbers.

8. One number is a multiple of another number if it is divisible by that number.

   Examples: 24 is a multiple of 4.
   24 is a multiple of 6.
   6 is a multiple of 2.

9. A factor of a number is any number that will divide into the first number without leaving a remainder.

   Examples: 4 is a factor of 24.
   2 is a factor of 6.
   8 is a factor of 8.

Quick Tests for Divisibility

Sometimes, in order to save valuable time on the SAT, it would be helpful to know whether or not one number is divisible by another, without, of course, going through the process of actually dividing and looking for a remainder. For example, is 234117 divisible by 9? Yes. 234117 = (26013)(9). Add the digits of 234117 to get, 2 + 3 + 4 + 1 + 1 + 7 = 18. Since 18 is divisible by 9, 234117 is divisible by 9. Neat, huh?

A number is divisible by	*if*
2	its last digit is divisible by 2. (86, 44, 92, 108, 30. Remember, $^0/_2 = 0$.)
3	the sum of its digits is divisible by 3. (27: 2 + 7 = 9. 9 is divisible by 3 so 27 is divisible by 3. 2115: 2 + 1 + 1 + 5 = 9, so 2115 is divisible by 3, etc.)
4	the number formed by its last two digits is divisible by 4. (90724: last two digits form the number 24, which is divisible by 4, so 90724 is divisible by 4).
5	its last digit is 0 or 5. (25, 605, 900, etc.)
6	it is divisible by 2 and by 3. (324 is divisible by 2 and by 3, so 324 is divisible by 6. If a number is divisible by $a, b, c, d, \ldots$ where these letters represent prime numbers, then the original number is divisible by any product that can be formed by these prime numbers. Example: 210 = (2)(3)(5)(7). So 210 is divisible by 2, 3, 5 and 7. It is also divisible by (2)(3) = 6, by (2)(3)(7) = 42, etc.)
7	no quick test
8	the number formed by the last 3 digits is divisible by 8. (237040: last 3 digits form the number 040, which is divisible by 8, so 237040 is divisible by 8.)
9	the sum of its digits is divisible by 9. (See the opening paragraph.)

10. 2 is the *only even* prime number. Any other even number has a factor of 2 and is therefore composite.
11. If k is an integer, then $k + 1, k + 2$, etc., represent the next few consecutive integers.
12. If k is an even integer, then the next few consecutive even integers are $k + 2$, $k + 4$, etc.
13. If k is an odd integer, then the next few consecutive odd integers are (also) $k + 2, k + 4$, etc.

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.

11.1 If $N = 4 \times 5 \times 12 \times 36$, then N is divisible by each of the following numbers *except:*

(A) 24 (B) 15 (C) 144 (D) 45 (E) 84

11.2 If the 5-digit number $3172p$ is divisible by 9, then $p =$

(A) 3 (B) 4 (C) 5 (D) 6 (E) 9

11.3 A boy can divide his baseball cards into equal piles of 3,4, or 6 cards so that every card is in at least one pile. What is the smallest number of baseball cards that he might have?

(A) 68 (B) 72 (C) 18 (D) 12 (E) 24

11.4 Of the following numbers, the one which can be written in the form $3Q$, where Q is an integer, is

(A) 22 (B) 2,222 (C) 22,222 (D) 222,222 (E) 2,222,222

11.5 The average of five consecutive even integers is greater than the least even integer by

(A) 3 (B) 4 (C) 5 (D) 6 (E) cannot be determined from the information given

11.6 If a, b, and c are three consecutive integers, then which of the following is *always* true?

(A) $a + b = c$ (B) $a + b > c$ (C) $a + b = c + 1$ (D) $a + c = 2b$
(E) $2(a + b) > 2c$

11.7 If N is a positive integer divisible by 7 without remainder, then which of the following must also be divisible by 7 without remainder?

(A) $\frac{N}{7}$ (B) $\frac{N}{7} + 7$ (C) $3N + 28$ (D) $\frac{77}{N}$ (E) $11N + 6$

11.8 If $\frac{x^4}{25}$ is not an integer, then x could be

(A) 5 (B) 10 (C) 16 (D) 25 (E) $5\sqrt{5}$

	Column A	*Column B*	
11.9	The average of 7 consecutive odd integers, a, b, c, d, e, f, g.	$\frac{c + e}{2}$	A B C D

	Column A	*Column B*	
	a, b, and c are consecutive odd integers		
11.10	abc	$a + b + c$	A B C D

ANSWERS AND EXPLANATIONS

11.1 (E)	11.2 (C)	11.3 (D)	11.4 (D)	11.5 (B)
11.6 (D)	11.7 (C)	11.8 (C)	11.9 (C)	11.10 (D)

11.1 If $N = (4)(5)(12)(36)$, then $N = (2 \times 2)(5)(2 \times 2 \times 3)(2 \times 2 \times 3 \times 3)$. Now look at the answers. (A) $24 = 2 \times 2 \times 2 \times 3$. Divide N by 24 and you get $\frac{\cancel{2} \times \cancel{2} \times 5 \times \cancel{2} \times 2 \times \cancel{3} \times 2 \times 2 \times 3 \times 3}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{3}} = 5 \times 2 \times 2 \times 2 \times 3 \times 3 = 360$. So N is divisible by 24. As long as N contains the same factors as (or more than) its divisor, it will be divisible by its divisor. Try (B). $15 = 3 \times 5$. There is a factor of 3 as well as a factor of 5 in N, so N is divisible by 15. Try (C). $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$. N contains all these factors, so N is divisible by 144. (D) $45 = 3 \times 3 \times 5$. N contains these factors, so N is divisible by 45. (E) must be the answer. Let's see why, though! $84 = 2 \times 2 \times 3 \times 7$. If you divide N by 84, then, that "7" will be left in the denominator because there is no combination of factors in the numerator, N, that will give you a 7. Therefore, N is *not* divisible by 84. Choice (E).

11.2 A number is divisible by 9 if the sum of its digits is divisible by 9. $3 + 1 + 7 + 2 + p = 13 + p$. If $p = 5$, then $13 + 5 = 18$, which is divisible by 9. Therefore, 31725 is divisible by 9. Choice (C).

11.3 Be careful here! What is the *smallest* number that is divisible by 3,4, and 6? The answer is 12, choice (D). He can thus make 4 piles of 3 cards each, 3 piles of 4 cards each, or 2 piles of 6 cards each. Many students see that "72" in choice (B) and since 3,4, and 6 are all factors of 72, they decide on this answer. However, the problem states, "What is the *smallest* number . . ." You should really *underline* important words like these when they come up in a problem.

11.4 Suppose $Q = -2,-1,0,3,7,9, \ldots$ Then $3Q = -6,-3,0,9,21,27, \ldots$, which means that $3Q$ is divisible by 3 (since it has 3 as a factor). Which of the answers is divisible by 3? *That* is really the question. A number is divisible by 3 if the sum of its digits is divisible by 3. Try the answers now. (A) 22; $2 + 2 = 4$. 4 is *not* divisible by 3. (B) 2,222: $2 + 2 + 2 + 2 = 8$. No. (C) 22,222: $2 + 2 + 2 + 2 + 2 = 10$. No. (D) 222,222: $2 + 2 + 2 + 2 + 2 + 2 = 12$. Since 12 *is* divisible by 3, then 222,222 is also divisible by 3, Choice (D). Of course, you probably "skipped" right to this answer as you "added up" all those 3's in your mind! Whatever you do, don't try dividing 3 into each of the answers. You'll probably take too much time.

11.5 Consecutive even (or odd) integers can be expressed as k, $k + 2$, $k + 4$, $k + 6$, $k + 8$. For example, if $k = 6$, then the 5 consecutive even integers could be 6, 8, 10, 12, 14. Their average is $\frac{6 + 8 + 10 + 12 + 14}{5} = \frac{50}{5} = 10$. Suppose $k = -8$. Then the 5 consecutive even integers would be -8, -6, -4, -2, 0, and the average would be $\frac{-8 + -6 + -4 + -2 + 0}{5} = \frac{-20}{5} = -4$. Do you notice any-

thing? Let's take 5 consecutive even (or odd) integers in general, k, $k + 2$, $k + 4$, $k + 6$, $k + 8$, and find their average: $\frac{k + k + 2 + k + 4 + k + 6 + k + 8}{5} = \frac{5k + 20}{5} = \frac{5(k + 4)}{5} = k + 4$ (the "middle" integer). So, the average of 5 consecutive even integers is always 4 greater than the least integer. $k + 4$ is 4 greater than k. Choice (B).

11.6 Let the consecutive integers be x, $x + 1$, and $x + 2$. Then choice (D) $x + (x + 2) = 2(x + 1)$ is correct since $x + (x + 2) = 2x + 2 = 2(x + 1)$. Of course, you could have substituted numbers. But be careful! $1 + 2 = 3$ choice (A) seems correct. But what about $2 + 3 = 4$? Likewise, in choice (B), $1 + 2$ is *not* greater than 3.

11.7 This is very tricky. If you look at choice (A), it brings about some sort of immediate recognition, doesn't it? But the problem *doesn't* state, "Which of the following shows that N is divisible by 7," it states, "Which of the following *is* divisible by 7?" Is $\frac{N}{7}$ divisible by 7? Well, if N is divisible by 7 then N could be $-14, 7, 21$, etc. Try some of those values to see whether or not $\frac{N}{7}$ is divisible by 7. Let $N = 7$. Then $\frac{N}{7} = 1$. Is 1 divisible by 7? No. Choice (A) is out! (B) Again, it is easy to let $N = 7$ (since 7 is certainly divisible by 7). So, is $\frac{1}{7} + 7$ divisible by 7? No. On to (C). Letting $N = 7$ again, is $3(7) + 28$ divisible by 7? $21 + 28 = 49$ and since 49 *is* divisible by 7, it *seems* that (C) is the answer. What if you let N = a number other than 7, though? Would $3N + 28$ *still* be divisible by 7? If you had the time to try other N's, you could make that decision. Since you are rushed for time your first inclination is probably right. But rather than try other N's, why not stick to $N = 7$ and see if you can eliminate the other answers. (D) $\frac{77}{N}$ is equal to 11 when $N = 7$. Since 11 is *not* divisible by 7, choice (D) is out. (E) $11N + 6 = 83$ when $N = 7$. 83 is *not* divisible by 7, so (E) is out! *So,* choice (C) *was* correct. You probably got nervous because you did not have time to try many sets of values for N, as you were warned to do throughout this book, right? Let's go back though to (C), $3N + 28$. If N is divisible by 7 then $3N$ is divisible by 7, because it contains a factor that is divisible by 7,(N). Since 28 is divisible by 7, the question is, "Is the *sum* of two numbers divisible by 7 if *each number* in the sum is divisible by 7?" The answer to that is yes. The statement is true even if the word "sum" is replaced by "difference" or "product." It is *not* true if the word is "quotient." You can see that in choices (A), (B) and (D).

11.8 If $\frac{x^4}{25}$ is *not* an integer then x^4 *can't* be divisible by 25. (A) 5: $5^4 = 5 \times 5 \times 5 \times 5 = 25 \times 25$, which *is* divisible by 25. (B) 10: $10^4 = 10 \times 10 \times 10 \times 10 = 100 \times 100$, which is certainly divisible by 25. (C) 16: $16^4 = 16 \times 16 \times 16 \times 16$. Numbers that are divisible by 25 must have 0 or 5 as their last digit. Examples: 25, 50, 75, 100, 125, 625. $16 \times 16 \times 16 \times 16$ will have a 6 as its last digit when multiplied out. So $16 \times 16 \times 16 \times 16$ is *not* divisible by 25. Another way of thinking about this is to see if $16 \times 16 \times 16 \times 16$ contains the factors of 25. $\frac{16 \times 16 \times 16 \times 16}{5 \times 5} = \frac{(2 \times 2 \times 2 \times 2)(2 \times 2 \times 2 \times 2)(2 \times 2 \times 2 \times 2)(2 \times 2 \times 2 \times 2)}{5 \times 5}$. There aren't "two 5's up there." So, $16 \times 16 \times 16 \times 16$ is *not* divisible by 25. What about choice (E), even though we have the answer? Well, $(5\sqrt{5})^4 = (5\sqrt{5})(5\sqrt{5})(5\sqrt{5})(5\sqrt{5}) = (5 \times 5 \times 5 \times 5)(\sqrt{5}\sqrt{5}\sqrt{5}\sqrt{5}) = (25 \times 25)(\sqrt{25}\sqrt{25}) = (25 \times 25 \times 5 \times 5) = 25 \times 25 \times 25$, which is divisible by 25!

11.9 Remember, in problem 11.5 *you* discovered that the average of an odd number of consecutive even (or odd) integers was indeed the middle integer. So the average of the 7 consecutive odd integers a, b, c, d, e, f, and g is d. So the problem here is really whether or not $\frac{c+e}{2}$ is less than, equal to, or greater than d, if in fact that determination can be made. Let's start with 3 odd integers, say, 3, 5, 7. The average is 5. Now, what is the average of the *two* integers around that average? In other words, what is $\frac{3+7}{2}$? How about that? Try 5 consecutive odd integers, say, 7, 9, 11, 13, 15. The average of these 5 integers is $\frac{55}{5} = 11$, and the average of 9 and 13 is $\frac{9+13}{2} = \frac{22}{2} = 11$. Let's take any 7 consecutive odd integers, say k, $k+2$, $k+4$, $k+6$, $k+8$, $k+10$, $k+12$. The average of these integers is $\frac{7k+42}{7} = \frac{7(k+6)}{7} = k+6$, the middle term again. The average of the two integers "surrounding" $k+6$ is $\frac{(k+4)+(k+8)}{2} = \frac{2k+12}{2} = \frac{2(k+6)}{2}$ $= k+6$. So, the quantities in Column A and Column B are equal. Choice (C). This whole argument works as well for consecutive *even* integers, and in fact consecutive integers (without being even or odd).

11.10 Substitute some consecutive odd integers: $a = 3$, $b = 5$, $c = 7$. $abc = 105$. $a+b+c = 15$. Again, $a = 1$, $b = 3$, $c = 5$. $abc = 15$. $a+b+c = 9$. So it looks like choice (A). *But* (this time your nervousness paid off) what if some of those odd integers were *negative?* Let $a = -1$, $b = 1$, and $c = 3$. Then $abc = -3$ and $a+b+c = 3$. Then you would have marked choice (B). Since there is confusion, you *must* mark choice (D); no relationship can be determined. (*Whenever* you're solving a problem by "plugging in," make sure you try negative as well as positive values, unless the conditions of the problem state that all values are positive.)

Section 12. Coordinate Geometry

There are only two formulas that you are responsible for:

The *distance* between two points whose coordinates are (x_1,y_1) and (x_2,y_2) is equal to $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. It doesn't matter which of the two points you label (x_1,y_1).

Example: What is the distance between (2,3) and (5,7)?

$$d = \sqrt{(2-5)^2+(3-7)^2} = \sqrt{(-3)^2+(-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

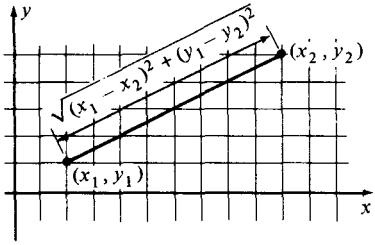
or

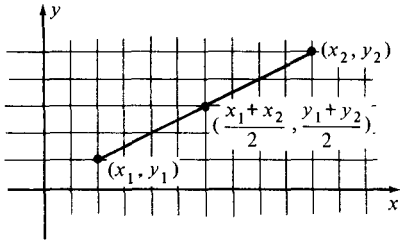
$$d = \sqrt{(5-2)^2+(7-3)^2} = \sqrt{(3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

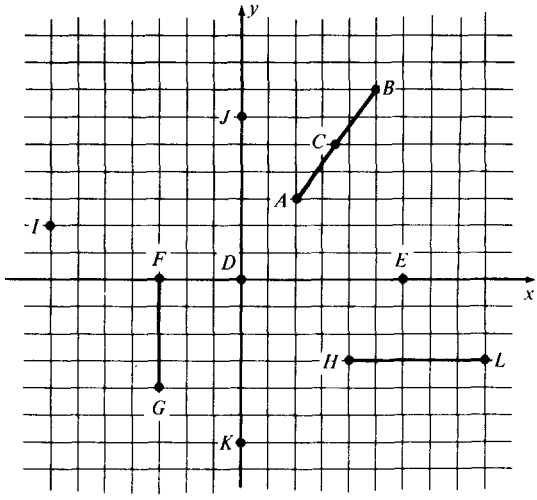
Remember: $\sqrt{a^2+b^2} \neq a+b$ for all values of a and b.
Your safest bet is to add the numbers under the square root symbol

before you take the square root of the sum. *Do not* take the square roots of the individual summands.

The *midpoint* of the line segment connecting the two points whose coordinates are (x_1, y_1) and (x_2, y_2) has the coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.







In the above diagram, the distance from $A(2,3)$ to $B(5,7)$ is 5. The coordinates of point C, the midpoint of AB, are $\left(\frac{2+5}{2}, \frac{3+7}{2}\right) = \left(\frac{7}{2}, \frac{10}{2}\right) = (3.5,5)$. The *origin* is point $D(0,0)$, where the horizontal x axis and the vertical y axis intersect. Those points located on either of these axes have a 0 for one of their coordinates; $E(6,0)$, $F(-3,0)$, $J(0,6)$, and $K(0,-6)$.

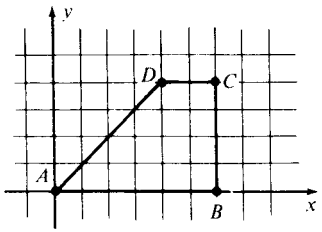
The distance from $H(4, -3)$ to $L(9, -3)$ can be found by the distance formula, $HL = \sqrt{(4-9)^2 + (-3--3)^2} = \sqrt{(-5)^2 + (0)^2} = \sqrt{25 + 0} = \sqrt{25} = 5$. *But*, if two points are on the same horizontal line segment, all you have to do is simply "count the boxes." From H to L is 5 boxes. Likewise, the distance from F to G *can* be found by the distance formula, but it would be a waste of time to use it. Again, just count the boxes. $FG = 4$. Just to make sure you know which coordinate comes first and which one comes second, find the coordinates of points H, L, F, G, and I. The answers are at the bottom of this page.

.....

$H(4,-3)$, $L(9,-3)$, $F(-3,0)$, $G(-3,-4)$, $I(-7,2)$

PRACTICE PROBLEMS

See p. 228 for instructions on marking your answers.



12.1 In the figure above, each box along the x-axis represents 2 units and each box along the y-axis represents 3 units. Using this scale, what is the area of *ABCD?*

(A) 16 (B) 32 (C) 48 (D) 64 (E) 96

12.2 The distance between point $A(5,0)$ and point B is 8. The coordinates of point B could be any of the following *except*

(A) (5,8) (B) (−3,0) (C) (13,0) (D) (3,8) (E) (5,−8)

12.3 Point $M(a,b)$ is the midpoint of the line segment connecting point $A(2a,b)$ and point $B(x,y)$. $x + y =$

(A) a (B) b (C) $a + b$ (D) $a - 2b$ (E) $-a$

12.4 If the endpoints of a diameter of a circle are located at (a,b) and $(3a, -b)$, then the area of the circle is

(A) $a^2\pi$ (B) $b^2\pi$ (C) $(a^2 + b^2)\pi$ (D) $4a^2\pi$ (E) $2b^2\pi$

12.5 The center of a circle is at point $A(3,2)$. The circle passes through the point $B(5,1)$. What is the area of this circle?

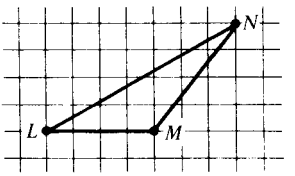
(A) $\sqrt{5}\pi$ (B) 5π (C) 25π (D) 16π (E) 36π

12.6 The points $P(0,0)$, $Q(0,b)$, and $S(a,0)$ are three vertices of rectangle $PQRS$. What must the coordinates of point R be?

(A) (a,b) (B) (b,a) (C) (b,0) (D) (−a,−b) (E) (−b,−a)

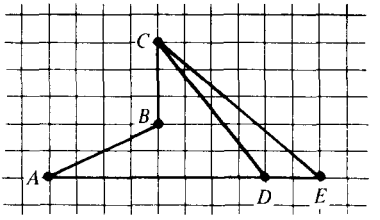
12.7 Find the area of the triangle PQR whose coordinates are $P(1,3)$, $Q(4,-1)$ and $R(1,-1)$.

(A) 6 (B) 8 (C) 10 (D) 12 (E) 24



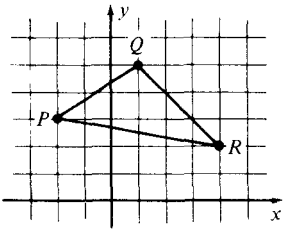
12.8 The area of triangle LMN in the figure above is

(A) 4 (B) 6 (C) 8 (D) 12 (E) 16



12.9 In the figure above, the area of *ABCD* is

(A) 12.5 (B) 14 (C) 17.5 (D) 18 (E) none of these



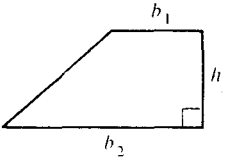
12.10 The area of triangle *PQR* is

(A) 12.5 (B) 18 (C) 9 (D) 7.5 (E) 11

ANSWERS AND EXPLANATIONS

12.1 (E)	12.2 (D)	12.3 (B)	12.4 (C)	12.5 (B)
12.6 (A)	12.7 (A)	12.8 (C)	12.9 (B)	12.10 (D)

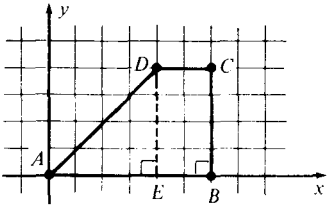
12.1 *If* you remember the formula for the area of a *trapezoid*,



$$A = \tfrac{1}{2}\,(\text{height})(\text{base}_1 + \text{base}_2),$$

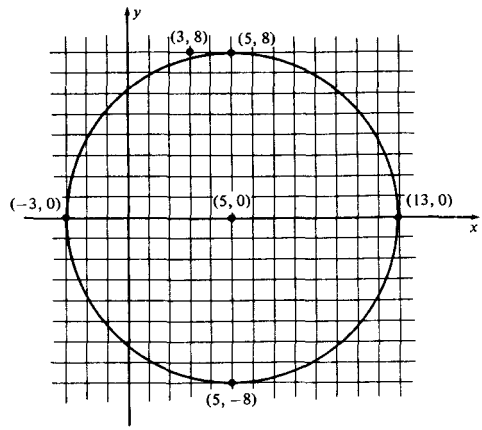
then this problem is fairly easy. Count the boxes from A to B. $AB = 6$. *But* since each box represents 2 units, $AB = 6 \times 2 = 12$. Likewise, $DC = 2 \times 2 = 4$. The height of the trapezoid, $BC = 4 \times 3 = 12$. Therefore, the area of *ABCD* is equal to $\tfrac{1}{2}(BC)(AB + DC) = \tfrac{1}{2}(12)(12 + 4) = 6(16) = 96$. Choice (E).

If you don't remember that formula, *don't worry*. You really don't need fancy formulas for the SAT. Break up the area of *ABCD* into "convenient figures," that is, figures whose areas are easy to find. In most cases, these "easy" figures are squares, rectangles, and right triangles. See the figure below, for one "break-up" of *ABCD*.



As you can see, $ABCD$ has been broken up into a rectangle, $BCDE$, and a right triangle, AED. $EB = 4$ (remember the *scale*) and $BC = 12$, so the area of rectangle $BCDE = 12 \times 4 = 48$. The area of right triangle $AED = \frac{1}{2}(AE)(ED) = \frac{1}{2}(8)(12) = 48$. So the area of $ABCD = 48 + 48 = 96$. Again, choice (E).

It is not that easy to count the boxes in order to find the area directly, because of that scaling feature of the problem; each box represents more than one unit!



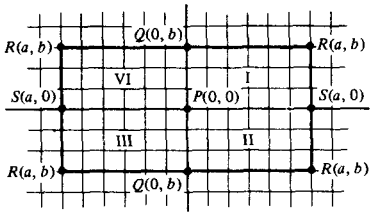
12.2 Every point that is 8 units from point A lies on a circle whose center is A and whose radius is 8. See the above figure. Since (5,8) is 8 units directly "over" (5,0), the choice (A) is 8 units from (5,0). So, (A) is *not* the answer. Since (−3,0) is 8 units to the left of (5,0), choice (B) is *not* the answer. Since (13,0) is 8 units to the right of (5,0), choice (C) is *not* the answer. Since (5, −8) is 8 units directly below (5,0), choice (E) is *not* the answer. So, by elimination, choice (D) is the answer. Of course, you could have used the distance formula here, but that would have taken quite a long time had you tested each of the choices that way. However, let's use the distance formula to prove that (3,8) is *not* 8 units from (5,0). $d = \sqrt{(3-5)^2 + (8-0)^2} = \sqrt{(-2)^2 + 8^2} = \sqrt{4 + 64} = \sqrt{68} > 8$.

12.3 Use the midpoint formula here. $(a,b) = \left(\frac{2a + x}{2}, \frac{b + y}{2}\right)$. So, $a = \frac{2a + x}{2}$ and $b = \frac{b + y}{2}$. "Cross-multiplying" in each case, we get $2a = 2a + x$; $0 = x$. $2b = b + y$; $b = y$. So, $x + y = 0 + b = b$. Choice (B). You can use the technique of substitution of numbers here, but *unless* you know the midpoint formula that method *won't help*.

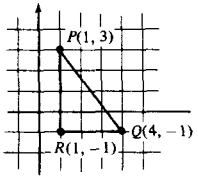
12.4 To find the area of the circle, we need to find the radius. The length of the radius is $\frac{1}{2}$ the length of the diameter. Since (a,b) and $(3a, -b)$ do *not* lie on either a horizontal or vertical line segment, there is no alternative but to use the distance formula here. The length of the diameter is $\sqrt{(a - 3a)^2 + (b - -b)^2} = \sqrt{(-2a)^2 + (2b)^2} = \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2 + b^2)} = \sqrt{4}\sqrt{(a^2 + b^2)} = 2\sqrt{a^2 + b^2}$. So the length of the radius is $\frac{1}{2}(2)\sqrt{a^2 + b^2}$ or $\sqrt{a^2 + b^2}$ The area of a circle is πr^2. So, in this case, the area of the circle is $\pi\sqrt{(a^2 + b^2)}^2 = \pi(a^2 + b^2)$. Choice (C).

12.5 This problem is easier than 12.4. To get the area of the circle we have to get the length of the radius again. This time, though, the approach is more straightforward. The radius is AB and its length is found by the distance formula: $r = \sqrt{(3-5)^2 + (2-1)^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{4 + 1} = \sqrt{5}$. So, the area of the

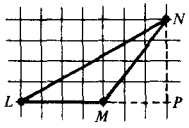
circle is πr^2, which is $\pi\sqrt{5}^2 = \pi\sqrt{5}\sqrt{5} = \pi\sqrt{25} = \pi 5 = 5\pi$. Choice (B). We think that you can see why the examiners put choices (A) and (C) in this problem. Don't be careless!



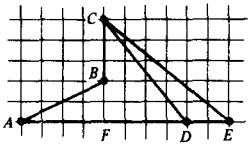
12.6 Given the three points P, Q, and S, you can see that 4 different rectangles can be formed (labeled I,II,III, and IV in the diagram above). Substituting numbers for a and b will illustrate this fact: Let $a = 6$ and $b = 3$. Then the coordinates of the rectangle will be $P(0,0)$, $Q(0,3)$, $S(6,0)$ and $R(6,3)$. This is rectangle I. If $a = -6$ and $b = 3$, you get rectangle IV. Point R in this case is located at $(-6,-3)$. If $a = 6$ and $b = -3$, you get rectangle II. But point R is *still* at (a,b), in other words at $(6,-3)$. We are sure that you can see that R is located at (a,b) in rectangle III also. Choice (A).



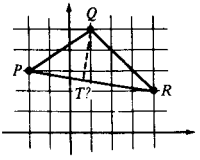
12.7 See the diagram above. Since the figure is a right triangle, its area is $\frac{1}{2}$(leg)(leg) = $\frac{1}{2}(PR)(RQ) = \frac{1}{2}(4)(3) = \frac{1}{2}(12) = 6$. Choice (A).



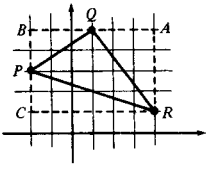
12.8 This is also a fairly easy problem. Since the area of any triangle is $\frac{1}{2}$(base)(height), and since LM can be considered a base and NP can be considered a height, then the area of triangle LMN is equal to $\frac{1}{2}(LM)(NP) = \frac{1}{2}(4)(4) = \frac{1}{2}(16) = 8$. Choice (C).



12.9. See the figure above. Perhaps the simplest way of doing this problem is to find the areas of the two right triangles, ABF and DCF. Area of triangle $ABF = \frac{1}{2}(AF)(BF) = \frac{1}{2}(4)(2) = \frac{1}{2}(8) = 4$. Area of triangle $DCF = \frac{1}{2}(DF)(CF) = \frac{1}{2}(4)(5) = \frac{1}{2}(20) = 10$. Therefore the area of $ABCD$ is $4 + 10 = 14$. Choice (B).



12.10 Unless triangle *PQR* is a right triangle, which would mean we could just take $\frac{1}{2}$(leg)(leg), finding the area of triangle *PQR* presents us with the difficulty of finding its height. (Even if we call *PR* a base, how can we find the precise measurement of height *QT*?) *However,* there is a simple method of finding the area of any "slanted" triangle, that is, a triangle whose base is *not parallel* to either the x or the y axis.



The area of triangle *PQR* is equal to the area of *rectangle RABC* minus the *sum* of the areas of the 3 right triangles, *RAQ*, *QBP*, and *PCR*. The area of *RABC* = (6)(3) = 18.

The area of triangle

$$\begin{aligned} RAQ &= \tfrac{1}{2}(3)(3) = \tfrac{1}{2}(9) = 4.5 \\ QBP &= \tfrac{1}{2}(3)(2) = \tfrac{1}{2}(6) = 3.0 \\ PCR &= \tfrac{1}{2}(1)(6) = \tfrac{1}{2}(6) = \underline{3.0} \\ & \qquad\qquad\qquad\qquad\quad 10.5 \end{aligned}$$

So the area of triangle *PQR* = 18 − 10.5 = 7.5. Choice (D).

This just *looks* like a lot of work. Since the diagram is drawn for you, it is only necessary to quickly draw the rectangle around the "slanted" triangle and get the areas of the right triangles formed!

SAMPLE TESTS
Mathematical Aptitude Test 2

Time—30 Minutes
25 Questions

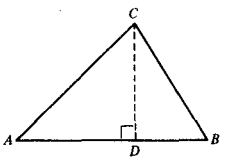
In this section solve each problem, using any available space on the page for scratchwork. Then indicate the *one* correct answer in the appropriate space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$.
The number of degrees in the arc of a circle is 360.
The measure in degrees of a straight angle is 180.

Definition of symbols:

$<$ is less than	$\leqq$ is less than or equal to
$>$ is greater than	$\geqq$ is greater than or equal to
$\perp$ is perpendicular to	$\parallel$ is parallel to

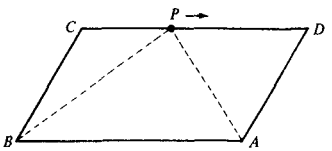


Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\measuredangle CDA$ is a right angle, then

(1) area of triangle $ABC = \dfrac{(AB)\cdot(CD)}{2}$

(2) $AC^2 = AD^2 + DC^2$

Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible *except* when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.



$AB \parallel CD$

1. In the figure above, as P moves from C to D on CD, the area of triangle APB

   (A) increases only (B) decreases only (C) increases, then decreases (D) decreases, then increases (E) remains the same

2. A piece of wire 64 inches long is cut into 2 pieces which are in the ratio 3 : 5. What is the length, in inches, of the longer piece?

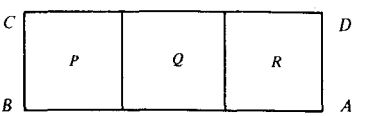
   (A) 16 (B) 24 (C) 8 (D) 40 (E) 48

3. A machine can make 12 widgets in 8 seconds. At the same rate, how many widgets can it make in 24 *minutes*?

   (A) 16 (B) 36 (C) 300 (D) 2160 (E) 4002

4. If Z and K are integers, and $0 < Z < 20$, how many integers, Z, can be written in the form $\dfrac{3K}{2}$?

   (A) 6 (B) 7 (C) 8 (D) 9 (E) 10



5. The perimeter of $ABCD$ is 80. If P,Q, and R are squares, what is the area of $ABCD$?

(A) 192 (B) 200 (C) 240 (D) 300 (E) 360

6. If $p = 3q/8$ and $q = 2n/3$, then $p =$

(A) $n/4$ (B) $9n/16$ (C) $4n$ (D) $25n/24$ (E) $2n$

7. If $\sqrt{XY} = 4$, where X and Y are positive integers, then which of the following could *not* be a value of $X - Y$?

(A) -6 (B) 6 (C) 9 (D) 0 (E) 15

8. If $.0003Y = 3$, then $\frac{Y}{100} =$

(A) 33 (B) 11 (C) 100 (D) 10 (E) 3.3

9. If $4^a = 8^{3b}$, then $a =$

(A) $9b$ (B) $4.5b$ (C) 2^{3b} (D) $6b$ (E) 4^{2b}

10. If a book is 1½ inches thick, what is the greatest number of books that can be placed on a 4-foot shelf?

(A) 2 (B) 3 (C) 12 (D) 32 (E) 60

11. If $(a,b) * (c,d) = (ac + bd, bc - ad)$, then if $(1,1) * (q,p) = (0,1)$, $q =$

(A) $^2/_5$ (B) $^1/_2$ (C) $^1/_3$ (D) $-^1/_2$ (E) $^1/_5$

12. If $p - 2 = q + 2$, then $q - p =$

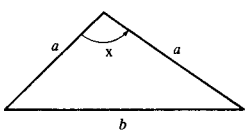
(A) -4 (B) 4 (C) 0 (D) -2 (E) 2

13. If $x > y$, then which of the following must be true?

(A) $x^2 > y^2$ (B) $x - y < 1$ (C) $xy > y^2$
(D) $x + y > 3y$ (E) $x + 1 > y$

14. If $x^2 = 5$, then $3x^6 =$

(A) 15 (B) 45 (C) 90 (D) 125 (E) 375



15. If $90° \leq x \leq 180°$, then in the above triangle, which of the following represents the *smallest* range of values for b?

(A) $0 < b < a$ (B) $0 < b < 2a$ (C) $0 < b < a\sqrt{2}$
(D) $a < b < 2a$ (E) $a\sqrt{2} < b < 2a$

16. $\dfrac{1}{\dfrac{1}{1 + \frac{1}{x}}} =$

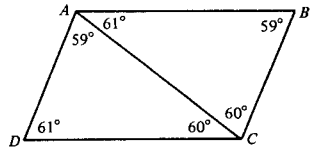
(A) $\frac{x+1}{x}$ (B) x (C) $\frac{1}{x}$ (D) $x(x + 1)$ (E) $\frac{x}{x+1}$

17. If the radii of two circles are in the ratio 9 : 4, what is the ratio of their areas?

(A) 3 : 2 (B) 9 : 4 (C) 81 : 16 (D) 4.5 : 2
(E) $9\sqrt{2} : 4\sqrt{3}$

18. If the radius of a circle is 6 and then is increased by 20%, the area of that circle is increased by

(A) 1.2π (B) 12π (C) 15.84π (D) 20.4π
(E) 51.84π



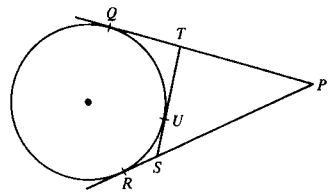
Note: Figure not drawn to scale.

19. If the angles of $ABCD$ have the measures indicated, which is the longest line segment?

(A) BC (B) AB (C) AC (D) AD
(E) cannot be determined from the given information

20. A company pays its salesmen a commission of 6% on all sales up to and including \$500 and then 10% on all sales above \$500. If one of the salesmen for the company made \$130 in commissions, what were his total sales?

(A) \$800 (B) \$1000 (C) \$1300 (D) \$1500
(E) \$2400



21. In the above figure, PQ and PR and TS are tangent to the circle at points Q, R, and U respectively. If $PQ = 12$, the perimeter of triangle PTS is

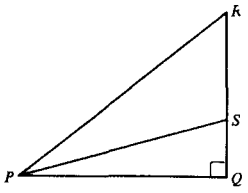
(A) 22 (B) 24 (C) 26 (D) 36 (E) cannot be determined from the given information

22. If $D(N)$ is defined to be the sum of the positive integer divisors of N, excluding N, so that, for example, $D(15) = 1 + 3 + 5 = 9$, then $D(D(6)) =$

(A) 6 (B) 7 (C) 9 (D) 12 (E) 24

23. If p and q are positive numbers, which of the following is not always true?

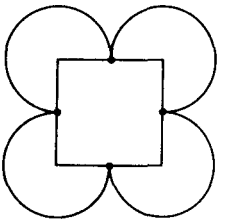
(A) $\sqrt{p}\sqrt{q} = \sqrt{pq}$
(B) $\sqrt{(p + q)^2} = p + q$
(C) $(\sqrt{p})^q = \sqrt{p^q}$
(D) $\sqrt{p + q} = \sqrt{p} + \sqrt{q}$
(E) $\dfrac{1}{\sqrt{p}} = \dfrac{q}{\sqrt{pq^2}}$



Note: Figure not drawn to scale.

24. In the figure $PS = SR$, $PQ = 12$, and $QS = 5$. $PR =$

(A) $6\sqrt{13}$ (B) 13 (C) $13\sqrt{2}$ (D) $13\sqrt{3}$ (E) 22



25. If a side of the square is 8, find the area of the entire region

(A) 48π (B) $32\pi + 64$ (C) $48\pi + 64$
(D) $64\pi + 108$ (E) $32\pi + 16$

STOP

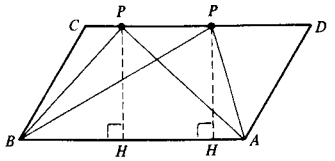
IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

Answer Sheet—Mathematical Aptitude Test 2

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
25	☐	☐	☐	☐	☐

ANSWERS AND EXPLANATIONS

1. (E)	2. (D)	3. (D)	4. (A)	5. (D)
6. (A)	7. (C)	8. (C)	9. (B)	10. (D)
11. (B)	12. (A)	13. (E)	14. (E)	15. (E)
16. (A)	17. (C)	18. (C)	19. (A)	20. (D)
21. (B)	22. (A)	23. (D)	24. (A)	25. (C)

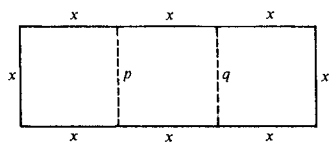


1. (E) Although the shape of triangle APB changes as point P moves from C to D, the base of the triangle, BA, and the height of the triangle, PH, remain unchanged. Therefore, the area remains the same. Since parallel lines are everywhere equidistant, PH remains constant. If two or more triangles have the same base and equal heights, they are equal in area.

2. (D) Let $3x$ = the length of the shorter piece. Then, $5x$ is the length of the larger piece, so that $3x + 5x = 64$; $8x = 64$; $x = 8$. The longer piece is then $5(8) = 40$. *Be careful.* Notice that choice (C) is the value of x. In many problems like this one, be sure that although you have found the value of a variable, it is indeed that variable that is the answer to the question.

3. (D) This is a direct proportion. The more time allowed, the more widgets made. The only catch here is the fact that you are asked for the answer in terms of widgets per *minute*. Change all times to seconds and set up a proportion: $\frac{12}{8} = \frac{w}{24 \times 60}$ (because there are 60 seconds in a minute). *Don't* multiply 24×60. It is a waste of time. "Cross-multiplying," now, $12 \times 24 \times 60 = 8 \times w$. Divide both sides of the equation by 8 to get $12 \times 3 \times 60 = w$; $2160 = w$.

4. (A) Let's start by letting $Z = 1$. Can 1 be written as $\frac{3K}{2}$? If so, then $\frac{3K}{2} = 1$; $3K = 2$. Since K is an integer, $K \neq \frac{2}{3}$. Therefore, 1 *cannot* be written in the form of $\frac{3K}{2}$. If $Z = 2$, then $\frac{3K}{2} = 2$, which means $3K = 4$; $K = \frac{4}{3}$ which is not allowed. Perhaps now you can see that the only permissible values of Z are going to be multiples of 3. Let $Z = 3$; can 3 be written in the form of $\frac{3K}{2}$? Let's see. If $3 = \frac{3K}{2}$, then $6 = 3K$ and $2 = K$. This is allowed, since K is an integer here. The multiples of 3 between 0 and 20 are 3, 6, 9, 12, 15, and 18.



5. (D) Let a side of a square be represented by x. The perimeter then is $x + x + x + x + x + x + x + x = 80$; $8x = 80$; $x = 10$. So the area of the rectangle is length $\times$ width $= (3x)(x) = (30)(10) = 300$. Or you can figure the area of one square and then multiply by 3. Be careful. Do *not* include the vertical line segments in calculating the perimeter (marked p and q in the diagram above). If you do that, then your equation would be: $10x = 80$; $x = 8$. Then, of course, you would get an area of $24 \times 8 = 192$ or choice (A). *ERROR!* Don't feel bad. A lot of students do make this error.

6. (A) $p = \frac{3}{8}(q) = \frac{\cancel{3}}{\cancel{8}_{4}}\left(\frac{\cancel{2}n}{\cancel{3}}\right) = \frac{n}{4}$ or $\frac{1}{4}n$. If you didn't know how to substitute algebraic terms, then you could have used the technique of substituting numbers. Let $q = 8$, then $p = \frac{3}{8}(8) = 3$. Since $q = \frac{2n}{3}$, then $8 = \frac{2n}{3}$; $24 = 2n$; $n = 12$. Now, look at the answers. You are looking for the case where 3 = some number times 12. Choice (A), $\frac{1}{4}(12) = 3$, suits your purpose. Algebra is a lot faster here!

7. (C) If $\sqrt{XY} = 4$, then $XY = 16$. Set up a table: There is *no* way to get a difference of 9.

X	Y	$X - Y$
1	16	−15
2	8	−6
4	4	0
8	2	6
16	1	15

8. (C) $.0003Y = 3$; move the decimal points on both sides of the equation 4 places to the right (multiply both sides by 10000) to get $3Y = 30000$; $Y = 10000$. Therefore, $\frac{Y}{100} = \frac{100\cancel{00}}{1\cancel{00}} = 100$.

9. (B) Change both sides of the equation to powers of 2: $4^a = 8^{3b}$ is equivalent to $(2^2)^a = (2^3)^{3b}$; $2^{2a} = 2^{9b}$, which implies that $2a = 9b$. To find a, divide both sides of the equation by 2 to get $a = \frac{9}{2}b$ or $a = 4.5b$. A very common mistake made by students who wrote choice (C), *ERROR*, was that they thought they could divide both sides of the equation by 4 at the start of the problem. When they did this, they got $a = 2^{3b}$. First of all, $\frac{4^a}{4} = \frac{4^a}{4^1} = 4^{a-1}$, which isn't necessarily the same number as a.

10. (D) Be careful. The widths of the books are in *inches* and the width of the shelf is in *feet*. 4 feet $= 4 \times 12 = 48$ inches. How many one-and-one-halves are there in 48? Divide 48 by $1\frac{1}{2}$ as follows: $\frac{48}{3/2} = \overset{16}{\cancel{48}} \times \frac{2}{\cancel{3}} = 16 \times 2 = 32$. Or, working with decimals: $48 \div 1.5 = 480 \div 15 =$

$$\begin{array}{r} 32. \\ 15.\overline{)480.} \\ \underline{45} \\ 30 \\ \underline{30} \end{array}$$

11. (B) In this example of an *invented* operation, $a = 1$, $b = 1$, $c = q$, and $d = p$. By simple substitution, then, $(1,1) * (q,p) = (1q + 1p, 1q - 1p) = (0,1)$.

Therefore, $q + p = 0$
$\underline{q - p = 1}$

Adding, $2q \quad = 1; q = \frac{1}{2}$

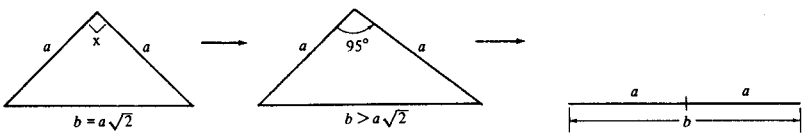
12. (A) If $p - 2 = q + 2$, then subtracting p from both sides of the equation we get $-2 = q + 2 - p$. Now, we have to "get rid" of that 2 on the right side of the equation, so subtracting 2 from both sides of the equation, we get $-2 - 2 = q - p$, so $-4 = q - p$.

13. (E) It is obvious to some that (E) must be true, because if x is bigger than y, $x + 1$ must also be bigger than y. If that doesn't occur to you immediately, you can substitute numbers. But be careful. Try *several* values of the variables. If $x > y$, then

is $x^2 > y^2$? If $x = 3, y = 2$, yes. If $x = 3, y = -5$, *no*.
is $x - y < 1$? If $x = 3, y = 2$, *no*.
is $xy > y^2$? If $x = 3, y = 2$, yes. If $x = 3, y = -5$, *no*.
is $x + y > 3y$? If $x = 2, y = 1$, *no*.

So choice (E) is the answer by elimination. Since $x > y$, then certainly when you add 1 to the bigger number, that new number will still be bigger than y.

14. (E) $x^6 = (x^2)^3 = (5)^3 = 5 \times 5 \times 5 = 125$. Therefore, $3x^6 = 3(125) = 375$.



15. (E) If $x = 90°$ then you have an *isosceles* right triangle. Therefore, $b = a\sqrt{2}$. (Remember the 45°-45°-90° right triangle?) As x increases to 180°, the triangle progressively "flattens" into a straight line with length of b. When x "hits" 180°, $b = 2a$. Choices (A), (B), and (C) allow b to be 0. This is impossible. Choice (D) allows b to be equal to a, making the triangle equilateral. Then *all* the angles must be 60°. But this is not the case, since x is "growing" from 90° to 180°. So, this problem could have been done by the technique of *elimination*.

16. (A) Work from the "bottom up," in problems like these:

$$\frac{1}{\dfrac{1}{\dfrac{1}{1} + \dfrac{1}{x}}} = \frac{1}{\dfrac{1}{\dfrac{x+1}{x}}} = \frac{1}{\dfrac{1x}{x+1}} = \frac{x+1}{x}$$

Remember, $\dfrac{1}{\frac{2}{3}} = \dfrac{3}{2}$, $\dfrac{1}{\frac{1}{q}} = \dfrac{q}{1}$, etc. 1 "over" any fraction is just that fraction inverted. Once again, if you are weak in algebra, perhaps the technique of substitution of numbers would come in handy here. Let $x = 1$. Then the original fraction,

$$\frac{1}{\frac{1}{1+\frac{1}{x}}} = \frac{1}{\frac{1}{1+\frac{1}{1}}} = \frac{1}{\frac{1}{1+1}} = \frac{1}{\frac{1}{2}} = 2$$

Now look at all the answers. Which one is equal to 2 after making the substitution of 1 for x?

(A) $\frac{1+1}{1} = \frac{2}{1} = 2$ (B) 1 (C) $\frac{1}{1} = 1$ (D) $1(1+1) = 1(2) = 2$

(E) $\frac{1}{1+1} = \frac{1}{2}$ *Trouble!* Choices (A) *and* (D) both give the value of 2. Perhaps this will convince you that you should always try several values of the variable when using this technique. *Or*, don't use the number 1. Too many silly things can happen when you do. Let's try 3 instead of 1. The original fraction will have a value of:

$$\frac{1}{\frac{1}{1+\frac{1}{3}}} = \frac{1}{\frac{1}{\frac{4}{3}}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

Now, which of the answers have a value of $\frac{4}{3}$ when you "plug in" 3 for x? (A) *does*, (B) doesn't, (C) doesn't, (D) doesn't, and (E) doesn't.

17. (C) $\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\text{radius}_1}{\text{radius}_2}\right)^2$. So, in this problem $\frac{A_1}{A_2} = \left(\frac{9}{4}\right)^2 = \frac{81}{16}$. Obviously, choice (A) is there in case you take the square root of $\frac{9}{4}$. Choice (B) would be correct if the word "area" was changed to "circumference." The rest of the choices are "garbage."

18. (C) The area of the original circle is $\pi 6^2 = \pi 36$ or 36π. 6 increased by 20% is equivalent to $6 + .20(6) = 6 + 1.2 = 7.2$. The area of this new circle is then $\pi(7.2)^2 = \pi 51.84$ or 51.84π. The *increase* is $51.84\pi - 36\pi = 15.84\pi$. The important word in this problem is "by." It didn't say "to." If the word "to" is substituted for "by," then the answer would have been (E). Be careful.

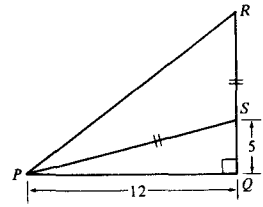
19. (A) The longest side of a triangle is opposite the greatest angle. In triangle *ADC*, then, *AC* is the longest side. In triangle *ABC*, *BC* is the longest side, so it is longer than *AC*. It certainly doesn't look that way from the diagram—but remember that the figure isn't drawn to scale.

20. (D) Let x represent the total sales. Then $.06(500) + .10(x - 500) = 130$. Multiply by 100 (move the decimal points 2 places to the right) in order to "get rid" of the decimal points to get $6(500) + 10(x - 500) = 13000$. Now, $3000 + 10x - 5000 = 13000$; $10x - 2000 = 13000$; adding 2000 to both sides of the equation, $10x = 15000$; dividing by 10 now, $x = 1500$. *Or*, work your way backwards from the answers. First subtract 500 from the total sales represented by each of the answers. For example, (C) $1300 - 500 = 800$. The salesman gets 10% of this or \$80. He also gets 6% of 500 or \$30. $80 + 30 = 110$. *No good.* We need 130. Obviously, then, the total sales have to be greater than \$1300. This eliminates (A), (B), and (C), now.

21. (B) We deliberately included this problem to see if you really *studied* Test 1. See the explanation there!

22. (A) $D(D(6))$, according to the definition, is $D(1 + 2 + 3) = D(6) = 1 + 2 + 3 = 6$.

23. (D) (A),(B), and (C) are theorems concerning square roots. Choice (D) is incorrect simply by *counterexample*. Let $p = 16$, $q = 9$. Then is it true that $\sqrt{16 + 9} = \sqrt{16} + \sqrt{9}$? Does the square root of 25 = the square root of 16 plus the square root of 9? $5 \neq 7$. *NO*!



24. (A) In right triangle PQR, $PR^2 = PQ^2 + QR^2$. We already know that $PQ = 12$. In order to find QR we need to know the length of RS. But, since $PS = RS$, if we can find PS we will have RS. In right triangle PQS, $PS^2 = QS^2 + PQ^2$, so $PS^2 = 5^2 + 12^2 = 25 + 144 = 169$. Therefore, $PS = \sqrt{169} = 13$. (It would have been faster had you recognized a 5-12-13 right triangle.) Now, $SR = 13$ also, so $QR = 5 + 13 = 18$. Back in a flash to right triangle PQR. $PR^2 = PQ^2 + QR^2 = 12^2 + 18^2 = 144 + 324 = 468$. Therefore, $PR = \sqrt{468} = \sqrt{36 \times 13} = 6\sqrt{13}$.

25. (C) The area of the square is $8 \times 8 = 64$. This fact alone seems to eliminate choices (A), (D), and (E). So, if you are going to guess because of lack of time, you have a 50% chance. These are good odds. Each of the circular regions represents $\frac{3}{4}$ of a complete circle. Since there are 4 of these, then $\cancel{4}\left(\frac{3}{\cancel{4}}\right) = 3$ full circles. The radius of each circle is one-half the side of the square $= \frac{1}{2}(8) = 4$. So the area of one circle is $\pi 4^2 = \pi 16$ or 16π. The area, then, of 3 such circles is equal to $3(16\pi) = 48\pi$.

Mathematical Aptitude Test 3

Time—30 Minutes
25 Questions

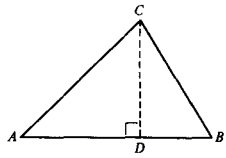
In this section solve each problem, using any available space on the page for scratchwork. Then indicate the *one* correct answer in the appropriate space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$.
The number of degrees in the arc of a circle is 360.
The measure in degrees of a straight angle is 180.

Definition of symbols:

$<$ is less than	$\leqq$ is less than or equal to
$>$ is greater than	$\geqq$ is greater than or equal to
$\perp$ is perpendicular to	$\parallel$ is parallel to



Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\measuredangle CDA$ is a right angle, then

(1) area of triangle $ABC = \dfrac{(AB)\cdot(CD)}{2}$

(2) $AC^2 = AD^2 + DC^2$

Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible *except* when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

1. If $r = s + 3$, then $2r - 6 =$

   (A) $2s$ (B) $2s - 3$ (C) $s - 9$ (D) $2s - 9$
   (E) $3 - 2s$

2. $\dfrac{1/3 + 1/3 + 1/3}{1/4 + 1/4 + 1/4 + 1/4} =$

   (A) $2/3$ (B) $4/3$ (C) 1 (D) $3/4$ (E) $5/12$

3. If s is the average of p and q, then which of the following must be true?

   (A) $p = q$ (B) $2(p + q) = s$ (C) $2p + q = s$
   (D) $2s + p = q$ (E) $p = 2s - q$

4. $2\sqrt{2}(3\sqrt{6}) =$

   (A) 72 (B) $12\sqrt{3}$ (C) 24 (D) 36 (E) $36\sqrt{2}$

5. If, on a map, the scale is $1/3$ inch = 1 mile, then a distance of $11\frac{1}{4}$ miles would be represented by how many inches?

   (A) $1\frac{1}{8}$ (B) $2\frac{1}{2}$ (C) $3\frac{3}{4}$ (D) 4 (E) $4\frac{1}{3}$

6. If the radius of a circle is increased by 10%, its area is increased by

   (A) 10% (B) 20% (C) 21% (D) 32% (E) 40%

7. The number of days in w weeks and h hours is

   (A) $\dfrac{w}{7} + 24h$ (B) $7w + \dfrac{h}{24}$ (C) $7w + 24h$
   (D) $7h + \dfrac{w}{24}$ (E) $\dfrac{h}{7} + 24w$

8. If $x:y = a:b$, then $x =$

(A) $\frac{ay}{b}$ (B) $\frac{by}{a}$ (C) aby (D) $\frac{ab}{y}$ (E) $\frac{b}{ay}$

9. If $0.02x - 0.3 = 0.104 - 2x$, then $x =$

(A) 0.2 (B) 0.02 (C) 0.002 (D) 2.02 (E) 2.2

10. Of the following numbers, the one which is the greatest is

(A) $^5/_8$ (B) $^3/_5$ (C) .627 (D) $^{16}/_{25}$ (E) $62^2/_3\%$

11. If two sides of a triangle are 3.5 and 7.5, which of the following may be the largest integer length of the third side?

(A) 4 (B) 7 (C) 8 (D) 10 (E) 11

12. If the price of an item was increased by 20% one week and then decreased by 10% the second week, what was the change, in percent, from the original price?

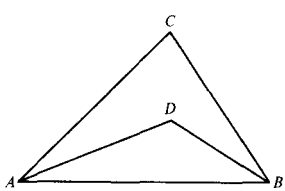
(A) 8% (B) 10% (C) 12% (D) 15% (E) 18%

13. The posts of a fence are 6 feet apart. The distance from the first post to the last post is 126 feet. How many posts are in the fence?

(A) 19 (B) 20 (C) 21 (D) 22 (E)23

14. If $\frac{p}{q} > t$, and p and q are both negative, then which of the following must be true?

(A) $p > qt$ (B) $p < qt$ (C) $p > q + t$
(D) $p > q - t$ (E) $p > t - q$



Note: Figure not drawn to scale.

15. In triangle ABC above, AD and BD bisect angles A and B respectively. If angle $C = 80°$, then angle $D =$

(A) 50° (B) 80° (C) 100° (D) 130° (E) 150°

16. If $Z*a = 10^a$, and $Z*b = 10^b$, then $\frac{Z*a}{Z*b} =$

(A) $\frac{a}{b}$ (B) 10^{a-b} (C) 1 (D) 10 (E) $a - b$

17. If a grams of substance 1 is mixed with b grams of substance 2, then the percent of substance 1 in the mixture is

(A) $\frac{100a}{b}$ (B) $\frac{100b}{a+b}$ (C) $\frac{a}{a+b}$ (D) $\frac{100a}{a+b}$
(E) $\frac{a+b}{100a}$

18. If $a^b = 3$, then $a^{b+2} =$

(A) 5 (B) $3a^2$ (C) 9 (D) $9b$ (E) $6a$

19. If $p \# q = 2(p + 2q)$, then if $a \# b = b \# a$, which of the following must be true?

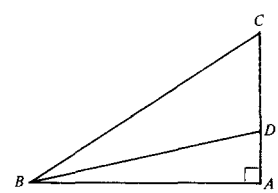
(A) $a = b$ (B) $a = -b$ (C) $a = b = 0$
(D) $a = b = 1$ (E) $a = 2b$

20. Which of the following can never have a value of 0 if x is real?

(A) $x^2 - 1$ (B) $x^3 + 1$ (C) $x^2 + 1$ (D) $\frac{x}{2}$
(E) $x^4 - 4^x$

21. If $(p + q)(ap - aq) = p^2 - q^2$, then $a =$

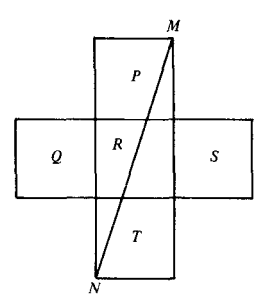
(A) p (B) q (C) 1 (D) 0 (E) -1



Note: Figure not drawn to scale.

22. In the figure above, $BC = 15$, $AB = 12$, and $DB = 13$. $CD =$

(A) 2 (B) 5 (C) 3 (D) 4 (E) 9

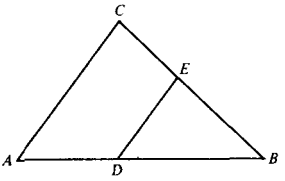


Note: Figure not drawn to scale.

23. In the figure above, P,Q,R,S, and T are squares.

If the areas of the squares are equal and the perimeter of the figure is 60, then $MN =$

(A) $5\sqrt{10}$ (B) 17 (C) 20 (D) $10\sqrt{5}$ (E) $15\sqrt{2}$



Note: Figure not drawn to scale.

24. In triangle ABC above, $DE \parallel AC$, $BE = 2$, $CE = 4$, and $DE = 3$. $AC =$

(A) 5.4 (B) 6 (C) 7.2 (D) 7.6 (E) 9

25. A cube of volume 1000 is cut into 125 cubes each of volume 8. The total surface area of the 125 cubes is how much larger than the surface area of the original cube?

(A) 600 (B) 1200 (C) 2400 (D) 2850 (E) 3000

STOP

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THIS TEST.

Answer Sheet—Mathematical Aptitude Test 3

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
25	☐	☐	☐	☐	☐

ANSWERS AND EXPLANATIONS

1. (A)	2. (C)	3. (E)	4. (B)	5. (C)
6. (C)	7. (B)	8. (A)	9. (A)	10. (D)
11. (D)	12. (A)	13. (D)	14. (B)	15. (D)
16. (B)	17. (D)	18. (B)	19. (A)	20. (C)
21. (C)	22. (D)	23. (A)	24. (E)	25. (C)

1. (A) By substitution, $2r - 6 = 2(s + 3) - 6 = 2s + 6 - 6 = 2s$.

2. (C) $\frac{3/3}{4/4} = \frac{1}{1} = 1$

3. (E) $s = \frac{p + q}{2}$; "cross-multiplying," $2s = p + q$; $2s - q = p$. The important word in the problem is "must." If $p = q = s = 0$, choices (A), (B), (C), and (D) would all be correct.

4. (B) $(2\sqrt{2})(3\sqrt{6}) = (2)(3)(\sqrt{2})(\sqrt{6}) = 6\sqrt{12}$. Since $\sqrt{12}$ is not rational, the answer cannot be (A), (C), or (D). So, if you must guess, your odds are 50%. $6\sqrt{12} = 6\sqrt{4 \times 3} = 6 \times 2\sqrt{3} = 12\sqrt{3}$.

5. (C) This is a direct proportion, for the more miles there are, the more inches there will be: Remember, $\frac{\text{inches}}{\text{miles}} = \frac{\text{inches}}{\text{miles}}$ $\frac{1/3}{1} = \frac{x}{11\frac{1}{4}}$. Change $11\frac{1}{4}$ to a fraction and then "cross-multiply" to find x: $x = \frac{1}{\cancel{3}} \times \frac{\overset{15}{\cancel{45}}}{4} = \frac{15}{4} = 3\frac{3}{4}$.

6. (C) % increase $= \frac{\text{actual increase}}{\text{original}}$. Use the technique of substitution of numbers here. Let the original radius be 10, then the new radius is $10 + .10(10) = 10 + 1 = 11$. The actual increase in the area then is $\pi(11)^2 - \pi(10)^2 = 121\pi - 100\pi = 21\pi$. The original area is 100π. So the percent increase is $\frac{21\pi}{100\pi} = \frac{21}{100} = .21 = 21\%$. Algebraically, if the original radius is r, then the new radius is $r + .10r = 1r + .10r = 1.10r$ or just $1.1r$. The old area is πr^2 and the new area is $\pi(1.1r)^2 = 1.21\pi$. The actual increase is $1.21\pi r^2 - 1\pi r^2 = .21\pi r^2$. The percent increase is then $\frac{.21\pi r^2}{1\pi r^2} = .21$ or 21%.

7. (B) The number of days in one week is 7. The number of days in 2 weeks is 2(7), etc. The number of days in w weeks is $w(7)$ or $7w$. We are down to that 50% guessing option between choices (B) and (C). Since there are 24 hours in one day, then 1 hour is $\frac{1}{24}$ of a day, 2 hours is $\frac{2}{24}$ of a day, etc. So, h hours is $\frac{h}{24}$ of a day. Therefore, the number of days in w weeks and h hours is $7w + \frac{h}{24}$.

8. (A) Read $x : y = a : b$ as "x is to y as a is to b" and then set up the proportion, $\frac{x}{y} = \frac{a}{b}$. "Cross-multiply" and get $bx = ay$. Divide both sides of the equation by b to "isolate" x: $x = \frac{ay}{b}$.

9. (A) To make .104 a whole number you must move the decimal point 3 places to the right. So, multiply both sides of the equation by 1000 (which moves *all* the decimal points 3 places to the right), to get $20x - 300 = 104 - 2000x$. Add $2000x$ to both sides now to get $2020x - 300 = 104$. Add 300 to both sides of the equation to "isolate" x: $2020x = 404$. Divide both sides of the equation by 2020 to find x:

$$\begin{array}{r} .2 \\ 2020\overline{)404.0} \\ \underline{404\ 0} \end{array}$$

10. (D) Change all the answers to decimals. $^5/_8 = 8\overline{)5.000}$; $^3/_5 = {}^6/_{10} = .6$; .627;

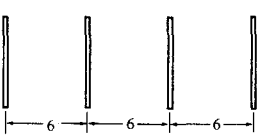
$$\begin{array}{r} .625 \\ 8\overline{)5.000} \\ \underline{4\ 8} \\ 200 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

$^{16}/_{25} = {}^{64}/_{100} = .64$; and since $^1/_3 = .333\ldots$ and $^2/_3 = .666\ldots$, so $62^2/_3\% = .62666\ldots$ The greatest number here is then .64.

11. (D) Remember, the third side of a triangle must be less than the sum of the other two sides and greater than the difference of the other two sides. So, $7.5 - 3.5 <$ third side $< 7.5 + 3.5$. Therefore, the third side must be less than 11 and greater than 4. So, the *largest* integer length the third side can have in this problem is 10.

12. (A) $\%\ \text{change} = \dfrac{\text{actual change}}{\text{original}}$. Let the original price be \$100. Then after it was increased by 20%, its new price is $\$100 + .20(\$100) = \$100 + \$20 = \$120$. If this new price is decreased by 10% now, the final price is $\$120 - .10(\$120) = \$120 - \$12 = \$108$. Therefore the net change is $\$108 - \$100 = \$8$. % change $= \dfrac{\$8}{\$100} = .08 = 8\%$.

13. (D) *Simplify* the problem so that the distance between the first and last posts is 18 feet:

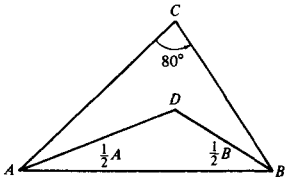


As you can see, dividing 18 by 6 gives you 3 but there are 4 posts. Now coming back to the real problem, if the distance from the first to the last post is 126 feet, then there are $\dfrac{126}{6} + 1$ posts, or $21 + 1 = 22$ posts. Be careful in problems such as these.

14. (B) Let $p = -5$ and $q = -4$ and $t = 1$. Then $\dfrac{-5}{-4} > 1$ $\left(\dfrac{5}{4} > 1\right)$. Multiplying both sides of the inequality by -4, $-5 < -4$. Remember, when you multiply or divide both sides of an inequality by a negative number you *must change the direction* of the inequality symbol. So, in this case, if $\dfrac{p}{q} > t$, then $p < qt$. You car

do this problem by the technique of substitution of numbers *and elimination*. Using the same values of p, q, and t above, let's test (E). $p > t - q$. $-5 > 1 - -4$. Is $-5 > 5$? *No*. (D) $-5 > -4 - 1$. Is $-5 > -5$? *No*. (C) $-5 > -4 + 1$. Is $-5 > -3$? *No*. So, the answer is either (A) or (B).

We know what you are thinking here. What if you chose other numbers for p, q, and t? *But*, the problem states, "Which of the following *must* be true?" We are producing *counterexamples* to show that there is at least *one* case where some of the examples are false. Our last choice is (A). $-5 > (-4)(1)$. Is $-5 > -4$? *No*. We have eliminated 4 choices out of 5!



15. (D) Since the sum of the measure of the angles of any triangle is 180° and we are given that $\measuredangle C = 80°$, then $\measuredangle A + \measuredangle B = 100°$. We can find angle D if we know the *sum* of angles DAB and DBA. Since angles CAB and CBA are bisected, $\measuredangle DAB = \frac{1}{2}\measuredangle CAB$ and $\measuredangle DBA = \frac{1}{2}\measuredangle CBA$. Therefore, since $\measuredangle A + \measuredangle B = 100°$, $\frac{\measuredangle A}{2} + \frac{\measuredangle B}{2} = \frac{100°}{2} = 50°$. So, angle $D = 180° - 50° = 130°$. No need for algebra here, though. You can try some numbers. Let $\measuredangle A = 40°$ and $\measuredangle B = 60°$. Then, the two angles in the smaller triangle are 20° and 30°. Therefore, since their sum is 50°, $\measuredangle D = 130°$. Try other values to convince yourself that $\measuredangle D = 130°$.

16. (B) By simple substitution, since $Z{*}a = 10^a$ and $Z{*}b = 10^b$, then $\frac{Z{*}a}{Z{*}b} = \frac{10^a}{10^b} = 10^{a-b}$. For those of you who marked choice (A), it's because you forgot that Z is *not* a number but an operation.

17. (D) Let's assume that there are 10 grams of substance 1 and 30 grams of substance 2. Then the percent of substance 1 in the *mixture* is $\frac{10}{10 + 30} = \frac{10}{40} = \frac{1}{4} = .25$. No matter what numbers you choose, the *fraction* that substance 1 is of the entire mixture is $\frac{a}{a + b}$. To change that fraction to a % we must then first change it to a decimal and then multiply that decimal by 100 (move the decimal point 2 places to the right). In order to change $\frac{a}{a + b}$ to a %, then, we must multiply it by 100 to get $\frac{100}{1} \times \frac{a}{a + b} = \frac{100a}{a + b}$. So in the case where $a = 10$ and $b = 30$, $\frac{10}{10 + 30} = \frac{1}{4}$. Then, $\frac{1}{4} = \left(100 \times \frac{1}{4}\right)\%$ or 25%.

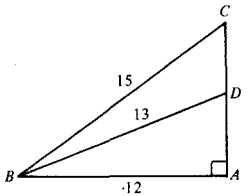
18. (B) $a^{b+2} = a^b a^2$. Since $a^b = 3$, then $a^{b+2} = 3a^2$. What did *you* do with that 2?

19. (A) According to this *invented* operation, #, $a \# b = 2(a + 2b) = 2a + 4b$. $b \# a = 2(b + 2a) = 2b + 4a$. Since $a \# b = b \# a$, $2a + 4b = 2b + 4a$. Subtracting $2a$ from both sides of the equation, $4b = 2b + 2a$. Subtracting $2b$ from both sides of the equation, we get $2b = 2a$. Dividing now by 2, we get $b = a$. Choices (C) and (D) may be true but not *necessarily* true. Choices (B) and (E) *can* be true if both a and b are 0. But the only conclusion about a and b that *must* be

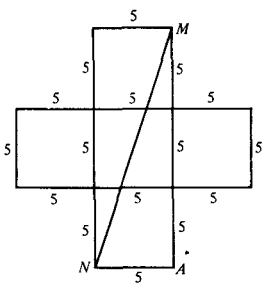
true is that *a must* equal b. So, in a case such as this, you can be led *astray* by the substitution of numbers.

20. (C) In choice (A), let $x = 1$. Then $1^2 - 1 = 1 - 1 = 0$. In choice (B), let $x = -1$. Then, $(-1)^3 + 1 = -1 + 1 = 0$. In choice (D), let $x = 0$, then $\frac{0}{2} = 0$. In choice (E), let $x = 2$. Then $2^4 - 4^2 = 16 - 16 = 0$. So the answer is (C). If x is negative, x^2 is positive and $x^2 + 1 \neq 0$. If $x = 0$, then $x^2 = 0$ and $x^2 + 1 \neq 0$. If x is positive, x^2 is positive and $x^2 + 1 \neq 0$.

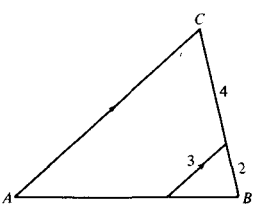
21. (C) By the distributive postulate, $(ap - aq) = a(p - q)$. So the problem can be rewritten as $(p + q)(ap - aq) = p^2 - q^2 = (p + q)(a)(p - q)$. Since $(p + q)(p - q) = p^2 - q^2$, $(p + q)(a)(p - q) = a(p^2 - q^2)$. Therefore, *a must* be 1.



22. (D) We can find AC quickly by using the Pythagorean Theorem: $BC^2 = BA^2 + AC^2$; $15^2 = 12^2 + AC^2$; $225 = 144 + AC^2$; $81 = AC^2$; $AC = 9$. Since $CD = AC - AD$ and since we know AC now, we only have to find AD to finish the problem. In right triangle BAD, $BD^2 = BA^2 + AD^2$; $13^3 = 12^2 + AD^2$; $169 = 144 + AD^2$; $25 = AD^2$; $AD = 5$. So, $CD = AC - AD = 9 - 5 = 4$. The solution would have been arrived at much quicker had you remembered the 5-12-13 right triangle and the 9-12-15 (a multiple of the 3-4-5) right triangle. You would not have had to use the Pythagorean Theorem at all then!

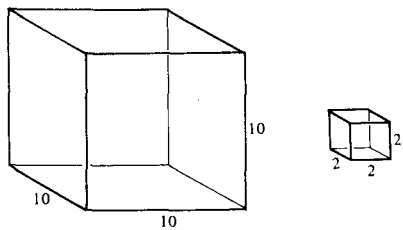


23. (A) If the perimeter of the figure is 60, then to get from M to M again you must "travel" over 12 equal line segments, each of which must be $^{60}/_{12} = 5$. (The fact that the areas are equal makes those line segments equal.) So, in right triangle NAM, $NM^2 = NA^2 + AM^2 = 5^2 + (5 + 5 + 5)^2 = 5^2 + 15^2 = 25 + 225 = 250$. $NM = \sqrt{250} = \sqrt{25 \times 10}\ \sqrt{25}\ \sqrt{10} = 5\sqrt{10}$.



24. (E) Since $DE \| AC$, angles C and DEB are equal and angles A and EDB are equal (corresponding angles). So, triangle ABC and DEB are similar (A.A.)

Therefore, the corresponding sides of those triangles are in proportion. $\frac{AC}{DE} = \frac{CB}{EB}$. Substituting the given values now, $\frac{AC}{3} = \frac{6}{2}$. "Cross-multiplying," we get, $2AC = 18$; $AC = 9$. Be careful. Many students get (B) as their answer because they forget that CE is not a *side* of either triangle, and they set up the following proportion: *ERROR* $\frac{AC}{3} = \frac{4}{2}$.



25. (C) If a cube has a volume of 1000, then each edge must be 10. (Volume = length × width × height. 10 × 10 × 10 = 1000.) The surface area of a cube is the area of one square "face" × 6 (because there are 6 "faces.") Since the area of one square face is 10 × 10 (length × width = area), the total surface area of the large cube is 100 × 6 = 600. Now, let's look at the small cubes. If the volume of one of the cubes is 8, then each edge is 2, since 8 = 2 × 2 × 2. The area of one square face is then 2 × 2 = 4. Six such faces gives a total of 24 for the surface area of one cube. Since there are 125 of these small cubes the total surface area = 125 × 24 = 3000. So, the total surface area of the 125 cubes is 3000 − 600 greater than the total surface area of the original big cube. 3000 − 600 = 2400.

Mathematical Aptitude Test 4

Time—30 Minutes
35 Questions

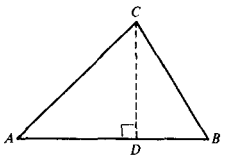
In this section solve each problem, using any available space on the page for scratchwork. Then indicate the *one* correct answer in the appropriate space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$.
The number of degrees in the arc of a circle is 360.
The measure in degrees of a straight angle is 180.

Definition of symbols:

$<$ is less than	$\leqq$ is less than or equal to
$>$ is greater than	$\geqq$ is greater than or equal to
$\perp$ is perpendicular to	$\parallel$ is parallel to



Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\measuredangle CDA$ is a right angle, then

(1) area of triangle $ABC = \dfrac{(AB)\cdot(CD)}{2}$

(2) $AC^2 = AD^2 + DC^2$

Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible *except* when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

1. If $x - 3 = 5$, then $2x - 3 =$

   (A) 5 (B) 7 (C) 8 (D) 11 (E) 13

2. On a number line, if the left end point of a segment is at -12 and the midpoint of the segment is at -4, then the right end point is at

   (A) -8 (B) -6 (C) 0 (D) 4 (E) 6

3. How many thirds are there in $^3/_4$?

   (A) 2 (B) $2^1/_4$ (C) 3 (D) $3^1/_4$ (E) 4

4. $\sqrt{12} + \sqrt{27} =$

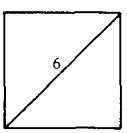
   (A) $5\sqrt{3}$ (B) $\sqrt{39}$ (C) $5\sqrt{6}$ (D) 15 (E) 18

5. If $a \# b = a^2 + b^2$, then $(1 \# 1)\#1 =$

   (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

6. If $n = 1$, then $(n + 1)^{n+1} + (n + 1)^{n-1} =$

   (A) 0 (B) 4 (C) 5 (D) 6 (E) 8

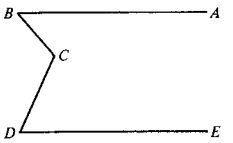


7. The area of the square above is

   (A) 10 (B) 12 (C) 18 (D) 36 (E) 40

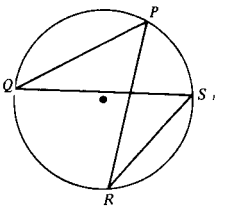
8. If $\measuredangle B = 50°$, $\measuredangle D = 65°$, and $AB \parallel DE$, then $\measuredangle BCD =$

(A) 115° (B) 90° (C) 100° (D) 85° (E) 120°

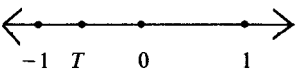


Note: See Test 1, p. 208, for instructions on marking your answers for problems 9–30.

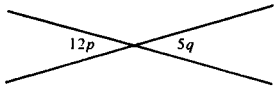
	Column A		*Column B*	
		$x < 0$		
9.	$x^5 + 1$		0	A B C D
10.	$a + 7$		$7 - a$	A B C D
		$p > 0$ $q > 0$		
11.	$p - q$		$p + q$	A B C D
		$a + b = 1$ $-1 < a < 1$		
12.	2		b	A B C D



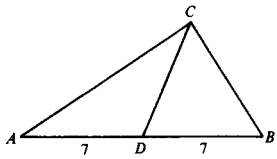
13.	$\measuredangle PQS$		$\measuredangle PRS$	A B C D
14.	$(p - q)^2$		$p^2 - q^2$	A B C D
		$m = 0.375$ $n = 0.75$		
15.	0.75		mn	A B C D



16.	$5T$		T^5	A B C D
17.	$505 \times 95 \times 2 \times 3$		$5 \times 95 \times 505 \times 1.2$	A B C D
18.	¼%		0.025	A B C D
		$\frac{1}{x} < 0$		
19.	$\frac{-1}{x}$		x	A B C D



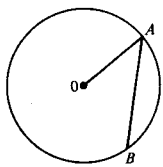
	Column A	Column B	
20.	p	q	A B C D
21.	The average of 0.55, 0.66, and 0.77	$\frac{2}{3}$	A B C D



22.	The area of triangle *ACD*	The area of triangle *BCD*	A B C D

$\sqrt{x} + 2 = 15$

23.	x	150	A B C D



Note: Figure not drawn to scale.

24.	*OA*	*AB*	A B C D

$A * B = \frac{1}{A} - \frac{1}{B}$

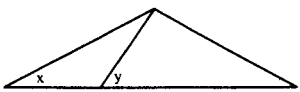
25.	The value of $A * B$ if $A = \frac{1}{2}$ and $B = \frac{1}{3}$	-1	A B C D

60 is $x\%$ of 30

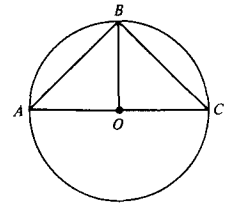
26.	$3x$	150	A B C D

$0 < a < b < c$

27.	$a + b$	$c - b$	A B C D



28.	x	$y - x$	A B C D



$AB = BC = \sqrt{2}$
0 is the center of the circle.

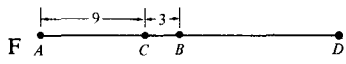
29.	$AB + BC$	$AO + OB + OC$	A B C D
30.	Area of a circle with radius 7.	Area of an equilateral triangle with side 14.	A B C D

31. In the following computer program,
Step 1 LET $X = 5$
Step 2 LET $Y = -3$
Step 3 IF $X < 2Y + 8$ THEN GO TO STEP 5: OTHERWISE GO TO STEP 7
Step 4 LET $A = X + 2$
Step 5 LET $A = X + 3$
Step 6 STOP
Step 7 LET $A = X + 4$
Step 8 STOP
what would be the value of A after the program STOPs?

(A) -2 (B) 0 (C) 7 (D) 8 (E) 9

32. If n is a positive integer, then which of the following is always odd?

(A) $19n + 6$ (B) $18n + 4$ (C) $19n + 5$
(D) $18n + 5$ (E) $19n^2 + 5$



33. In the figure below, $AC = 9$, $BC = 3$, and point D is 3 times as far from A as from B. $BD =$

(A) 9 (B) 12 (C) 6 (D) 18 (E) 15

34. A cubic foot of water is poured into a rectangular box whose base is 12 inches by 18 inches. How high up will the water rise?

(A) 4 inches (B) 6.5 inches (C) 8 inches
(D) 12.5 inches (E) 18 inches

35. If the volume of a cube is 8, then the sum of the lengths of its edges is

(A) 8 (B) 24 (C) 16 (D) 32 (E) 64

STOP

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THIS TEST.

Answer Sheet—Mathematical Aptitude Test 4

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
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26	☐	☐	☐	☐	☐
27	☐	☐	☐	☐	☐
28	☐	☐	☐	☐	☐
29	☐	☐	☐	☐	☐
30	☐	☐	☐	☐	☐
31	☐	☐	☐	☐	☐
32	☐	☐	☐	☐	☐
33	☐	☐	☐	☐	☐
34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐

ANSWERS AND EXPLANATIONS

1. (E)	2. (D)	3. (B)	4. (A)	5. (E)
6. (C)	7. (C)	8. (A)	9. (D)	10. (D)
11. (B)	12. (A)	13. (C)	14. (D)	15. (A)
16. (B)	17. (C)	18. (B)	19. (A)	20. (B)
21. (B)	22. (C)	23. (A)	24. (D)	25. (C)
26. (A)	27. (D)	28. (D)	29. (B)	30. (A)
31. (E)	32. (D)	33. (C)	34. (C)	35. (B)

1. (E) If $x - 3 = 5$, then $x = 8$. $2(8) - 3 = 16 - 3 = 13$.

A	M	B
-12	-4	4

2. (D) By the midpoint formula, $-4 = \frac{-12 + B}{2}$. "Cross-multiplying," we get $-8 = -12 + B$; adding 12 to both sides of the equation, $-8 + 12 = -12 + B + 12$; $4 = B$.

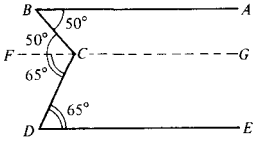
3. (B) Change the problem so that it reads, "How many twos are there in 6?" Obviously, the answer is 3, which you get by *division*. Now, go back to the original problem. "How many thirds are there in $^3/_4$?" So, we must divide $^3/_4$ by $^1/_3$; $^3/_4 \div {}^1/_3 = {}^3/_4 \times {}^3/_1 = {}^9/_4 = 2^1/_4$. So, there are $2^1/_4$ thirds in $^3/_4$. (A little-known fact!)

4. (A) $\sqrt{12} + \sqrt{27} = \sqrt{4 \times 3} + \sqrt{9 \times 3} = \sqrt{4}\ \sqrt{3} + \sqrt{9}\ \sqrt{3} = 2\ \sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$. Remember, it is generally *not* true that $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$, for those of you who marked choice (B).

5. (E) Do what is in the parentheses first: $(1 \# 1) \# 1 = (1^2 + 1^2) \# 1 = (1 + 1) \# 1 = 2 \# 1$. Now, $2 \# 1$ means $2^2 + 1^2 = 4 + 1 = 5$.

6. (C) Substitute 1 for n to get $(1 + 1)^{1+1} + (1 + 1)^{1-1} = 2^2 + 2^0 = 4 + 1 = 5$. Remember, if $a \neq 0$, a^0 is defined as 1. For example, $2^0 = 1$, *not* 0.

7. (C) The area of a square is equal to $(\text{side})^2$. If you represent a side of the square by x, then by the Pythagorean Theorem, $x^2 + x^2 = 6^2 : 2x^2 = 36; x^2 = 18$. Therefore, $(\text{side})^2 = (x)^2 = 18$. It would have been much quicker had you remembered the formula, Area $= {}^1/_2\,(\text{diagonal})^2$. Then, area $= {}^1/_2(6^2) = {}^1/_2(36) = 18$.



8. (A) Draw $FG \parallel AB$. Then $BCF = ABC = 50$ and $FCD = CDE = 65$ (alternate interior angles). Therefore, $BCD = BCF + FCD = 50 + 65 = 115$.

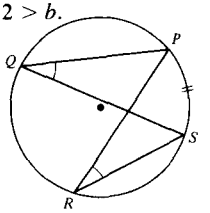
Note: In answers 9-30, A = B means that the quantities in Columns A and B are equal; A > B means that the quantity in Column A is greater than the quantity in Column B; and B > A means that the quantity in Column B is greater than the quantity in Column A.

9. (D) Let $x = -1$. Then $x^5 + 1 = (-1)^5 + 1 = -1 + 1 = 0$. Now let $x = -2$. Then $(-2)^5 + 1 = -32 + 1 = -31$. So A = B *or* B > A. Since there are *at least* 2 possibilities, we must mark choice (D) because the relationship *cannot* be determined from the information given.

10. (D) If $a = 1$, then $a + 7 > 7 - a$. If $a = 0$, then $a + 7 = 7 - a$. Since A > B *or* A = B, we must mark choice (D) again.

11. (B) In this problem, it doesn't matter which number, p or q, is the greater one. The difference of two positive numbers is always less than their sum. B > A.

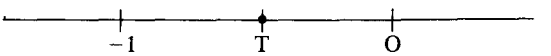
12. (A) Pick some values of a that are between -1 and 1. If $a = -1/2$, then b must be $3/2$ since $a + b = 1$. Therefore, $2 > b$. If $a = 1/2$, then $b = 1/2$ also and again, $2 > b$. If $a = 0$, then $b = 1$ and $2 > b$ *again*. So, it *seems* that A > B. Algebraically, if $a + b = 1$, then $a = 1 - b$. We are given that $-1 < a < 1$. Substitute $a = 1 - b$ into that inequality to get $-1 < 1 - b < 1$. This inequality is equivalent to the *two* separate inequalities, $-1 < 1 - b$ *and* $1 - b < 1$. In the *first* inequality, if we add b to both sides we get $-1 + b < 1 - b + b$ or $-1 + b < 1$. Then, adding 1 to both sides, we get $-1 + b + 1 < 1 + 1$ or $b < 2$. In the second inequality, $1 - b < 1$, add b to both sides to get $1 - b + b < 1 + b$ or $1 < 1 + b$. Now, subtract 1 from both sides to get $1 - 1 < 1 + b - 1$ or $0 < b$. Therefore, $0 < b < 2$. So, it *is* true that $2 > b$.



13. (C) An inscribed angle is equal to $1/2$ (its intercepted arc). $\measuredangle PQS = 1/2\ (\widehat{PS})$ and $\measuredangle PRS = 1/2\ (\widehat{PS})$. Therefore, $\measuredangle PQS = \measuredangle PRS$. (If $x = y$ and $z = y$, then $x = z$.)

14. (D) If $p = q$, then $(p - q)^2 = (0)^2 = 0$ and $p^2 - q^2 = q^2 - q^2 = 0$. So, A = B. If $p = 2$ and $q = 3$, then $(p - q)^2 = (2 - 3)^2 = (-1)^2 = 1$ and $p^2 - q^2 = 2^2 - 3^2 = 4 - 9 = -5$. In this case, A > B. Therefore, we must mark choice (D).

15. (A) $mn = (.375)(.75)$ = (a number less than 1)(.75) = a number less than .75. Therefore, A > B. Choice (A).



16. (B) Let's try some values for T. Since T is between -1 and 0, it must be a negative fraction. Let $T = -1/2$; then $T^5 = (-1/2)^5 = -1/32$. $5T = 5(-1/2) = -5/2$, so T^5 is greater (it is closer to 0 on the number line). Let T be $-1/10$ (very close to 0), then $T^5 = (-1/10)^5 = -1/100000$. $5T$, on the other hand, is $5(-1/10) = -5/10 = -1/2$. Again, T^5 is closer to 0 and is greater.

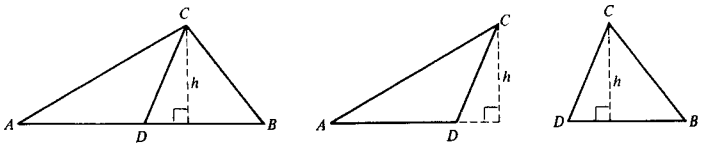
17. (C) Rearrange the factors so that the product in Column A is $\underbrace{505 \times 95}$ $\times$ 2×3 and the product in Column B is $\underbrace{505 \times 95}$ $\times$ 1.2×5. Since $2 \times 3 = 1.2 \times 5$, A = B. In arithmetic problems such as these, the answer can *never* be (D).

18. (B) ¼% = .25% = .0025. Since .025 > .0025, B > A.

19. (A) If $\frac{1}{x}$ is less than 0, then, since the numerator is positive, the denominator, x, *must* be negative. Therefore, the quantity in Column A, $\frac{-1}{x}$, represents a positive number (a negative divided by a negative is a positive). Since a positive number is always greater than a negative number, A > B.

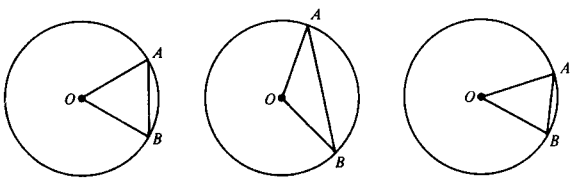
20. (B) The two angles indicated in the problem are vertical angles, so they are equal. $12p = 5q$. Dividing both sides of the equation by 12, we get $p = 5q/12$. Therefore, $q > p$ or B > A.

21. (B) $\frac{.55 + .66 + .77}{3} = .66$ (the average of an odd number of numbers which are equally spaced is the middle number). In Column B, $\frac{2}{3} = .6666\ .\ .\ .$ Therefore, B > A.



22. (C) If two triangles have equal bases and the same height (altitude), then they have equal area. Therefore, A = B.

23. (A) If $\sqrt{x} + 2 = 15$, then $\sqrt{x} = 13$. Squaring both sides of the equation, we get $(\sqrt{x})^2 = 13^2 = 169$. Therefore, $x = 169$, so A > B.

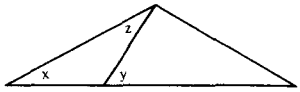


24. (D) $\overline{OA} = \overline{OB}$ (radii of the same circle are equal). If $\measuredangle AOB = 60°$, then $\measuredangle A = \measuredangle B = 60°$ ($\measuredangle A = \measuredangle B$ because $\overline{OA} = \overline{OB}$), and the triangle, AOB, is equilateral. Therefore, $\overline{OA}$ *could* equal $\overline{AB}$. *But,* if $\measuredangle AOB$ is other than 60°, then $OA > AB$ or $OA < AB$. You should mark choice (D).

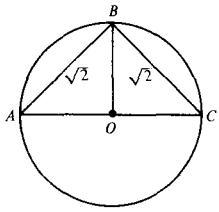
25. (C) If $A = ½$ and $B = ⅓$, then by definition, $½ * ⅓ = \frac{1}{½} - \frac{1}{⅓} = 2 - 3 = -1$. So, A = B.

26. (A) If 60 is x% of 30, then we can write this as $60 = \left(\frac{x}{100}\right)(30) = \frac{30x}{100}$. Therefore, $60 = \frac{3x}{10}$. Now, "cross-multiply" to get $3x = 600$; therefore $3x > 150$, so A > B.

27. (D) Let $a = 1$, $b = 2$, and $c = 3$. Then $a + b = 3$ and $c - b = 1$. So, A > B. *But,* if $a = 1$, $b = 2$, and $c = 10$, then $a + b = 3$ and $c - b = 8$. In this case, B > A. So you must mark choice (D).



28. (D) Let $x = 20°$ and $z = 40°$. Then $y = x + z = 60°$. (An exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles.) So, $y - x = 60° - 20° = 40°$. B > A. *But,* if you let $x = z = 50°$, then $y = x + z = 100°$ and $y - x = 100° - 50° = 50°$. In this case, A = B. Once again, our advice is to substitute several values of the variables, if you have time, before making your conclusion.



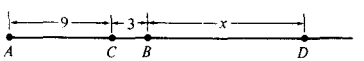
29. (B) Triangle ABC is an isosceles *right* triangle. ($\measuredangle B = \frac{1}{2}(\widehat{AC}) = \frac{1}{2}(180°) = 90°$.) By the Pythagorean Theorem, then, $AC^2 = AB^2 + BC^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 2 + 2 = 4$. So, $AC = 2$. Since AC is a diameter, the radii shown, $AO = OB = OC = 1$. Now we are ready to decide which quantity is the greater one. $AB + BC = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$. $AO + OB + OC = 1 + 1 + 1 = 3$. Since $\sqrt{2}$ is approximately 1.4, then $2\sqrt{2}$ is approximately 2.8. Therefore, B > A.

30. (A) The area of a circle is πr^2. So, the area of the circle described in Column A is $\pi 7^2$ or 49π. The area of an equilateral triangle is $\frac{(\text{side})^2}{4}\sqrt{3}$. So, the area of the equilateral triangle described in Column B is $\frac{14^2}{4}\sqrt{3} = \frac{196}{4}\sqrt{3} = 49\sqrt{3}$. Since $\pi > \sqrt{3}$, $49\pi > 49\sqrt{3}$, so (A) is the correct choice.

31. (E) After step 2, the computer has "put" 5 into X and -3 into Y. At step 3, the computer "compares" X with $2Y + 8$. Since $X = 5$ and $2Y + 8 = -6 + 8 = 2$, X is *not* less than $2Y + 8$ so the computer "goes" to step 7 and "puts" the value of $X + 4$ "into" A. Therefore, $A = 5 + 4 = 9$. Then the computer "goes" to step 8 and STOPS.

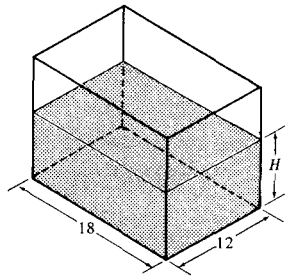
32. (D) In (A), let $n = 2$. Then $19n + 6 = 38 + 6 = 44$. Even.
In (B), let $n = 1$. Then $18n + 4 = 18 + 4 = 22$. Even.
In (C), let $n = 1$. Then $19n + 5 = 19 + 5 = 24$. Even.
In (E), let $n = 1$. Then $19n^2 + 5 = 19 + 5 = 24$. Even.

So, the answer must be (D) by *elimination.* A much *quicker* method, of course, is to realize that since 18 is an even number, $18n$ is always even (try $n = 1, 2, 3$, etc. and see for yourself). $18n + 5$, therefore, is an odd number because an even + an odd = an odd, always!

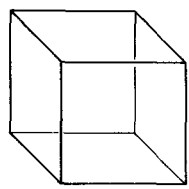


Note: Figure not drawn to scale.

33. (C) We are given that $DA = 3(DB)$. Therefore, $DA = 12 + x$ and $DB = x$. So, $12 + x = 3(x)$. Subtracting x from both sides of the equation, we get $12 = 2x$; $6 = x$.



34. (C) One cubic foot measures 12″ × 12″ × 12″. If this cubic foot of water is poured into a box (shown above) whose base measures 12″ × 18″, then the water will reach a height, H. Since it is assumed that no water is lost in the transfer, $12 \times 12 \times 12 = 12 \times 18 \times H$. Divide both sides of the equation by 12×18 to "isolate" H: $\frac{\cancel{12} \times \cancel{18} \times H}{\cancel{12} \times \cancel{18}} = \frac{\cancel{12} \times \overset{2}{\cancel{12}} \times \overset{4}{\cancel{12}}}{\cancel{12} \times \underset{\cancel{3}}{\cancel{18}}}$, so $H = 8''$.



35. (B) The volume of a cube is $(\text{edge})^3$. In this problem, then, $(\text{edge})^3 = 8$, so edge $= 2$. Since there are 12 edges in a cube, (4 for the top, 4 for the bottom, and 4 for the sides) the sum of the edges is $12 \times 2 = 24$.

Mathematical Aptitude Test 5

Time—30 Minutes
25 Questions

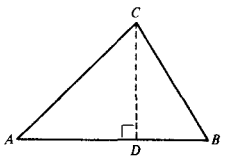
In this section solve each problem, using any available space on the page for scratchwork. Then indicate the *one* correct answer in the appropriate space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$.
The number of degrees in the arc of a circle is 360.
The measure in degrees of a straight angle is 180.

Definition of symbols:

$<$ is less than	$\leqq$ is less than or equal to
$>$ is greater than	$\geqq$ is greater than or equal to
$\perp$ is perpendicular to	$\parallel$ is parallel to



Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\measuredangle CDA$ is a right angle, then

(1) area of triangle $ABC = \dfrac{(AB)\cdot(CD)}{2}$

(2) $AC^2 = AD^2 + DC^2$

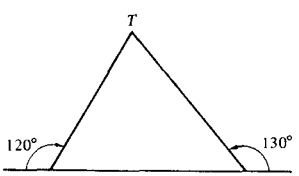
Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible *except* when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

1. A telephone pole is 24 feet high. What is the length, in feet, of a wire which extends from the top of the pole to a point that is 10 feet from the base of the pole?

   (A) 10 (B) 13 (C) 26 (D) 30 (E) 54

2. The angles of a triangle are in the ratio 1 : 2 : 3. If the smallest side of the triangle is 8, then the largest side of the triangle is

   (A) 10 (B) 12 (C) 14 (D) 16 (E) 24

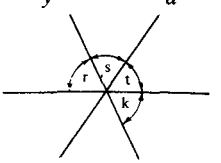


3. In the figure above, angle T =

   (A) 30° (B) 50° (C) 60° (D) 70° (E) 85°

4. If x pencils cost d dollars, then y pencils cost how many dollars?

   (A) $\dfrac{dy}{x}$ (B) $\dfrac{(x+d)}{y}$ (C) $\dfrac{(x+y)}{d}$ (D) $\dfrac{xy}{d}$ (E) $\dfrac{xd}{y}$



Note: Figure not drawn to scale.

5. If $r = 2s$ and $t = 3r$, then k =

   (A) 20° (B) 30° (C) 40° (D) 60° (E) 80°

6. If it takes m men h hours to complete a certain job, then how long will it take n men working at the same rate?

   (A) $\frac{mh}{n}$ (B) $\frac{nh}{m}$ (C) $\frac{(m+n)}{h}$ (D) $\frac{h}{(m+n)}$ (E) $\frac{2h}{m+n}$

7. If it costs \$1.50 for the first mile and 50¢ for each additional mile, then how many miles can a passenger travel in a taxicab for \$6.00?

   (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

8. 15% of what number is K?

   (A) $15K$ (B) $\frac{3K}{20}$ (C) $\frac{20K}{3}$ (D) $0.15K$ (E) $0.85K$

9. The surface area of a cube is 54. What is the volume of this cube?

   (A) 18 (B) 27 (C) 48 (D) 64 (E) 125

10. One man can do a job in 180 minutes and another man can do the same job in 60 minutes. If they work together, how many minutes will it take them to complete the job?

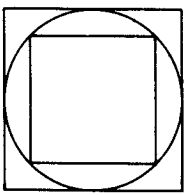
    (A) 30 (B) 45 (C) 60 (D) 120 (E) 150

11. Last night, Jack read from page 156 to page 200 inclusive in his science book. How many pages did he read?

    (A) 43 (B) 44 (C) 45 (D) 46 (E) 47

12. If p represents the perimeter of an equilateral triangle, and if $\frac{2}{3}(p) = \frac{8}{15}$, then each side of the triangle is

    (A) $\frac{4}{15}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{11}{15}$ (E) $\frac{1}{5}$



13. In the figure above, if the area of the larger square is 64, then the area of the smaller square is

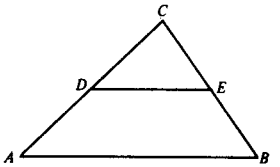
    (A) 32 (B) $16\sqrt{3}$ (C) $16\sqrt{2}$ (D) 28 (E) $28\sqrt{2}$

14. If $\frac{x}{8} = \frac{1}{5}$, then $\frac{x}{6} =$

    (A) $\frac{1}{12}$ (B) $\frac{5}{48}$ (C) $3\frac{3}{4}$ (D) $\frac{4}{15}$ (E) $9\frac{3}{5}$

15. A class contains twice as many boys as girls. What percent of the class is girls?

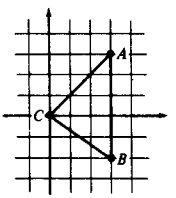
    (A) 25% (B) 33⅓% (C) 40% (D) 50% (E) 66⅔%



Note: Figure not drawn to scale.

16. If $DE \parallel AB$ and $3CD = AD$, what is the ratio of the area of triangle CDE to the area of triangle ABC?

    (A) 1:3 (B) 1:6 (C) 1:4 (D) 1:16 (E) cannot be determined from the information given



17. What is the area of triangle ABC?

    (A) 4.5 (B) 5 (C) 5.5 (D) 6 (E) 7.5

18. If $P + Q = 9$ and $P - Q = 3$, then $P^2 + Q^2 =$

    (A) 81 (B) 75 (C) 45 (D) 17 (E) 9

19. If $x^2 - y^2 = 1$ and $x - y = 1$, then $x + y =$

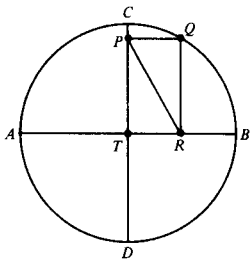
    (A) 1 (B) 0 (C) −1 (D) 2 (E) −2

20. If one bell rings every 12 minutes and another bell rings every 10 minutes, then the first time that both bells will ring at the same time is after how many minutes?

    (A) 140 (B) 120 (C) 80 (D) 60 (E) 48

21. How many degrees are there in the angle formed by the hands of a clock at 2:30 P.M.?

    (A) 95 (B) 100 (C) 105 (D) 110 (E) 120

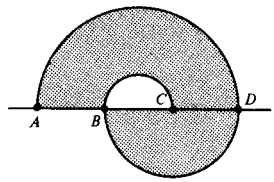


22. In the circle above, $PQRT$ is a rectangle, and AB and CD are diameters. If $TR = RB = 5$, then $PR =$

(A) 5 (B) 8 (C) 9 (D) 10 (E) cannot be determined from the information given

23. If $q\&t = q^t - 1$, then $2\&(2\&3) =$

(A) 12 (B) 127 (C) 181 (D) 255 (E) 331



24. In the figure above, if $AB = BC = CD = 4$ and all the arcs are semicircles, find the area of the shaded region.

(A) 36π (B) 24π (C) 52π (D) 60π (E) 72π

25. A 39-foot ladder is placed against a vertical wall of a building so that the ladder is flush with the wall. How far down the wall does the ladder slip after the base of the ladder is 15 feet from the base of the building?

(A) 3 feet (B) 4 feet (C) 4.2 feet (D) $4\sqrt{2}$ feet (E) 5.6 feet

STOP

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THIS TEST.

Answer Sheet—Mathematical Aptitude Test 5

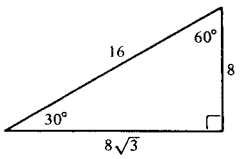
	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
11	☐	☐	☐	☐	☐
12	☐	☐	☐	☐	☐
13	☐	☐	☐	☐	☐
14	☐	☐	☐	☐	☐
15	☐	☐	☐	☐	☐
16	☐	☐	☐	☐	☐
17	☐	☐	☐	☐	☐
18	☐	☐	☐	☐	☐
19	☐	☐	☐	☐	☐
20	☐	☐	☐	☐	☐
21	☐	☐	☐	☐	☐
22	☐	☐	☐	☐	☐
23	☐	☐	☐	☐	☐
24	☐	☐	☐	☐	☐
25	☐	☐	☐	☐	☐

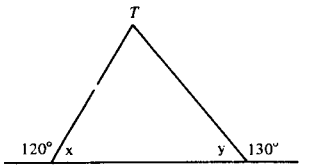
ANSWERS AND EXPLANATIONS

1. (C)	2. (D)	3. (D)	4. (A)	5. (C)
6. (A)	7. (B)	8. (C)	9. (B)	10. (B)
11. (C)	12. (A)	13. (A)	14. (D)	15. (B)
16. (D)	17. (E)	18. (C)	19. (A)	20. (D)
21. (C)	22. (D)	23. (B)	24. (B)	25. (A)

1. (C) $x^2 = 10^2 + 24^2 = 100 + 576 = 676; x = 26$. (If you remember the 5-12-13 right triangle, then the 10-24-26 right triangle "springs out" at you.)

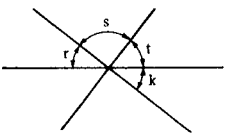
2. (D) $1x + 2x + 3x = 180°$; $6x = 180°$; $x = 30°$. Therefore, $2x = 60°$ and $3x = 90°$. So, we are dealing with a 30°-60°-90° right triangle. The smallest side is opposite the smallest angle, 30°, and the largest side, the hypotenuse, is twice the smallest side. Since the smallest side is given as 8, the largest side must be 16. The side opposite the 60° angle is, of course, $8\sqrt{3}$.





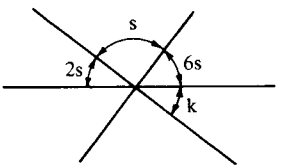
3. (D) $120 + x = 180°$ and $130 + y = 180°$ (they are pairs of supplementary angles). Therefore, $x = 60°$ and $y = 50°$. Since the sum of the measures of the angles of any triangle is 180°, $50° + 60° + T = 180°$; $110° + T = 180°$; $T = 70°$.

4. (A) This is a *direct* proportion. The cost increases (decreases) as the number of pencils increases (decreases). $\frac{\text{pencils}}{\text{dollars}} = \frac{\text{pencils}}{\text{dollars}}$ $\frac{x \text{ pencils cost}}{d \text{ dollars}}$ then $\frac{y \text{ pencils cost}}{h \text{ (how many?) dollars}}$. We have to solve for h in terms of x, y, and d. "Cross-multiply" to get $xh = dy$. Then divide both sides of the equation by x to get $h = \frac{dy}{x}$.



Note: Figure not drawn to scale.

5. (C) Since r and k are vertical angles, they are equal. If we can find r we will then have k. First, $r + s + t = 180°$ (their sum forms a straight angle). Since we are given that $r = 2s$ and $t = 3r$, then $t = 3(2s) = 6s$. Now, all the angles are in terms of one variable, s.



Therefore, $2s + s + 6s = 180°$; $9s = 180°$; $s = 20°$. So, $2s = 40° = k$.

Note: Figure not drawn to scale.

6. (A) Be careful here. This is *not* a direct proportion because as the number of men increases (decreases), the number of hours it takes to do the job decreases (increases). This is an *inverse* proportion. The easiest way to think about this type of problem is in terms of "man-hours" it takes to complete the job. If there are m men and it takes them h hours, then it takes mh "man-hours." Since it takes 5 men 4 hours to do the job, it takes 5×4 or 20 man-hours. If the number of man-hours remains a constant, then as you *increase* the number of men to 10, let's say, then it will take $10 \times 2 = 20$ man-hours *again*. Notice that the number of hours has *decreased*. In examples such as this, men $\times$ hours = men $\times$ hours. So, in our problem, $mh = nx$ and we have to solve for x. Divide both sides of the equation by n to get $\frac{mh}{n} = x$.

7. (B) Let x = the number of miles a passenger can travel for \$6. Then \$1.50 + \$0.50$(x - 1)$ = \$6.00. Move the decimals 2 places to the right (multiply each term by 100) to get $150 + 50(x - 1) = 600$; $150 + 50x - 50 = 600$; $100 + 50x = 600$; $50x = 500$; $x = 10$. Let's check. The first mile is \$1.50 and the other 9 cost 50¢ each or \$4.50. \$1.50 + \$4.50 = \$6.00. You could work "backwards" from the answers in this problem but it would take a bit longer.

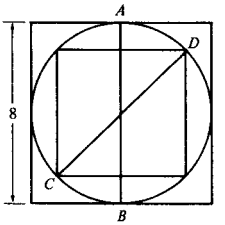
8. (C) Translate the sentence into $.15x = K$; $15x = 100K$; $x = \frac{100K}{15}$; $x = \frac{20K}{3}$.

9. (B) The surface area of a cube is equal to $6 \times$ (area of one square face). We were given that the surface area was 54, therefore, the area of one square face = $\frac{54}{6} = 9$. Then, each side of the square must be 3, since the area of a square is $(\text{side})^2$. So the volume of the cube is $3 \times 3 \times 3 = 27$.

10. (B) Do everything in terms of 1 minute. If a man can do a job in 180 minutes then he can do $\frac{1}{180}$ of that job in 1 minute. The other man can do $\frac{1}{60}$ of the job in 1 minute. If they work together, for 20 minutes, let's say, then the first man can do $\frac{20}{180}$ or $\frac{1}{9}$ of the job and the other man can do $\frac{20}{60}$ or $\frac{1}{3}$ of the job. So, in 20 minutes the total *part* of the job they can do together is $\frac{1}{9} + \frac{1}{3} \cdot \frac{1}{9} + \frac{1}{3} = \frac{1}{9} + \frac{3}{9} = \frac{4}{9}$. So, in 20 minutes they do *not* finish the job. Instead of trial and error, let's call the number of minutes they work together x. Then $x\left(\frac{1}{180}\right) + x\left(\frac{1}{60}\right) = 1$. The number 1 on the right side of the equation is there because we want the sum of the two fractions to represent 1 whole job (one *complete* job). Multiply each term by 180 to "clear" the denominators in this case so we get $180\left(\frac{x}{180}\right) + 180\left(\frac{x}{60}\right) = 180(1)$; $x + 3x = 180$; $4x = 180$; $x = 45$. So, it takes the two men working together 45 minutes to complete the job. Check it! In 45 minutes, the first man can do $\frac{45}{180}$ or $\frac{1}{4}$ of the job. The second man can do $\frac{45}{60}$ or $\frac{3}{4}$ of the job in the same time. $\frac{1}{4} + \frac{3}{4} = 1$.

11. (C) Let's look at a comparable problem using smaller numbers: "How many pages are there from page 1 to page 10 inclusive?" If you subtract, you get $10 - 1 = 9$. But, there are 10 pages inclusive, not 9. So you can see the method. Subtract and then add one to the difference. In our problem, then, $200 - 156 = 44$; $44 + 1 = 45$.

12. (A) Each side of an equilateral triangle is $\frac{1}{3}$ the perimeter. If $\frac{2p}{3} = \frac{8}{15}$, then $\frac{1p}{3} = \frac{4}{15}$. Of course, some of you probably solved for p and then took $\frac{1}{3}$ of that answer. If $\frac{2p}{3} = \frac{8}{15}$, then, "cross-multiplying," you get $30p = 24$; dividing both sides of the equation by 30 now, you get $p = \frac{24}{30}$ or $\frac{4}{5}$. Each side of the triangle is $\frac{1}{3}$ of $\frac{4}{5}$ or $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$. Can you see that sometimes a little "insightful" reasoning can save you a lot of time?

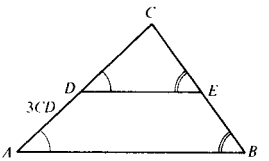


$AB = CD = 8$

13. (A) The area of the large square = 64, so each side is 8. Then the diameter of the circle is 8. If you "rotate" the diameter a bit, it becomes the diagonal of the smaller square. Since the area of a square can be found by the formula $\frac{1}{2}(\text{diagonal})^2$, then the area of the smaller square is $\frac{1}{2}(8^2) = \frac{1}{2}(64) = 32$.

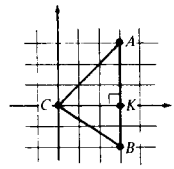
14. (D) If $\frac{x}{8} = \frac{1}{5}$, then "cross-multiply" to get $5x = 8$; $x = \frac{8}{5}$. The problem asks for $\frac{x}{6}$. So, $\frac{1}{6}$ of $x = \frac{1}{\cancel{6}\,3} \times \frac{\cancel{8}\,4}{5} = \frac{4}{15}$.

15. (B) Let x = the number of girls. Then $2x$ = the number of boys.
$\frac{\text{girls}}{\text{total students}} = \frac{x}{x + 2x} = \frac{x}{3x} = \frac{1}{3}$, which is $33\frac{1}{3}\%$.



16. (D) The triangles are similar by A.A. (corresponding angles are equal), so we can set up a proportion, $\left(\frac{CD}{CA}\right)^2 = \frac{\text{area of } \triangle CDE}{\text{area of } \triangle ABC}$. Since $CA = CD + DA = CD + 3CD = 4CD$, $\left(\frac{CD}{4CD}\right)^2 = \frac{\text{area of } \triangle CDE}{\text{area of } \triangle ABC}$; $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$. (Remember, the

ratio of the area of two similar triangles is the square of the ratio of any two corresponding sides.)

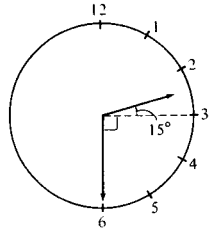


17. (E) Let AB be the base of the triangle and CK be the height. Then, simply by counting boxes, $AB = 5$ and $CK = 3$. The area of the triangle is then $\frac{1}{2}(5)(3) = \frac{1}{2}(15) = 7.5$.

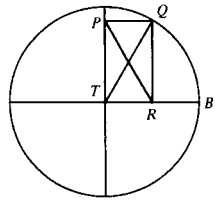
18. (C)
$$\begin{array}{rl} P + Q &= 9 \\ \underline{+P - Q} &\underline{= 3} \\ 2P &= 12 \end{array}$$
So, $P = 6$. Q must then be 3. So, $P^2 + Q^2 = 6^2 + 3^2 = 36 + 9 = 45$.

19. (A) $x^2 - y^2 = (x - y)(x + y)$. Since we are given that $x^2 - y^2 = 1$ and $x - y = 1$, then $1 = 1(x + y)$. Dividing both sides of the equation by 1, we get $1 = x + y$.

20. (D) Find the least number that is divisible by 12 and 10. $12 \times 5 = 60 = 10 \times 6$. The bells will ring at the same time after 60 minutes (also after 120 minutes, etc.). Be careful. The important word in this problem is "first."

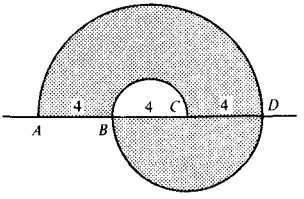


21. (C) Be careful. The little hand is *on* the 2 only when the big hand is on the 12 (at 2 o'clock). Remember, as the big hand moves *from* 12 around *to* 12 again, the little hand moves (ever so slowly) from 2 to 3. At 2:30 (in this problem) the big hand has gone ½ of the distance around the circle (clock). So, the little hand must have gone ½ the distance between 2 and 3. The angle between any two consecutive hours is 30°. $\left(\frac{360°}{12} = 30°\right)$. Therefore, the little hand has moved $\frac{1}{2}(30°) = 15°$. So, as you can see in the diagram above, the answer is $90° + 15° = 105°$. Many of you, who forgot that the little hand moves too, probably marked choice (E).

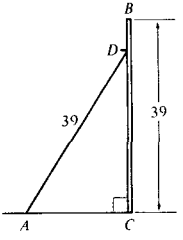


22. (D) Draw the other diagonal, TQ. Since $TB = 5 + 5 = 10$, then $TQ = 10$. TB and TQ are *radii* of the same circle. Sneaky? Since the diagonals of a rectangle are equal, $PR = TQ = 10$.

23. (B) This is another "invented" operation. Do what is in the parentheses first, so that $2 \,\&\, (2 \,\&\, 3) = 2 \,\&\, (2^3 - 1) = 2 \,\&\, (8 - 1) = 2 \,\&\, (7) = 2^7 - 1 = 128 - 1 = 127$.



24. (B) The area of the shaded region is equal to the area of the semicircle whose arc is AD + the area of the semicircle whose arc is BD − the area of the semicircle whose arc is BC. Since $AD = 4 + 4 + 4 = 12$, the radius of the greatest semicircle is 6, so that its area is $\frac{\pi 6^2}{2} = \frac{36\pi}{2} = 18\pi$. The radius of the semicircle BD is 4, so that its area is $\frac{\pi 4^2}{2} = \frac{16\pi}{2} = 8\pi$, and the radius of the smallest semicircle is $\frac{1}{2}(4) = 2$, so that its area is $\frac{\pi 2^2}{2} = \frac{4\pi}{2} = 2\pi$. Therefore, the area of the shaded region is $18\pi + 8\pi - 2\pi$ or 24π.



25. (A) See the diagram above. Since $BC = 39$ (the ladder is "flush" against the wall), we can find BD if we know DC. In right triangle ACD, $AD^2 = AC^2 + DC^2$; $39^2 = 15^2 + DC^2$; $1521 = 225 + DC^2$; $1296 = DC^2$; $DC = 36$. Therefore, $BD = BC - DC = 39 - 36 = 3$. Did you recognize a 5-12-13 right triangle? 15-36-39 is a multiple (3, in fact) of 5-12-13.

Mathematical Aptitude Test 6

Time 30—Minutes
25 Questions

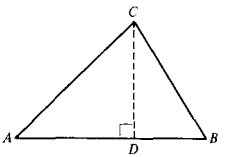
In this section solve each problem, using any available space on the page for scratchwork. Then indicate the *one* correct answer in the appropriate space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$.
The number of degrees in the arc of a circle is 360.
The measure in degrees of a straight angle is 180.

Definition of symbols:

$<$ is less than	$\leqq$ is less than or equal to
$>$ is greater than	$\geqq$ is greater than or equal to
$\perp$ is perpendicular to	$\parallel$ is parallel to



Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\measuredangle CDA$ is a right angle, then

(1) area of triangle $ABC = \dfrac{(AB)\cdot(CD)}{2}$

(2) $AC^2 = AD^2 + DC^2$

Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible *except* when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

1. $^2/_3 \times {}^3/_4 \times {}^4/_5 \times {}^5/_6 =$

   (A) 1 (B) $^2/_5$ (C) $^1/_3$ (D) $^7/_9$ (E) $^8/_{15}$

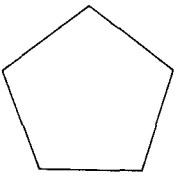
2. If $\sqrt{27} + \sqrt{12} = N$, then N = ?

   (A) $5\sqrt{3}$ (B) $\sqrt{39}$ (C) 15 (D) $5\sqrt{6}$ (E) $7\sqrt{3}$

3. Jack was R years old M years ago. His age B years from now will be

   (A) R + M + B (B) R + M − B
   (C) R + B − M (D) R − M + B (E) R − M − B

4. If $p > q$, then which of the following must be true?
   (A) $p > q + 1$ (B) $p > 2q$ (C) $p > 3q + 3$
   (D) $p > \dfrac{(p+q)}{2}$ (E) $p^2 > q^2$



5. If all the diagonals are drawn in the figure above, then *ABCDE* will be divided into how many regions?

   (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

6. If $\frac{160}{360} = \frac{p}{q}$, then the least integer $p + q$ could be is

(A) 5 (B) 9 (C) 13 (D) 97 (E) 52

7. If the product of 113 integers is positive, what is the greatest number of integers that could be negative?

(A) 109 (B) 110 (C) 111 (D) 112 (E) 113

8. If $.001x = 1$, then $1.1x =$

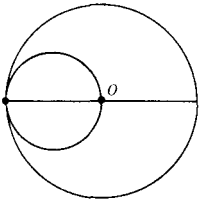
(A) 1100 (B) 1010 (C) 1001 (D) 1101 (E) 1110

9. If $4x + 8y = 12$, then $(x + 2y)^2 =$

(A) 16 (B) 9 (C) 25 (D) 36 (E) 24

10. A clown can wear any combination of 5 hats, 5 coats, and 5 shirts. How many different outfits can he wear?

(A) 15 (B) 75 (C) 125 (D) 50 (E) 5



Point 0 is the center of the larger circle

11. In the above figure, if the area of the larger circle is 36π, then the area of the smaller circle is

(A) 9π (B) 18π (C) 12π (D) 16π (E) 20π

12. Which of the following fractions has the greatest value if P and Q are positive integers?

(A) $\frac{P+1}{Q+1}$ (B) $\frac{P+1}{Q}$ (C) $\frac{P}{Q}$ (D) $\frac{P-1}{Q}$ (E) $\frac{P}{Q+1}$

13. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, then if $\begin{vmatrix} 2 & x \\ 3 & 5 \end{vmatrix} = \begin{vmatrix} x & 2 \\ 1 & 1 \end{vmatrix}$, $x =$

(A) -3 (B) 1 (C) 0 (D) 3 (E) 4

14. If one tube out of 120 is defective, then in a case containing 1000 dozen tubes, how many are defective?

(A) 50 (B) 100 (C) 120 (D) 3000 (E) 1000

15. By how much is the maximum value of $\frac{P}{Q}$ greater than the minimum value of $\frac{P}{Q}$ if $1 \leqq P \leqq 5$ and $5 \leqq Q \leqq 10$?

(A) $\frac{9}{10}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{1}{2}$ (E) $\frac{3}{4}$

16. A box contains 3 red marbles, 4 green marbles, and 5 blue marbles. What is the least number of marbles a blindfolded person should withdraw from the box so as to be sure that he has 3 of the same color?

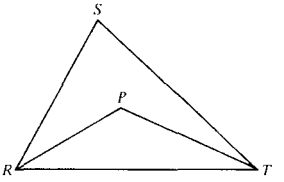
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

17. If $\frac{1}{p} + \frac{1}{q} = t$, and $p + q = y$, then $pq =$

(A) $\frac{y}{t}$ (B) $\frac{t}{y}$ (C) yt (D) $y + t$ (E) $t(t + y)$

18. If Jane was p years old n years ago, how old will she be t years from now?

(A) $p + n + t$ (B) $p + n - t$ (C) $p - n + t$ (D) $n - p + t$ (E) $p - n - t$



19. In the above figure, $\measuredangle R > \measuredangle T$. RP and TP are the bisectors of $\measuredangle R$ and $\measuredangle T$ respectively. Which of the following must be true?

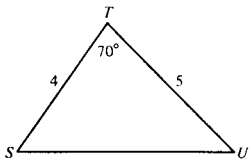
(A) $PT < RP$ (B) $PT = RP$ (C) $RP + PT > RS + ST$ (D) $PT > RP$ (E) no relationship between PT and RP can be determined from the information given

20. The gas gauge of a car shows that it is $\frac{1}{3}$ full. After g gallons of gas are poured in, the gauge shows that the gas tank is now $\frac{7}{9}$ full. How many gallons, in terms of g, does the tank hold when full?

(A) $9g$ (B) $\frac{9g}{7}$ (C) $\frac{3g}{7}$ (D) $\frac{9g}{4}$ (E) $\frac{4g}{9}$

21. If x students attain an average of p, and y students attain an average of q, then the average of $x + y$ students is

(A) $\frac{x+y}{2}$ (B) $p + q$ (C) $\frac{p+q}{2}$ (D) $\frac{px+qy}{x+y}$
(E) $\frac{p+q}{x+y}$



Note: Figure not drawn to scale.

22. In the triangle above, $ST = 4$, $TU = 5$, and $\measuredangle T = 70°$. Which of the following must be true?

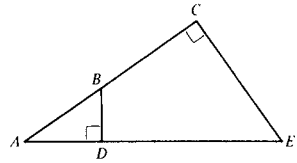
(A) $\measuredangle U = 55°$ (B) $\measuredangle S > 55°$ (C) $SU = 6$
(D) $\measuredangle U > 55$ (E) $SU > 5$

23. In a game of chance, Bob wins 10% of his first bet and loses 10% of his second bet on his first two bets. If he started the game with \$100 and bet all the money he had on each bet, then after the second bet he had

(A) \$100 (B) \$110 (C) \$90 (D) \$99 (E) \$105

24. The diameter of a wheel is 4. How many *complete* revolutions will the wheel make if it rolls a distance of 90π?

(A) 11 (B) 12 (C) 18 (D) 22 (E) 23



25. In the figure above, $AB = 6$, $AD = 4$, and $DE = 14$. $BC =$

(A) 6 (B) 7 (C) 9 (D) 10 (E) 12

STOP

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THIS TEST

Answer Sheet—Mathematical Aptitude Test 6

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	6	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	7	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	8	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	9	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	10	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐

ANSWERS AND EXPLANATIONS

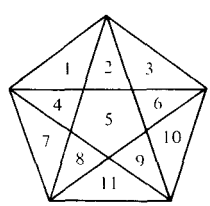
1. (C)	2. (A)	3. (A)	4. (D)	5. (C)
6. (C)	7. (D)	8. (A)	9. (B)	10. (C)
11. (A)	12. (B)	13. (D)	14. (B)	15. (A)
16. (C)	17. (A)	18. (A)	19. (D)	20. (D)
21. (D)	22. (B)	23. (D)	24. (D)	25. (A)

1. (C) $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{2 \times \cancel{3} \times \cancel{4} \times \cancel{5}}{\cancel{3} \times \cancel{4} \times \cancel{5} \times 6} = \frac{2}{6} = \frac{1}{3}$.

2. (A) $\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$ and $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$. So $\sqrt{27} + \sqrt{12} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$. Choice (A). Choice (B) is incorrect because $\sqrt{27} + \sqrt{12} \neq \sqrt{39}$. You can't add radicals like that! For example, $\sqrt{16} + \sqrt{9} = \sqrt{25}$ $4 + 3 \neq 5$. See?

3.(A) Substitute numbers. Jack was 10 years old (R = 10) 3 years ago (M = 3). So now he is 10 + 3 or R + M. 5 years from now (B = 5) he will be 10 + 3 + 5 or R + M + B.

4. (D) Remember, the word "must" means you should *not* be able to produce even *one* counterexample.
If $p = 2$ and $q = 1$, then in (A), $2 > 1 + 1$? *No.* $2 = 2$
If $p = 2$ and $q = 1$, then in (B), $2 > 2(1)$? *No.* $2 = 2$
If $p = 3$ and $q = 0$, then in (C), $3 > 3(0) + 3$? *No.* $3 = 3$
If $p = 2$ and $q = -3$, then in (E), $(2)^2 > (-3)^2$? *No.* $4 < 9$
So, the answer must be (D), by elimination.



5. (C) See the diagram above.

6. (C) $^{160}/_{360}$ reduced to lowest terms is $^{4}/_{9}$. Therefore, $p + q = 4 + 9 = 13$.

7. (D) If the product of n integers is positive, then there must be an even number of negative integers. (Of course, there doesn't have to be *any* negative integer.) However, in this problem, the greatest even integer is 112.

8. (A) Move the decimal points 3 places to the right (multiply by 1000) to get $1x = 1000$. Then $1.1x = 1.1(1000) = 1100$.

9. (B) Algebraically, if $4x + 8y = 12$, then dividing both sides of the equation by 4 we get $x + 2y = 3$. Therefore, $(x + 2y)^2 = (3)^2 = 9$.

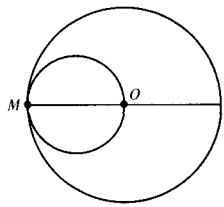
10. (C) If the clown had 2 hats, 1 coat and 1 shirt then he could wear 2 possible outfits: $2 \times 1 \times 1 = 2$

H	*C*	*S*
A	*X*	*W*
B	*X*	*W*

If he had 2 hats, 2 coats, and 2 shirts, then he could wear 8 possible outfits: $2 \times 2 \times 2 = 8$

H	*C*	*S*
A	X	W
A	X	Z
A	Y	W
A	Y	Z
B	X	W
B	X	Z
B	Y	W
B	Y	Z

Intuitively, then, with 5 hats, 5 coats, and 5 shirts, the answer is $5 \times 5 \times 5 = 125$.



11. (A) The diameter of the smaller circle is the radius of the larger circle. Since the area of the larger circle is $\pi r^2 = 36\pi$, then r must be 6. So, in the figure above, OM is 6. Therefore, the radius of that smaller circle must be 3, which means that its area must be $\pi 3^2$ or 9π.

12. (B) The greatest fraction is the one with the greatest numerator and the least denominator. Since P and Q are both positive integers, $P + 1$ is the greatest numerator and Q is the least denominator. Therefore, $\frac{P+1}{Q}$ is the greatest fraction.

13. (D) If $\begin{vmatrix} 2 & x \\ 3 & 5 \end{vmatrix} = \begin{vmatrix} x & 2 \\ 1 & 1 \end{vmatrix}$, then by definition, $(2)(5) - (x)(3) = (x)(1) - (2)(1)$. Therefore, $10 - 3x = x - 2$; $12 = 4x$; $3 = x$.

14. (B) Set up a direct proportion: $\frac{1}{120} = \frac{x}{12(1000)}$. "Cross-multiply" to get $120x = (12 \times 1000)$. Divide both sides of the equation by 120 and you get $x = \frac{(12)(1000)}{120} = \frac{(\cancel{12})(1000)}{(\cancel{12})(10)} = 100$.

15. (A) Once again, the greatest fraction will have the greatest numerator and the least denominator. Pick $P = 5$ and $Q = 5$ from the allowable values so that $\frac{P}{Q} = \frac{5}{5} = 1$. Now, the least valued fraction will have the least numerator and the greatest denominator. So pick $P = 1$ and $Q = 10$ from the allowable values, so that in this case, $\frac{P}{Q} = \frac{1}{10}$. The problem asks for the difference, $1 - \frac{1}{10} = \frac{9}{10}$.

16. (C) Some of the possible combinations a person could draw are as follows:

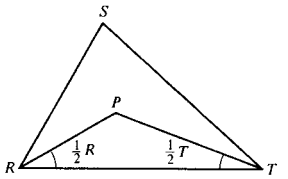
R	B	G	number of marbles
1	1	1	3
2	1	1	4
2	2	1	5
2	2	2	6
2	2	3	7

With 7 marbles, he *must* have 3 of one color. 7 is the *least* amount, then, he should withdraw.

17. (A) If $\frac{1}{p} + \frac{1}{q} = t$, then adding the two fractions on the left side of the equation we get $\frac{p+q}{pq} = \frac{t}{1}$. Since we have to solve for pq, "invert" both fractions to get $\frac{pq}{p+q} = \frac{1}{t}$. Since $p + q = y$, $\frac{pq}{y} = \frac{1}{t}$. Multiply both sides of the equation now by y to get $pq = \frac{y}{t}$.

This problem could have been done by substituting numbers for p, q, and t, and then looking at all the answers to see which one had the same value as pq. It does take a little longer, though.

18. (A) "Plug in" numbers for p, n, and t so that the problem could read, "If Jane was $\overset{10}{p}$ years old $\overset{3}{n}$ years ago, how old will she be $\overset{5}{t}$ years from now?" So, in this example, if Jane was 10 years old 3 years ago, then she is now 13 or $p + n$ and 5 years from now, she will be $13 + 5$ $(p + n + t)$, or 18.

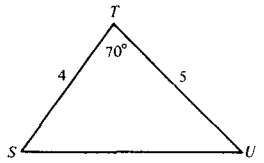


19. (D) Since $\measuredangle R > \measuredangle T$, then $\frac{1}{2}\measuredangle R > \frac{1}{2}\measuredangle T$. In any triangle, if 2 angles are unequal, then the side opposite the greater angle is the greater side. So, in this problem, $PT > RP$. This *must* be true.

20. (D) We need to find the total capacity of the tank. Represent that total capacity by T. Then, the tank is $\frac{1}{3}T$ full. After g gallons are poured in, the tank contains $\frac{1}{3}T + g$ gallons. Since it is now $\frac{7}{9}$ full, $\frac{1}{3}T + g = \frac{7}{9}T$. Multiply both

sides of the equation by 9 to "clear" the denominators, to get $9\left[\frac{1}{3}T + g\right] = \cancel{9}\left[\frac{7}{\cancel{9}}T\right]$, or $\overset{3}{\cancel{9}}\left(\frac{1}{\cancel{3}}T\right) + 9g = 7T$; $3T + 9g = 7T$; $9g = 4T$; $\frac{9g}{4} = T$. (Good grief!)

21. (D) If $\frac{10}{x}$ students had an average of $\frac{70}{p}$ and $\frac{20}{y}$ students had an average of $\frac{40}{q}$, then the $\frac{30}{x+y}$ students will have a combined average of $\frac{10(70) + 20(40)}{10 + 20} = \frac{700 + 800}{30} = \frac{1500}{30} = 50$. Algebraically, then, the answer is $\frac{x(p) + y(q)}{x + y}$.



22. (B) If ST was equal to TU, then the triangle would have been isosceles. In that case, $\measuredangle U = 55°$. ($55° + 55° + 70° = 180°$). Since $TU > ST$, though, $\measuredangle S$ must be >55.

Choice (A) is incorrect because if $\measuredangle U = 55°$, then $\measuredangle S = 55°$ and the triangle *must* be isosceles. But $ST \neq TU$.

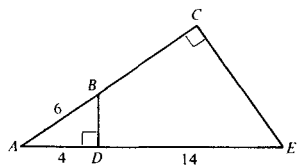
Choice (C) is incorrect because SU *could* equal 6. It doesn't have to be 6, though!

Choice (D) is incorrect because if $\measuredangle U > 55°$, then $\measuredangle S < 55°$, therefore $\measuredangle U > \measuredangle S$. But $\measuredangle U < \measuredangle S$ from the diagram.

Finally, choice (E) is incorrect because SU *could* equal 5 *if* $\measuredangle S = 70°$. In this case, we have an isosceles triangle where $TU = US$.

23. (D) $100 + .10(100) = 100 + 10 = 110.$
$110 - .10(110) = 110 - 11 = 99.$

24. (D) Since the diameter of the wheel is 4, its circumference is πd or 4π. If you "cut" the wheel and make a straight "line" out of it, it will then "stretch" a distance of 4π. Since the circle rolled 90π, it must have made $\frac{90\pi}{4\pi}$ revolutions. $\frac{90\pi}{4\pi} = 22.5$. Therefore, the wheel made 22 *complete* revolutions.



Note: Figure not drawn to scale.

25. (A) Since $\measuredangle ADB = \measuredangle C$ (right angles are equal) and $\measuredangle A = \measuredangle A$, then triangles ADB and ACE are similar (A.A). Therefore their corresponding sides are in proportion: $\frac{AB}{AE} = \frac{AD}{AC}$. Now, since $AE = AD + DE$ and $AC = AB + BC$, the proportion is $\frac{6}{4 + 14} = \frac{4}{6 + BC}$; $\frac{6}{18} = \frac{4}{6 + BC}$; "cross-multiplying," we get $(18)(4) = 6(6 + BC)$; $72 = 36 + 6BC$; $36 = 6BC$; $6 = BC$.

Mathematical Aptitude Test 7

Time—30 Minutes
35 Questions

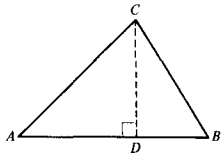
In this section solve each problem, using any available space on the page for scratchwork. Then indicate the *one* correct answer in the appropriate space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$.
The number of degrees in the arc of a circle is 360.
The measure in degrees of a straight angle is 180.

Definition of symbols:

$<$ is less than	$\leqq$ is less than or equal to
$>$ is greater than	$\geqq$ is greater than or equal to
$\perp$ is perpendicular to	$\parallel$ is parallel to



Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\measuredangle CDA$ is a right angle, then

(1) area of triangle $ABC = \dfrac{(AB)\cdot(CD)}{2}$

(2) $AC^2 = AD^2 + DC^2$

Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible *except* when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.

1. If $(4)(8)(16)(K) = (16)(16)(12)$, then $K =$

   (A) 6 (B) 8 (C) 12 (D) 16 (E) 36

2. The center of a circle is located at point (5,0). If one endpoint of a diameter of this circle is point (11,0), then the other endpoint of this diameter is point

   (A) (6,0) (B) (0,6) (C) (−1,0) (D) (1,1) (E) (1,−1)

3. The center of a circle is at (0,0) and the circle passes through (5,12). What is the area of this circle?

   (A) 25π (B) 49π (C) 144π (D) 169π (E) 225π

4. If J has a value between 2 and 5 and K has a value between 1 and 6, then $\dfrac{K}{J}$ has a value between

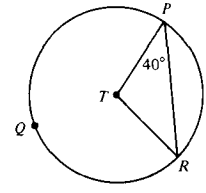
   (A) $1/2$ and $6/5$ (B) $1/5$ and $1/3$ (C) $1/5$ and 3 (D) 3 and 5 (E) $1/2$ and 5

5. If $H = \dfrac{68 \times 994}{34998}$, then of the following, the one which is the closest approximation to H is

   (A) 2 (B) 20 (C) 200 (D) 2000 (E) 20,000

6. 20% of 8 is what % of 16?

   (A) 10 (B) 25 (C) 40 (D) 60 (E) 80



Point T is the center of the circle

7. The measure in degrees of arc PQR, in the circle on page 347 is

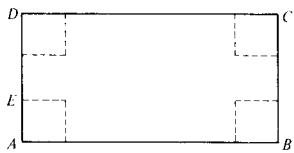
(A) 80 (B) 160 (C) 220 (D) 260 (E) 280

8. If the radius of a circle is increased by 50%, the circumference is increased by

(A) 25% (B) 50% (C) 75% (D) 100% (E) 125%

9. In the below figure, $ABCD$ represents a rectangular piece of tin from which 4 equal squares have been cut out as indicated. What is the volume of the open box that can be formed by turning up its sides if $AB = 10$, $BC = 6$, and $AE = 2$?

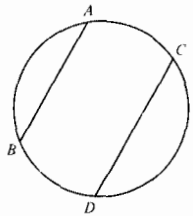
(A) 120 (B) 96 (C) 64 (D) 24 (E) 72



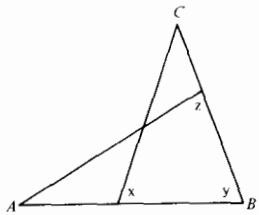
See Test 1, p. 208, for instructions on marking your answers for problems 10–30.

	Column A		Column B	
		$x^2 - 6x + 9 = 0$ $y^2 + 8y + 16 = 0$		
10.	x		y	A B C D
		$0 < x < 8$ $0 < y < 10$		
11.	x		y	A B C D
		$p < q < r$		
12.	$p + r$		q	A B C D
		$2p - 2q = 10$		
13.	p		q	A B C D
		$\frac{P}{Q} = \frac{3}{4}$		
14.	$\frac{4P}{3}$		Q	A B C D
15.	$3^0(5 - 2)^3$		$3(3)^2$	A B C D
16.	The number of degrees between the hands of a clock at 3:30		90°	A B C D
		$0 < x < y$		
17.	$\frac{1}{x}$		y	A B C D
18.	3% of 4%		0.12%	A B C D
19.	6 divided in half		6 divided by one-half	A B C D
20.	2^{50}		4^{25}	A B C D

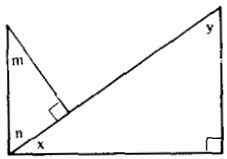
	Column A		*Column B*	
21.	x	$0.2x - 1 = 1.1$	10	A B C D
22.	$p + q$	$p < 0$ $q < 0$	$p - q$	A B C D



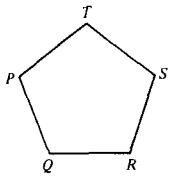
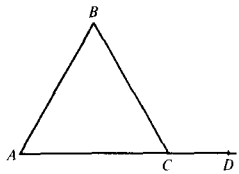
23.	$\widehat{AC}$	$AB \parallel CD$	$\widehat{AB}$	A B C D
24.	$x(x - y)$	$x = y$	$x(x + y)$	A B C D
25.	The volume of a cube with surface area 12		$2\sqrt{2}$	A B C D



26.	AB	$x = y = z$	BC	A B C D



27.	$y + n$	$n + x = 90°$	$x + m$	A B C D
28.	$\frac{b}{a}$	$a = 2b$	$\frac{a}{b} - 1$	A B C D
29.	The product of the integers	The sum of 5 consecutive integers is 0.	The average of the integers	A B C D

	Column A	Column B	
	$PQ = QR = RS = ST = TP = PQ$ $\measuredangle PTS = \measuredangle TSR = \measuredangle SRQ = \measuredangle RQP = \measuredangle QPT$	$AB = BC = CA$	
			
30.	Angle PQR	Angle BCD	A B C D

31. If the altitude of a triangle is increased by 25%, then by what percent must the base be decreased by in order to keep the area the same?

(A) 15% (B) 20% (C) 25% (D) 30% (E) 50%

32. If p and q are integers, and $\frac{p}{q}$ is negative, then which of the following must be negative?

I. $p - q$ II. pq III. $p^3 + q^3$

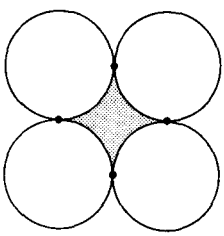
(A) I only (B) II only (C) III only
(D) I and II only (E) II and III only

33. The average of A numbers is B. If C is added to each number the average will be

(A) $\frac{BC}{A}$ (B) $\frac{AB}{C}$ (C) $AB - C$ (D) $A - C$ (E) $B + C$

34. If it takes 2 men 6 hours to cut down 3 trees, then how many hours will it take 4 men to cut down 6 trees?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12



35. The diameter of each circle in the above figure is 1 foot. What is the area of the shaded region?

(A) π (B) $\frac{\pi}{4}$ (C) $1 - \pi$ (D) $\frac{4 - \pi}{4}$ (E) $1 - 4\pi$

STOP

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THIS TEST.

Answer Sheet—Mathematical Aptitude Test 7

	A	B	C	D	E
1	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐
6	☐	☐	☐	☐	☐
7	☐	☐	☐	☐	☐
8	☐	☐	☐	☐	☐
9	☐	☐	☐	☐	☐
10	☐	☐	☐	☐	☐
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32	☐	☐	☐	☐	☐
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34	☐	☐	☐	☐	☐
35	☐	☐	☐	☐	☐

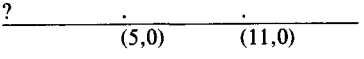
ANSWERS AND EXPLANATIONS

1. (A)	2. (C)	3. (D)	4. (C)	5. (A)
6. (A)	7. (D)	8. (B)	9. (D)	10. (A)
11. (D)	12. (D)	13. (A)	14. (C)	15. (C)
16. (B)	17. (D)	18. (C)	19. (B)	20. (C)
21. (A)	22. (B)	23. (D)	24. (D)	25. (C)
26. (D)	27. (D)	28. (B)	29. (C)	30. (B)
31. (B)	32. (B)	33. (E)	34. (C)	35. (D)

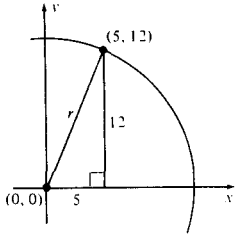
1. (A) Divide both sides of the equation by $4 \times 8 \times 16$ in order to "isolate" K so that $K = \frac{\cancel{16} \times \cancel{16} \times \cancel{12}}{\cancel{4} \times \cancel{8} \times \cancel{16}} = 6$.

2. (C) Let (x,y) be the other endpoint of the diameter. Since the center of any circle is the midpoint of its diameters, $(5,0) = \left(\frac{x+11}{2}, \frac{y+0}{2}\right)$ by the midpoint formula. Therefore, $5 = \frac{x+11}{2}$ and $0 = \frac{y+0}{2}$; multiply both equations by 2 to "clear" the denominators, to get $10 = x + 11$ and $0 = y + 0$. Therefore, $x = -1$ and $y = 0$, or $(-1,0)$.

A much *faster* method in this case comes about if you realize that (5,0) and (11,0) are both points on the x-axis. The distance from (5,0) to (11,0) is 6; therefore, the distance from (5,0) to the "left" endpoint must also be 6. If you count 6 boxes to the left of (5,0), you "arrive" at $(-1,0)$.



3. (D) Since the circle passes through the point (5,12), the length of the radius is the length from (0,0) to (5,12). You can use the distance formula here: $r = \sqrt{(5-0)^2 + (12-0)^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. Therefore, the area of the circle is $\pi r^2 = \pi 13^2 = 169\pi$. Of course, if you drew a diagram, then you probably realized that you had a 5-12-13 right triangle. Therefore, the radius of the circle had to be 13, etc.



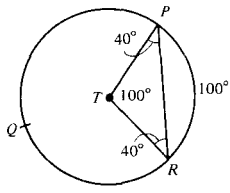
4. (C) The *least-valued* fraction has the least numerator and the greatest denominator. So, in this example, let $K = 1$ and $J = 5$ to get the fraction $\frac{K}{J} = \frac{1}{5}$. The *greatest* fraction is one in which the numerator is the greatest and the denominator the least number from the allowable set. So, in this case, let $K = 6$ and $J = 2$ to get the fraction $\frac{K}{J} = \frac{6}{2} = 3$. Therefore, the answer is choice (C).

5. (A) Round off the given integers: $\frac{68 \times 994}{34998}$ is approximately $\frac{70 \times 1\cancel{000}}{35\cancel{000}} = \frac{70}{35} = 2$.

6. (A) "Translate" the sentence into $.20(8) = \frac{x}{100}(16)$.

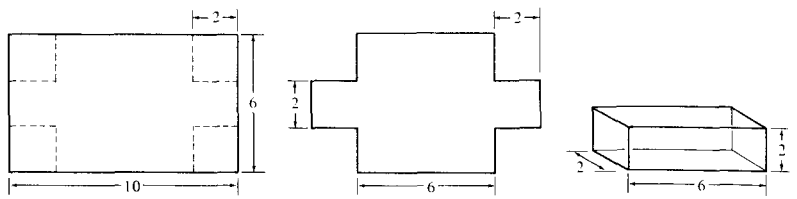
If you want to work with decimals, change $\frac{16x}{100}$ into $\left(\frac{16}{100}\right)x = .16x$ so that the equation now reads $.20(8) = .16(x)$. Move the decimal points 2 places to the right (multiply both sides of the equation by 100) to get $20(8) = 16x$; $160 = 16x$; $10 = x$.

If you want to work with fractions, change .20 into $\frac{20}{100} = \frac{1}{5}$, so that the equation reads $\frac{1}{5}(8) = \frac{(16)(x)}{100} = \frac{4x}{25}$. Then "cross-multiply" to get $(8)(25) = (4x)(5)$; divide by 4 now to get $(2)(25) = x(5)$; divide by 5 now to get $2(5) = x = 10$.



7. (D) Arc $PQR = 360° -$ arc PR. So, the real problem is to find the measure in arc degrees of arc PR. We can do this if we can find central angle PTR. Since $PT = TR$ (radii of the same circle are equal), triangle PTR is isosceles, so $\measuredangle P = \measuredangle R = 40°$. Therefore, $\measuredangle T = 100°$, so arc $PQ = 100°$. Then, arc $PQR = 360° - 100° = 260°$.

8. (B) Try some numbers. Let the radius of the circle be 10. Then its circumference is $2\pi r$ or $2\pi(10) = 20\pi$ in this case. If the radius is increased by 50% or $\frac{1}{2}$, then the new radius is $10 + \frac{1}{2}(10) = 10 + 5 = 15$. So, the new circumference is $2\pi(15) = 30\pi$. Since % increase $= \frac{\text{actual increase}}{\text{original amount}}$, then $\frac{30\pi - 20\pi}{20\pi} = \frac{10\pi}{20\pi} = \frac{1}{2} = 50\%$. Algebraically: If $C_1 = 2\pi r$, then $C_2 = 2\pi\left(r + \frac{1}{2}r\right) = \cancel{2}\pi\left(\frac{3r}{\cancel{2}}\right) = 3\pi r$. So, % increase $= \frac{3\pi r - 2\pi r}{2\pi r} = \frac{1\pi r}{2\pi r} = \frac{1}{2} = 50\%$. So, r can be *any* positive number.



9. (D) The volume of the box is length × width × height. After cutting out squares from each corner that are 2 × 2, the length of the box will be $10 - (2 + 2) = 10 - 4 = 6$; the width of the box will be $6 - (2 + 2) = 6 - 4 = 2$; and the height of the box will be 2. The volume of the box is, then, $6 \times 2 \times 2 = 24$.

Note: In answers 10–30, A = B means that the quantities in Columns A and B are equal; A > B means that the quantity in Column A is greater; and B > A means that the quantity in Column B is greater.

10. (A) If $x^2 - 6x + 9 = 0$, then $(x - 3)(x - 3) = 0$. If the product of two quantities is 0, then either one or both of them may be 0. So, $x - 3 = 0$ and $x = 3$. If $y^2 + 8y + 16 = 0$, then $(y + 4)(y + 4) = 0$, so $(y + 4) = 0$, which means $y = -4$. Therefore, the quantity in Column A is greater. Choice (A).

11. (D) Since x *could* equal y, then both could be 5, let's say, *or* 6, *or* 6.5, etc. However, if $x = 6$ when $y = 3$, then A > B. If $x = 2$ when $y = 7$, then B > A. So, you must mark choice (D), since no relationship can be determined between the quantities.

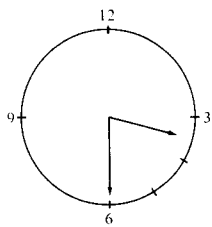
12. (D) $p < q < r$
$1 < 2 < 10 \qquad p + r = 11$. Since $q = 2$, A > B

$p < q < r$
$-10 < 1 < 2 \qquad p + r = -10 + 2 = -8$. Since $q = 1$, B > A.
Watch out for those pesky negative numbers when working with inequalities.

13. (A) If $2p - 2q = 10$, then $2(p - q) = 10$. Dividing both sides of the equation by 2, we get $p - q = 5$, and $p = q + 5$. Therefore, p is 5 more than q; $p > q$. Choice (A).

14. (C) Since $\frac{P}{Q} = \frac{3}{4}$, then, "cross-multiplying," we get $4P = 3Q$, so $\frac{4P}{4} = \frac{3Q}{4}$ and $P = \frac{3Q}{4}$. Of course, if you just multiplied both sides of the equation by Q, you would have solved for P more quickly: $\frac{P}{Q} = \frac{3}{4}$; $P = \frac{3Q}{4}$. So, $\frac{4P}{3} = \frac{4}{3}\left(\frac{3Q}{4}\right) = \frac{\cancel{4} \times \cancel{3} \times Q}{\cancel{3} \times \cancel{4}} = Q$. Therefore, A = B. (Remember, this "shorthand" means that the quantity in Column A is equal to the quantity in Column B.) You must mark choice (C) when this is the case.

15. (C) The only important fact here is that $3^0 = 1$. Be careful. (*ERROR:* $3^0 = 0$.) So, $3^0(5 - 2)^3 = 1(3)^3 = 1(27) = 27.3(3)^2 = 3(9) = 27$. So, once again mark choice (C), since A = B. Before we leave this problem we should point out another common *ERROR:* $3(3)^2 = 9^2$. Remember, do what is in the parentheses *first;* $3(3)^2 \neq 9^2$.



16. (B) Many students forget that the little hand of a clock is also in constant motion. The only time that the little hand is on the 3 is *exactly* at 3 o'clock. At *one* second after 3, the little hand has moved ever so slightly. At 3:30, the big hand has moved $\frac{1}{2}$ the distance around the circular clockface. Therefore, the little hand has

moved $\frac{1}{2}$ the distance from 3 to 4. Since there are 12 hour-markings on the clockface, each hour represents $\frac{360°}{12} = 30°$. $\frac{1}{2}(30°) = 15°$. Looking at the diagram above, you can see (for the purposes of this problem) that the angle made by the two hands at 3:30 is *less* than 90°. It would not be too difficult to find the exact number of degrees but it is *unnecessary* here. So, B > A.

17. (D) Pick values for x and y that satisfy the inequality: $x = 2$, $y = 3$. Then $\frac{1}{2} < 3$ so B > A. *HOWEVER* (shouting here!) what if x is a fraction, say, $\frac{1}{5}$? Then $\frac{1}{x} = \frac{1}{\frac{1}{5}} = 5$. In this case, although $0 < x < y$, $\frac{1}{x} > y$ $(5 > 3)$. Therefore, A > B. So, you *must* try *several* values of the variables before you reach your conclusion. Just like negative numbers, fractions are also pesky. Mark choice (D), since no relationship can be determined from the information given.

18. (C) 3% of 4% means (.03)(.04) = .0012; 0.12% is equivalent to .0012 also (remember how to change a % to a decimal?). Therefore A = B. Mark choice (C).

19. (B) 6 divided *in* half is equal to 3. 6 divided *by* one-half is equal to $\frac{6}{\frac{1}{2}} = 6 \times 2 = 12$. Therefore, although the two statements "sound" alike, they are *not*. B > A.

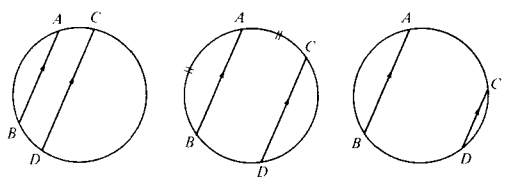
20. (C) $4^{25} = (2^2)^{25} = 2^{50}$. Therefore, A = B. Remember, $(a^b)^c = a^{bc}$.

21. (A) If $.2x - 1 = 1.1$, then "clear" the decimals by multiplying both sides of the equation by 10 (move the decimal points 2 places to the right) so that you get $2x - 10 = 11$; adding 10 to both sides of the equation now, you get $2x - 10 + 10 = 11 + 10$; $2x = 21$; $x = 10.5$. Therefore, A > B.

22. (B)

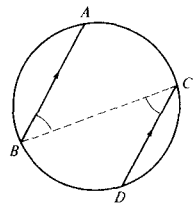
	p	q	$p + q$	$p - q$
$p > q$	-1	-2	$(-1) + (-2) = -3$	$(-1) - (-2) = -1 + 2 = 1$
$p < q$	-2	-1	$(-2) + (-1) = -3$	$(-2) - (-1) = -2 + 1 = -1$
$p = q$	-2	-2	$(-2) + (-2) = -4$	$(-2) - (-2) = -2 + 2 = 0$

Since we have covered the three possibilities for p and q, namely, $p > q$, $p < q$, or $p = q$, then we can see that $p - q > p + q$ if p and q are negative numbers. $(1 > -3$, $-1 > -3$, and $0 > -4$.)



$\widehat{AC} < \widehat{AB}$ $\quad$ $\widehat{AC} = \widehat{AB}$ $\quad$ $\widehat{AC} > \widehat{AB}$

23. (D) Draw several diagrams using the information given. The *only* relationship that *can* be determined is that $\widehat{AC} = \widehat{BD}$, parallel chords in a circle cut off equal arcs *between* the parallel chords.



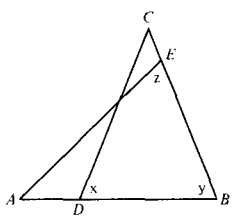
In the diagram above, $\measuredangle B = \measuredangle C$ (alternate interior angles). Since $\measuredangle B = \frac{1}{2}\,\widehat{AC}$ and $\measuredangle C = \frac{1}{2}\,\widehat{BD}$, then $\frac{1}{2}\,\widehat{AC} = \frac{1}{2}\,\widehat{BD}$ or $\widehat{AC} = \widehat{BD}$.

24. (D)

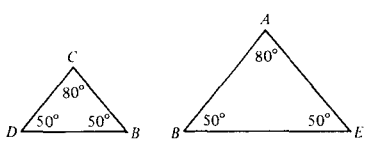
x	y	A $x(x - y)$	B $x(x + y)$
3	3	$3(3 - 3) = 3(0) = 0$	$3(3 + 3) = 3(6) = 18$
0	0	$0(0 - 0) = 0(0) = 0$	$0(0 + 0) = 0(0) = 0$
−3	−3	$-3(-3 - -3) =$ $-3(-3 + 3) =$ $-3(0) = 0$	$-3(-3 + -3) =$ $-3(-6) =$ 18

So, A < B, A = B, or A > B. You must mark choice (D). Of course, you didn't have to test three values for each variable here; after seeing that A < B or A = B, you could have made the conclusion that no relationship could be determined from the information given.

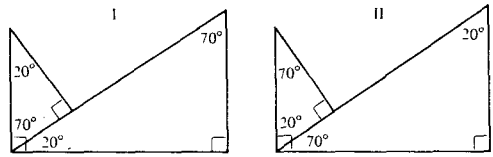
25. (C) The surface area of a cube is equal to 6 times (the area of one of its square "faces"). Therefore, 12 = 6 times (area of one of the square "faces"), or the area of one square "face" = 2. Since area = (side)^2, 2 = (side)^2 or $\sqrt{2}$ = side. The *volume* of a cube is (side)^3. Therefore, in this case, $V = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$. Therefore, the quantities in both columns are equal.



26. (D) Since $x = y = z$, triangles CDB and ABE are both isosceles. So, $CD = CB$ and $AB = AE$. *But*, no relationship can be determined between AB and BC. $AB > BC$, $AB = BC$, or $AB < BC$.



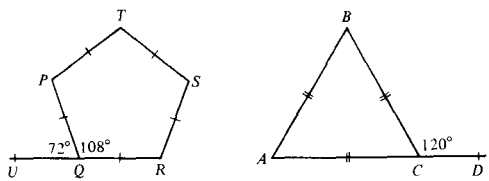
You can see here that two isosceles triangles can have all of their angles equal respectively, yet the triangles do not have to be congruent. They are, of course, similar.



27. (D) See the diagrams above. Remember, the acute angles of any right triangle are complementary (their sum is 90°). In I, $y + n = 140°$ and $x + m = 40°$. In II, $y + n = 40°$ and $x + m = 140°$. Therefore, you must mark choice (D).

28. (B) Since $a = 2b$, then $\frac{b}{a} = \frac{b}{2b} = \frac{1}{2}$. Therefore, $\frac{a}{b} = \frac{2}{1}$. So, $\frac{a}{b} - 1 = 2 - 1 = 1$, and the quanity in Column B is the greater quantity.

29. (C) Let the integers be x, $x + 1$, $x + 2$, $x + 3$, $x + 4$. Therefore, $\frac{x + x + 1 + x + 2 + x + 3 + x + 4}{5} = 0$. Multiplying both sides of the equation by 5 to "clear" the denominator, we get $x + x + 1 + x + 2 + x + 3 + x + 4 = 5(0) = 0$. So, $5x + 10 = 0$; $5x = -10$; $x = -2$. The product of the integers is then $(-2)(-1)(0)(1)(2) = 0$, and the average of the integers is $\frac{-2 + -1 + 0 + 1 + 2}{5} = \frac{0}{5} = 0$. Therefore, the quantities in both columns are equal.



30. (B) Since the exterior angles of any *regular* polygon (all angles equal; all sides equal) have a sum of 360°, then *each* exterior angle is equal to $\frac{360°}{n}$, where n represents the number of sides (and angles, of course). If you extend RQ to U, then $\measuredangle PQU = \frac{360}{5} = 72°$, so $\measuredangle PQR = 180 - 72 = 108$ (an interior and exterior angle of any polygon are supplementary). In the equilateral triangle above, $\measuredangle BCD = \frac{360°}{3} = 120°$. Therefore, the quantity in Column B is the greater quantity.

31. (B) Assume that the altitude of a triangle is 12 and the base is 10. Then, its area = ½(base)(height) = ½(10)(12) = 5(12) = 60. If we increase the altitude of this triangle by 25%, then the new altitude will be 12 + .25(12) = 12 + ¼(12) = 12 + 3 = 15. (We chose 12 because it is easy to take ¼ of 12 rather than work with decimals.) Since the area of this new triangle is equal to the area of the old triangle, 60 = ½(base)(15); multiplying both sides of the equation by 2 to "clear" the fraction, we get 120 = 15(base); dividing now by 15, we get: base = 8.

Now, % decrease in base = $\frac{\text{old base} - \text{new base}}{\text{old base}}$. Remember, % decrease or increase = $\frac{\text{difference between old and new}}{\text{old}}$. Therefore, in this case, $\frac{10 - 8}{10} = \frac{2}{10} = \frac{1}{5} = 20\%$.

Algebraically: $A_{\text{old}} = {}^1/_2 b_1 h$. Then $A_{\text{new}} = {}^1/_2 b_2(h + {}^1/_4 h) = {}^1/_2 b_2({}^5/_4 h)$. Since the areas are given as equal, ${}^1/_2 b_1 h = {}^1/_2 b_2({}^5/_4 h)$; multiplying both sides of this equation by 2, we get $b_1 h = {}^5/_4 b_2 h$. Divide both sides of the equation by h now to get $b_1 = {}^5/_4 b_2$. Therefore, the % change in the base $= \frac{b_1 - b_2}{b_1} = \frac{{}^5/_4 b_2 - b_2}{{}^5/_4 b_2} = \frac{{}^1/_4 b_2}{{}^5/_4 b_2} = \frac{{}^1/_4}{{}^5/_4} = \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{5} = \frac{1}{5} = 20\%$. So, it *doesn't matter* what numbers you start with for the old base and height. The answer will always be 20%. Many students mark choice (C) *ERROR* because they "feel" that a decrease of 25% "offsets" an increase of 25% in *every* type of problem.

32. (B) If the quotient of two integers is negative, then one must be positive and one must be negative:

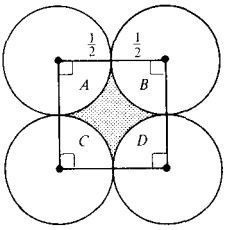
p	q	$p - q$	pq	$p^3 + q^3$
2	-1	$2 - (-1) = 2 + 1 = 3$	$2(-1) = -2$	$2^3 + (-1)^3 = 8 + -1 = 7$

So, you can see that $p - q$ doesn't have to be negative, nor does $p^3 + q^3 \cdot pq$, on the other hand, is always negative because the product of two numbers with different signs is negative.

33. (E) Assume that the numbers are 50 and 60. The average of these numbers is $\frac{50 + 60}{2} = \frac{110}{2} = 55$. If 10 is added to each of these numbers, then the new average will be $\frac{(50 + 10) + (60 + 10)}{2} = \frac{60 + 70}{2} = \frac{130}{2} = 65$. So, in this case, $A = 2$ (numbers), $B = 55$ (average) and $C = 10$ (added to each number). The new average is $55 + 10 = 65 = B + C$.

Algebraically: let $\frac{x + y}{A} = B$, then if C is added to each of the A numbers the new average is $\frac{x + y + AC}{A} = \frac{x + y}{A} + \frac{\cancel{A}C}{\cancel{A}} = B + C$.

34. (C) If it takes 2 men 6 hours to cut down 3 trees, then it takes $2 \times 6 = 12$ man-hours to cut down 3 trees. Obviously it will take 24 man-hours to cut down 6 trees. If there are 4 men, then it will take them 6 hours (4 men $\times$ 6 hours = 24 man-hours). Although the number of men is doubled, so is the number of trees, therefore the time remains *constant*.



35. (D) In the above figure, the area of the shaded region is equal to the area of the square minus $(A + B + C + D)$. Since the area of $A = \frac{90°}{360°} = \frac{1}{4}$ (the area of the circle), therefore $A + B + C + D =$ the area of one *whole* circle whose radius is $\frac{1}{2}$ (because the diameters are all equal to 1). So, area of square $-$ area of a circle $= 1^2 - \pi\left(\frac{1}{2}\right)^2 = 1 - \frac{1}{4}\pi = \frac{1}{1} - \frac{\pi}{4} = \frac{4}{4} - \frac{\pi}{4} = \frac{4 - \pi}{4}$.

Mathematical Aptitude Test 8

Time—30 Minutes
25 Questions

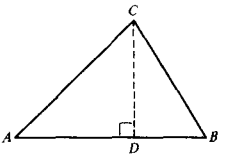
In this section solve each problem, using any available space on the page for scratchwork. Then indicate the *one* correct answer in the appropriate space on the answer sheet.

The following information is for your reference in solving some of the problems.

Circle of radius r: Area $= \pi r^2$; Circumference $= 2\pi r$.
The number of degrees in the arc of a circle is 360.
The measure in degrees of a straight angle is 180.

Definition of symbols:

$<$ is less than	$\leqq$ is less than or equal to
$>$ is greater than	$\geqq$ is greater than or equal to
$\perp$ is perpendicular to	$\parallel$ is parallel to

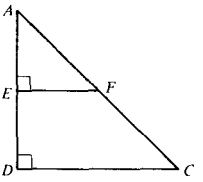


Triangle: The sum of the measures in degrees of the angles of a triangle is 180.
If $\measuredangle CDA$ is a right angle, then

(1) area of triangle $ABC = \dfrac{(AB)\cdot(CD)}{2}$

(2) $AC^2 = AD^2 + DC^2$

Note: Figures which accompany problems in this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible *except* when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated. All numbers used are real numbers.



Note: Figure not drawn to scale.

1. In the figure above, $AE = 2$, $ED = 6$, and $DC =$ $EF =$

   (A) [illegible] (B) 2.5 (C) $3\frac{1}{3}$ (D) 3.5 (E) 4

2. The radii of two circles are in the ratio 4 : 3. If the area of the larger circle is 16π, then the area of the smaller circle is

   (A) 9π (B) 12π (C) $10\sqrt{2}\pi$ (D) 14π (E) $9\sqrt{2}\pi$

3. If $3x = 2y$, and $8y = 3z$, then $\dfrac{x}{z} =$

   (A) 4 (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$ (E) $\frac{1}{4}$

4. $\dfrac{a(p-q) - b(p-q)}{a-b}$ is equivalent to

   (A) $p + q$ (B) 0 (C) 1 (D) $p - q$ (E) $2p - 2q$

5. If $p = q - 4$, then $(p - q)^3 =$

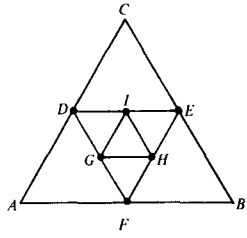
   (A) 12 (B) 81 (C) -64 (D) -12 (E) 48

6. If a storekeeper can buy books which have a price range of \$2.00 to \$5.00 and then mark them with prices between \$4.00 and \$8.00, what would be his maximum profit if he sold all of the 100 books that he bought?

   (A) \$200.00 (B) \$300.00 (C) \$400.00 (D) \$500.000 (E) \$600.00

7. In a box, there are x red marbles, y white marbles, and z blue marbles. If a blindfolded person reaches into the box and selects one marble, what is the probability that it is white?

(A) $\frac{1}{x+y+z}$ (B) $\frac{x+z}{y}$ (C) $\frac{y}{x+z}$
(D) $\frac{y}{x+y+z}$ (E) $\frac{1}{y}$



8. In the figure above, each point represents the midpoint of a side of a triangle. If triangle ABC is equilateral, what is the ratio of the area of triangle GHI to the area of triangle ABC?

(A) $1/4$ (B) $1/6$ (C) $1/8$ (D) $1/16$ (E) $1/7$

9. If pipe A can fill a tank in 3 hours and pipe B can empty it in 4 hours, how long would it take for the tank to be one-half full when both pipes are open?

(A) 4 (B) 4.5 (C) 6 (D) 8 (E) 8.4

10. A man travels a certain distance at a rate of 50 mph and then returns over the same route at a rate of 60 mph. If R represents his average rate for the round trip, then

(A) $49 < R < 53$ (B) $50 < R < 55$ (C) $R = 55$ (D) $55 < R < 60$ (E) it cannot be determined from the given information

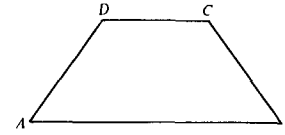
11. If P and Q represent *different* positive integers and $\sqrt{PQ} = 10$, then which of the following could *not* be a value of $P + Q$?

(A) 101 (B) 52 (C) 25 (D) 29 (E) 20

$$\begin{array}{r} AB \\ C4 \\ \hline 144 \\ 36 \\ \hline 504 \end{array}$$

12. In the multiplication problem above, each letter represents a digit from 1 to 9. $A + B + C =$

(A) 4 (B) 6 (C) 10 (D) 14 (E) 20



13. In the above figure, $AB \| DC$, $DC = 8$, $AB = 12$, and $\measuredangle A = \measuredangle B = 45°$. The perimeter of $ABCD$ is

(A) 24 (B) $24\sqrt{2}$ (C) $20 + 4\sqrt{2}$ (D) 26 (E) 30

14. In the formula $P = Qr + 5$, when $r = 3$, $P = 11$. What is the value of P when r is equal to 2?

(A) 2 (B) 3 (C) 4 (D) 9 (E) 15

15. The length of a rectangle is twice its width. If the perimeter of the rectangle is represented by P, then the area of the rectangle is represented by

(A) $2P^2$ (B) $\frac{P^2}{2}$ (C) $\frac{P^2}{18}$ (D) $\frac{P^2}{12}$ (E) $6P^2$

16. If the price of an item is reduced 10%, then to bring it back to its original price the discounted price must be increased by what percent?

(A) 10% (B) $11\frac{1}{9}\%$ (C) $12\frac{1}{8}\%$ (D) 10.8% (E) $14\frac{6}{7}\%$

17. If $0 < q < p$ and $r \neq 0$, then which of the following is *not* necessarily true?

(A) $p + r > q + r$ (B) $pr > qr$ (C) $pr^2 > qr^2$
(D) $p - r > q - r$ (E) $\frac{p}{r^2} > \frac{q}{r^2}$

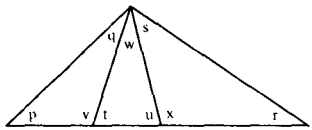
18. The senior class at North High School bought pencils from a dealer who was offering them at 5 for 10¢. The senior class sold them at 3 for 10¢ in order to raise money for a dance. If the senior class had to raise \$160.00, how many pencils did they have to sell?

(A) 12000 (B) 3000 (C) 800 (D) 15000 (E) 120

x	0	1	2	3	4	5
y	0	3	12	27	48	?

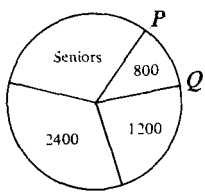
19. In the above table, what is the missing number?

(A) 62 (B) 75 (C) 81 (D) 96 (E) 105



20. In the triangle above, which of the following is always true?

(A) $p + t = w + u$ (B) $p + v + u = w + t + x$
(C) $s + r = p + v$ (D) $p + q + u = s + r + t$
(E) $q + v = s + r$



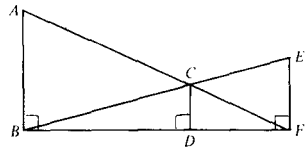
Note: Figure not drawn to scale.

21. In the above circle graph, the 4 sections represent the number of students at Local College. If there are 1200 freshmen, 800 sophomores, and 2400 juniors at this college and arc $PQ = 45°$, then how many seniors are at this college?

(A) 3600 (B) 2000 (C) 2200 (D) 3000 (E) 1800

22. In a right triangle, the square of the hypotenuse is equal to twice the product of the legs. One of the acute angles of this triangle must be

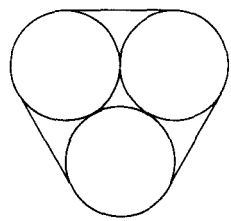
(A) 15° (B) 30° (C) 45° (D) 60° (E) 75°



Note: Figure not drawn to scale.

23. In the above figure, $AB \| CD \| EF$, $AB = 80$, $EF = 20$, and $BF = 100$. $CD =$

(A) 8 (B) 10 (C) 12 (D) 14 (E) 16



24. In the above figure, a wire is wrapped around three equal tangent circles. What is the length of this wire, in terms of r, if the radius of one of these circles is represented by r?

(A) $6r + 2\pi r$ (B) $8r + 2\pi\sqrt{2}r$
(C) $6\sqrt{2}r + 6\pi$ (D) $3r + \pi r$ (E) $6r + \pi r$

25. The XYZ Real Estate Corporation sold two parcels of property for $12,000 each. Based on the cost, XYZ realized a profit of 20% on one parcel and a loss of 20% on the other parcel. The net effect of these two transactions was that XYZ

(A) broke even (B) made $1500 (C) lost $1500
(D) made $2500 (E) lost $1000

STOP

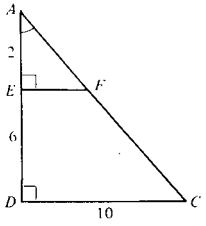
IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT WORK ON ANY OTHER SECTION IN THE TEST.

Answer Sheet—Mathematical Aptitude Test 8

	A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E		A	B	C	D	E
1	☐	☐	☐	☐	☐	6	☐	☐	☐	☐	☐	11	☐	☐	☐	☐	☐	16	☐	☐	☐	☐	☐	21	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	7	☐	☐	☐	☐	☐	12	☐	☐	☐	☐	☐	17	☐	☐	☐	☐	☐	22	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	8	☐	☐	☐	☐	☐	13	☐	☐	☐	☐	☐	18	☐	☐	☐	☐	☐	23	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	9	☐	☐	☐	☐	☐	14	☐	☐	☐	☐	☐	19	☐	☐	☐	☐	☐	24	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	10	☐	☐	☐	☐	☐	15	☐	☐	☐	☐	☐	20	☐	☐	☐	☐	☐	25	☐	☐	☐	☐	☐

ANSWERS AND EXPLANATIONS

1. (B)	2. (A)	3. (E)	4. (D)	5. (C)
6. (E)	7. (D)	8. (D)	9. (C)	10. (B)
11. (E)	12. (C)	13. (C)	14. (D)	15. (C)
16. (B)	17. (B)	18. (A)	19. (B)	20. (D)
21. (B)	22. (C)	23. (E)	24. (A)	25. (E)



Note: Figure not drawn to scale.

1. (B) Since $\measuredangle A = \measuredangle A$, and $\measuredangle AEF = \measuredangle ADC$, triangles AEF and ADC are similar. (A.A.) Therefore, their corresponding sides are in proportion: $\frac{AE}{AD} = \frac{EF}{DC}$. Now, $AD = AE + ED = 2 + 6 = 8$, so $\frac{2}{8} = \frac{EF}{10}$; $\frac{1}{4} = \frac{EF}{10}$; "cross-multiply" to get $4EF = 10$; divide both sides of the equation by 4 now to get $EF = \frac{10}{4} = 2.5$.

Some of you probably marked choice (C). *ERROR*. That is because you set up the *incorrect* proportion, $\frac{AE}{ED} = \frac{EF}{DC}$. Remember, ED is *not* a side of either similar triangle.

2. (A) $\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\text{radius}_1}{\text{radius}_2}\right)^2$; so, $\frac{16\pi}{A} = \left(\frac{4}{3}\right)^2$; $\frac{16\pi}{A} = \frac{16}{9}$; "cross-multiplying," we get $(\cancel{16}\pi)(9) = (\cancel{16})(A)$. Dividing both sides of the equation by 16, we get $9\pi = A$. If you marked choice (B), you probably forgot to *square* the ratio of the radii.

3. (E) If $3x = 2y$ and $3z = 8y$, then $\frac{3x}{3z} = \frac{2y}{8y}$; $\frac{x}{z} = \frac{1}{4}$.

4. (D) $\frac{a(p-q) - b(p-q)}{a-b} = \frac{(p-q)(a-b)}{(a-b)} = p - q$. Can you see the use of the *distributive postulate* at work here? $y(x - z) = yx - yz$. If you marked choice (B), then you were a "gung-ho" reducer . . . a person who "slashes out" everything in sight!

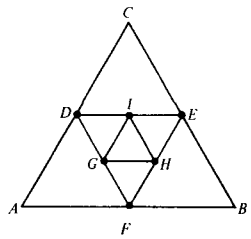
5. (C) If $p = q - 4$, then $p - q = -4$. Therefore, $(p - q)^3 = (-4)^3 = -64$. *Or*, you can substitute numbers, say, $p = 1$ and $q = 5$ (so that $p = q - 4$). Then, $(p - q)^3 = (1 - 5)^3 = (-4)^3 = -64$.

6. (E) Profit = selling price − cost. The *maximum* profit is obtained when the storekeeper *buys at minimum* price and *sells at maximum* price. Therefore, 100(\$8.00) − 100(\$2.00) = \$800 − \$200 = \$600.

7. (D) Assume that there are the following amounts of the different colored marbles:

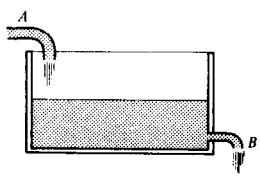
R	W	B	Probability of 1 white marble being picked
1	1	1	$\frac{1}{1+1+1} = \frac{1}{3}$ (or 1 out of 3)
1	2	1	$\frac{2}{1+2+1} = \frac{2}{4} = \frac{1}{2}$ (or 1 out of 2)
6	5	9	$\frac{5}{6+5+9} = \frac{5}{20} = \frac{1}{4}$ (or 1 out of 4)

Intuitively, then, the "odds" of picking a white marble turns out to be the ratio of white marbles to total marbles. In this problem, since there are y white marbles, and the total number of marbles is $x + y + z$, then the "odds" of picking a white marble are $\frac{y}{x+y+z}$.



8. (D) Since D, E, and F are midpoints of the equal sides AC, BC, and AB respectively, $AD = DC = CE = EB = BF = FA$. Also, since the triangle is equilateral, $\measuredangle A = \measuredangle B = \measuredangle C = 60°$. Therefore, triangles CDE and ADF are congruent (S.A.S.). So, $DE = AF$. (Corresponding sides of congruent triangles are congruent.) Since $AF = \frac{1}{2}(AB)$, then $DE = \frac{1}{2}(AB)$. Now, the same type of argument can be applied to triangle GHI in order to get $GH = \frac{1}{2}(DE)$. Therefore, $GH = \frac{1}{2}(DE) = \frac{1}{2}\left(\frac{1}{2}(AB)\right) = \frac{1}{4}(AB)$. Since the ratio $\frac{GH}{AB} = \frac{1}{4}$, the ratio of the areas of triangles GHI and ABC is $\left(\frac{1}{4}\right)^2$ or $\frac{1}{16}$.

Obviously, you don't have time to do all this! Perhaps you remember the theorem in Geometry which states, "If a line segment connects the midpoints of two sides of a triangle, then the line segment is parallel to the third side and *equal to one-half of it*." That saves a lot of time. *Or,* assuming that your memory failed you, you could have just "seen" that "business" of one-half by looking at the diagram. Besides, it would have been an excellent guess, right?



9. (C) This problem and the work problem in a previous test are close relatives. Instead of two people working together, though, we have two pipes working "against" each other. While one is filling the tank, the other is draining it. Do everything in terms of 1 hour. In *one* hour, pipe A can do $\frac{1}{3}$ of the job, i.e., it can fill $\frac{1}{3}$ of the tank. In *one* hour, pipe B can "undo" $\frac{1}{4}$ of the job, i.e., it can drain $\frac{1}{4}$ of the tank.

Therefore, in *one* hour, the tank will be $\left(\frac{1}{3} - \frac{1}{4}\right)$ full or $\frac{4}{12} - \frac{3}{12} = \frac{1}{12}$ full. In two hours it will be $\left(\frac{2}{3} - \frac{2}{4}\right)$ full or $\frac{8}{12} - \frac{6}{12} = \frac{1}{6}$ full, etc. In x hours, therefore, it will be $\left(\frac{x}{3} - \frac{x}{4}\right)$ full or $\frac{4x}{12} - \frac{3x}{12} = \frac{x}{12}$ full. So, you can see that in 3 hours ($x = 3$) it will be $\frac{3}{12}$ or $\frac{1}{4}$ full, in 6 hours ($x = 6$) it will be $\frac{6}{12}$ full or $\frac{1}{2}$ full, which is what was requested in the problem.

Of course, all this reasoning really boils down to an equation that you should have started with in the first place: $x\left(\frac{1}{3}\right) - x\left(\frac{1}{4}\right) = \frac{1}{2}$. Then, in order to "clear" the denominators, multiply by 12 to get $4x - 3x = 6$; $x = 6$.

10. (B) Many students mark choice (C) for this problem because they take the average of 50 and 60. Many other students mark choice (E) because no distance is given. Let's do this problem by picking some numbers and then we will do it algebraically. Assume that he goes 300 miles "out," at 50 mph, and the same 300 miles "back," at 60 mph. Then, since distance = rate × time ($D = rt$), the time $= \frac{\text{distance}}{\text{rate}}$, $\left(t = \frac{D}{r}\right)$. So, the time "out" is $\frac{300}{50} = 6$ hours and the time "back" is $\frac{300}{60} = 5$ hours. Therefore, the *average* rate is the $\frac{\text{total distance}}{\text{total time}} = \frac{300 + 300}{6 + 5} = \frac{600}{11} = 54.54\ldots$ which is *less* than 55. Suppose he went a total distance of 1200 miles. Then his time "out" is $\frac{600}{50} = 12$ and his time "back" is $\frac{600}{60} = 10$. Then his average rate is $\frac{1200}{10 + 12} = \frac{1200}{22} = 54.54\ldots$ again!

Algebraically: Let D represent his distance "out" *and* the distance "back." (Remember, they are equal.) Therefore his average rate is $\frac{D + D}{\text{total time}}$. Now, the time it took to go "out" is $\frac{D}{r_1}$ where r_1 represents his rate "out." The time it took to come "back" we will represent by $\frac{D}{r_2}$. Therefore, his average rate is

$$\frac{2D}{\frac{D}{r_1} + \frac{D}{r_2}} = \frac{2D}{\frac{Dr^2 + Dr_1}{r_1r_2}} = \frac{2D}{\frac{D(r_2 + r_1)}{r_1r_2}} = \frac{(2\cancel{D})(r_1r_2)}{\cancel{D}(r_2 + r_1)} = \frac{2(r_1r_2)}{r_1 + r_2}.$$

So, amazingly, the distance "disappears" in the derivation of this formula and all you need to solve a problem like this one, where the distances are equal but the rates are different, is to take $\frac{\text{twice the product of the rates}}{\text{sum of the rates}}$. In our example, then,

$$\frac{2(50)(60)}{50 + 60} = \frac{2(3000)}{110} = \frac{6000}{110} = \frac{600}{11} = 54.54\ldots$$

11. (E) If $\sqrt{PQ} = 10$, then $PQ = 100$. Since P and Q are integers, the possible combinations are:

P	Q	$P + Q$
1	100	101
2	50	52
4	25	29
5	20	25
10	10	20
20	5	25
25	4	29
50	2	52
100	1	101

But, since P and Q have to be *different* integers, the case where $P = Q = 10$ must be eliminated, so the sum *cannot* be 20.

12. (C)

$$\begin{array}{r} AB \\ \times C4 \\ \hline 144 \\ \cdot\cdot \\ \hline \cdot\cdot\cdot \end{array}$$

Assume that $B = 1$. Then,

$$\begin{array}{r|c} A & \boxed{1} \\ \times C & \boxed{4} \\ \hline 14 & \boxed{4} \\ \cdot\cdot & \\ \hline \cdot\cdot\cdot & \end{array}$$

Then $4A = 14$. *However,* each letter represents an *integer*. Since $A \neq {}^{14}/_{4}$, something is wrong with the hypothesis that $B = 1$. Therefore, the only other integer value for B is 6. Then

$$\begin{array}{r|c} A & \boxed{6} \\ \times C & \boxed{4} \\ \hline 14 & \boxed{4} \\ \cdot\cdot & \\ \hline \cdot\cdot\cdot & \end{array}$$

Since $6 \times 4 = 24$, there is a "carry" of 2. So, $4A + 2 = 14$; $4A = 12$; $A = 3$. Then,

$$\begin{array}{r} 36 \\ \times C4 \\ \hline 144 \\ \cdot\cdot \\ \hline \cdot\cdot\cdot \end{array}$$

Now, let's look at some of the integers we have been leaving out to avoid confusion:

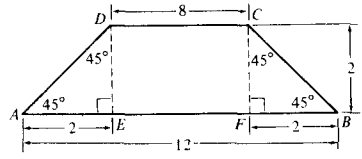
$$\begin{array}{r} 36 \\ C4 \\ \hline 144 \\ 36 \\ \hline 504 \end{array}$$

The possible values of C are 1 and 6 in order to get that "6" in the fourth row. Try $C = 1$. Then,

$$\begin{array}{r} 36 \\ 14 \\ \hline 144 \\ 36 \\ \hline 504 \end{array}$$

Therefore, $A + B + C = 3 + 6 + 1 = 10$. Of course, once you have $36(C4) = 504$, then you can divide 504 by 36 to find $C4$.

$$\begin{array}{r} 14 = C4;\ C = 1 \\ 36\overline{)504} \\ 36 \\ \hline 144 \end{array}$$

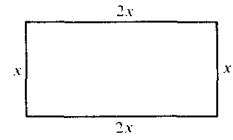


Note: Figure not drawn to scale.

13. (C) In order to get the perimeter of $ABCD$ we will need to find AD or BC. (They are equal, since the legs of an isosceles trapezoid are equal. It *is* an isosceles trapezoid because the base angles are equal.) Draw DE and CF. Since $EFCD$ is a rectangle now, $EF = 8$. Since $AB = 12 = AE + EF + FB$, then $12 = AE + 8 + FB$; $4 = AE + FB$. Since $\measuredangle A = \measuredangle B = 45°$ and $\measuredangle DEA = \measuredangle BFC = 90°$, then $\measuredangle ADE = \measuredangle BCF = 45°$ also in order to make the sum of the angles in each of these triangles 180°. Therefore, triangles ADE and BCF are congruent (S.A.S.) and $AE = FB$ (corresponding sides of congruent triangles are congruent). Since $AE = 2$, then $DE = 2$ and $AD = 2\sqrt{2}$. Likewise, $CB = 2\sqrt{2}$. Therefore, the perimeter of $ABCD$ is $8 + 12 + 2\sqrt{2} + 2\sqrt{2} = 20 + 4\sqrt{2}$.

You, of course, are *not* expected to produce any "proofs" on the SAT. Rather, you should use your intuition, and some carefully drawn diagrams!

14. (D) When $r = 3$ and $P = 11$, then $11 = Q(3) + 5$; $11 = 3Q + 5$; subtracting 5 from both sides of the equation, we get $6 = 3Q$; $Q = 2$. Therefore, the original formula was $P = 2r + 5$. So, when $r = 2$, $P = 2(2) + 5 = 4 + 5 = 9$.



15. (C) Let the width of the rectangle be represented by x. Then the length of the rectangle is represented by $2x$ and the perimeter, P, by $x + 2x + x + 2x = 6x$. The area of this rectangle is length × width $= (x)(2x) = 2x^2$. Since the question calls for the area in terms of P, we have to find x in terms of P and then substitute in the relationship $A = 2x^2$. Since $P = 6x$, then $\frac{P}{6} = x$. So, Area $= 2x^2 = 2\left(\frac{P}{6}\right)^2 = \cancel{2}\left(\frac{P^2}{\cancel{36}}\right) = \frac{P^2}{18}$.

18

You could have done this problem by substituting some numbers. Let the width be 5 and the length 10. Then $P = 30$ and the area $= 50$. Now look at all the

answers, substitute 30 for P in each of them, and try to find which one is equal to an area of 50. (A) $50 = 2(30)^2$ *No.* (B) $50 = \frac{30^2}{2}$ *No.* (C) $50 = \frac{30^2}{18} = \frac{900}{18} = 50$ *Yes.*

16. (B) Assume that the price of the article is \$100.00. Then after it is reduced 10% the discounted price is \$100 − .10(\$100) = \$100 − \$10 = \$90. To bring it back to its original price, \$90 + ? = \$100; i.e., it must obviously be increased by \$10. The % increase, then, is $\frac{\text{total increase}}{\text{discounted price}} = \frac{\$10}{\$90} = \frac{1}{9}$. To change this fraction to a percent, set up the proportion $\frac{1}{9} = \frac{x}{100}$; "cross-multiply" to get $9x = 100$; $x = \frac{100}{9} = 11\frac{1}{9}$. So, $\frac{1}{9} = \frac{11\frac{1}{9}}{100} = 11\frac{1}{9}\%$.

17. (B) If $p > q > 0$ and $r > 0$, then choices (A) − (E) are all true. Experiment with some numbers. Let $p = 5$, $q = 3$, and $r = 2$. Then (A) $5 + 2 > 3 + 2$; (B) $(5)(2) > (3)(2)$; (C) $5(2^2) > 3(2^2)$; (D) $5 - 2 > 3 - 2$; (E) $\frac{5}{2^2} > \frac{3}{2^2}$. *However,* if $r < 0$, say, -2, then (B) becomes $(5)(-2) < (3)(-2)$. The other choices are still correct: (A) $5 + -2 > 3 + -2$; (C) $(3)(-2)^2 > (2)(-2)^2$; (D) $5 - -2 > 3 - -2$; (E) $\frac{5}{(-2)^2} > \frac{3}{(-2)^2}$.

18. (A) Since each pencil costs $\frac{10}{5} = 2¢$ and was sold for $\frac{10}{3}¢$, the profit per pencil was selling price − cost $= \frac{10}{3} - 2 = \frac{10}{3} - \frac{6}{3} = \frac{4}{3}¢$. Therefore, letting x represent the total number of pencils sold, we get the equation $x\left(\frac{4}{3}\right)¢ = 16000¢$. Multiplying both sides of this equation by 3 to "clear" the fraction, we get $4x = 48000$; $x = 12000$. Be careful. If you marked choice (E), then you forget to change dollars to cents.

19. (B) There are a few ways of doing this problem. One way is to take the ratios of $\frac{x}{y}$ and try to see a pattern (except for $\frac{0}{0}$, which we shall ignore). $\frac{1}{3}, \frac{2}{12}, \frac{3}{27}, \frac{4}{48}, \frac{5}{?}$. Let's simplify these ratios: $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}$. ? is obviously equal to 15. Therefore, $\frac{5}{?} = \frac{1}{15}$; "cross-multiply" to get, ? = (5)(15) = 75.

Another method is to look for patterns:

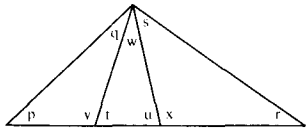
x	0	1	2	3	4	5
y	0	3	12	27	48	?

1 1 1 1 1

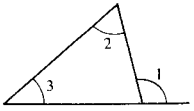
3 9 15 21 j

6 6 6 6

Can you see that $j = 27$ so that $48 + 27 = 75$?



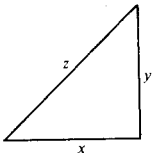
20. (D) If you try to do this problem by substituting numbers, you will probably get bogged down with all the possibilities. The only theorem you need here (although you may not see it immediately) is the exterior angle theorem, "An exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles."



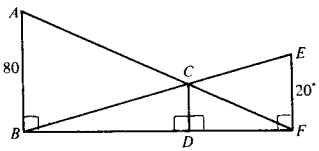
$$\measuredangle 1 = \measuredangle 2 + \measuredangle 3$$

In the diagram above, $p + q = t$ and $s + r = u$. Therefore, $\overbrace{p + q}^{t} + u = t + u = t + \overbrace{s + r}^{u}$.

21. (B) Since arc $PQ = 45°$, then 800 represents $\frac{45°}{360°}$ of the circle. Therefore, $800 = \frac{1}{8}$ (total number of students); multiplying both sides of the equation by 8, we get 6400 = total number of students. Now, $800 + 1200 + 2400 = 4400$; $6400 - 4400 = 2000$ seniors.



22. (C) In the above figure, according to the given information, $z^2 = 2xy$. Also, by the Pythagorean Theorem, $z^2 = x^2 + y^2$; subtracting $2xy$ from both sides of the equation, we get $0 = x^2 - 2xy + y^2$; $0 = (x - y)^2$. Taking the square root of both sides of the equation now, we get $0 = x - y$; $x = y$. Therefore the triangle is an isosceles right triangle and each of the acute angles must be 45°.



Note: Figure not drawn to scale.

23. (E) In the figure above, $\measuredangle EBF = \measuredangle CBD$ and $\measuredangle CDB = \measuredangle EFB$. Therefore, triangles CDB and EFB are similar (A.A). So their corresponding sides are in proportion:

$$\frac{CD}{EF} = \frac{BD}{BF}; \qquad \frac{CD}{20} = \frac{BD}{100} \tag{1}$$

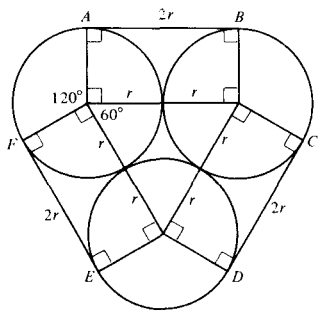
Likewise, $\measuredangle AFB = \measuredangle CFD$ and $\measuredangle ABF = \measuredangle CDF$. Therefore, triangles AFB and CFD are similar. So *their* corresponding sides are in proportion:

$$\frac{CD}{AB} = \frac{DF}{BF}; \qquad \frac{CD}{80} = \frac{DF}{100} \tag{2}$$

Multiply equation (1) by 20 to "isolate" CD: $CD = \frac{20BD}{100} = \frac{BD}{5}$. Multiply equation (2) by 80 to "isolate" CD: $CD = \frac{80DF}{100} = \frac{4DF}{5}$. Therefore, $\frac{BD}{5} = \frac{4DF}{5}$. Multiplying now by 5 to "clear" the denominators, we get,

$$BD = 4DF. \tag{3}$$

Since $BD + DF = 100$, $BD = 100 - DF$. Substitute this value of BD in equation (3) to get $100 - DF = 4DF$; $100 = 5DF$; $20 = DF$. *Now,* substitute $DF = 20$ in equation (2) to get $\frac{CD}{80} = \frac{20}{100} = \frac{1}{5}$; "cross-multiply" to get $5CD = 80$; divide by 5 to get $CD = 16$. *This is not one of the quickies!*



24. (A) In the above figure, the length of the wire is $\overline{AB} + \widehat{BC} + \overline{CD} + \widehat{DE} + \overline{EF} + \widehat{FA}$. But $AB = CD = EF = 2r + 2r + 2r = 6r$. This eliminates choices (B), (C), and (D). So you are down to that 50% guess option! However, this problem *can* be done. From the given diagram, the triangle joining the centers of the equal circles is equilateral, so each angle is 60°. Each right angle of the rectangles is 90° by definition. This means that each of the arcs = 120° or ⅓ of the circumference of the circles. Since there are 3 such circles, the 3 arcs together form one complete circle of circumference $2\pi r$. So, the answer to this problem is $6r + 2\pi r$.

25. (E) Since profit = selling price − cost, then selling price = profit + cost. On the *first* parcel, XYZ Corporation realized a profit of 20% on the cost, therefore \$12,000 = $.20C + C$ (where C represents the original cost). Then, \$12,000 = $.20C + 1C = 1.20C$; multiply each side of the equation by 10 to "get rid" of the decimals to get $120000 = 12C$; $10{,}000 = C$.

On the *second* parcel, XYZ Corporation suffered a loss of 20% of the cost. Since loss = cost − selling price, selling price = cost − loss. Therefore, \$12,000 = $C - .20C = 1C - .20C = .80C$; multiply each side of the equation by 10 to "get rid" of the decimal point to get $120000 = 8C$; $15000 = C$.

So, the original *cost* of the two parcels was \$10,000 + \$15,000 = \$25,000. XYZ *received* \$24,000 from the sale, so \$25,000 − \$24,000 = \$1,000 *loss*.